Thus $f_n$, the $O(4)$ partial wave amplitude of $f$, has a possible pole at $n=\alpha(0)$ in the complex $n$ plane. A $f^0(n)$ may be replaced by $c[n-\alpha(0)]^{-1}$ in getting the small $x$ limit. From (1) we get $F^b(x)$ in the Regge region; $F^b(x)\sim x^{-\alpha(0)+1}$ as $x\to 0$. Comparing this with (3), we find that the contribution of Fig. 1(b) is dominant in the Regge limit.

In sum our model calculation leads to the reasonable results for $F(x)$ in the two limiting regions: $F(x\to 1)\sim (1-x)^3$ and $F(x\to 0)\sim x^{-\alpha(0)+1}$. It must be mentioned that the Drell-Yan-West relation holds exactly in this model, because the electromagnetic form factor behaves as $(-q^2)^{-2}$ in the large-momentum-transfer limit.

Finally let us mention the renormalizable theories. We have shown by considering the specific model that $A(n)$ has poles as function of $n$ corresponding to the Regge pole. In the renormalizable theories it may also be reasonable to assume that $A(n)$ has the Regge pole because the correspondence between the operator expansion and the $O(4)$ symmetry is supposed to persist irrespective of the model. It should be noticed that the non-Regge behavior pointed out by De Rujula et al. is inevitably reproduced, because the essential singularity of the moment due to the anomalous dimension prevails over the Regge pole.

The authors would like to thank Professor T. Takabayasi for kind encouragement.

1) M. Ninomiya and K. Watanabe, Nagoya University preprint-20 (1975).

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On Origin of Temporal Asymmetry in Steady State Far from Thermal Equilibrium

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Recently, a new concept of irreversible circulation of fluctuation was proposed, which characterizes an asymmetry with respect to time reversal in non-equilibrium open system. In this note, this temporal asymmetry is explained as an asymmetry in the way of prediction and retrodiction in the theory of information.

Let us consider a diffusion process subject to the Fokker-Planck equation of the form

$$\frac{\partial}{\partial t}P(\vec{X}, t) = -\frac{\partial}{\partial \vec{X}}(\mathbf{K}\vec{X}P(\vec{X}, t))$$

$$+ \frac{\partial}{\partial \vec{X}} \left( \langle D \rangle \frac{\partial}{\partial \vec{X}} P(\vec{X}, t) \right),$$

(1)

where $\vec{X}$ is a state vector, and $\langle k \rangle$ and
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(D) are drift and diffusion matrices, respectively. A predictive probability is given by the transition probability $P(\mathbf{X}, t|\mathbf{X}', t')$ for $t < t'$, which is a solution of Eq. (1) satisfying an initial condition,

$$\lim_{t' \to t} P(\mathbf{X}, t|\mathbf{X}', t') = \delta(\mathbf{X}' - \mathbf{X}).$$  \hspace{1cm} (2)

Hereafter, we assume that initial state is stationary with a probability distribution,

$$P_s(\mathbf{X}) = \frac{1}{2\pi \sigma_0^2 \prod_{i=1}^n \sigma_i^2} \exp \left\{ -\frac{1}{2} \mathbf{X}'(g_i) \mathbf{X}' \right\},$$  \hspace{1cm} (3)

where $(g_i) = (\sigma_i)^{-1}$ and $(\sigma_i)$ is a stationary variance matrix determined from

$$(\mathbf{K}) (\sigma_i) + (\sigma_i) (\mathbf{K}) + (D) = 0.$$  \hspace{1cm} (4)

Using the fact that transition probability obeys Eq. (1), we have

$$\frac{\partial}{\partial t} \mathbf{X} = \lim_{t' \to t} \int d\mathbf{X}' \frac{\partial}{\partial t} P(\mathbf{X}, t|\mathbf{X}', t') d\mathbf{X}'$$

$$= (\mathbf{K}) \mathbf{X}$$

$$= \frac{1}{2} (D) + (\alpha) \frac{\partial}{\partial \mathbf{X}} \ln P_s(\mathbf{X}).$$  \hspace{1cm} (5)

Here the matrix $(\alpha)$ is an irreversible circulation of fluctuation defined by;

$$(\alpha) = \frac{1}{2} \{ - (\mathbf{K}) (\sigma_i) + (\sigma_i) (\mathbf{K}) \}.$$  \hspace{1cm} (6)

and a probability flow $\mathbf{G}$ in steady state far from thermal equilibrium is expressed in terms of $(\alpha)$ as

$$\mathbf{G} = (\alpha) \frac{\partial}{\partial \mathbf{X}} P_s(\mathbf{X}),$$  \hspace{1cm} (7)

where $(\mathbf{K})$ is a transposed matrix of $(\mathbf{K})$.

On the other hand, according to Bays' formula, a retrodictive process is made with the use of a posteriori probability,

$$P_r(\mathbf{X}) P(\mathbf{X}, t|\mathbf{X}', t') / P_s(\mathbf{X}).$$

The important point here is that in the retrodictive process we have to know a prior or priori probability $P_s(\mathbf{X})$ with respect to initial state. On the other hand, in the predictive process, we need not know such priori probability about initial state.

Through straightforward calculation with the help of Eq. (1), we have

$$\frac{\partial}{\partial t} P_s(\mathbf{X}) P(\mathbf{X}, t|\mathbf{X}', t')$$

$$= - \frac{\partial}{\partial \mathbf{X}'} \left[ (\mathbf{K}) - (D) \frac{\partial}{\partial \mathbf{X}} \ln P_s(\mathbf{X}) \right]$$

$$\times P(\mathbf{X}, t|\mathbf{X}', t') P_s(\mathbf{X}')$$

$$= \frac{1}{2} (D) \frac{\partial}{\partial \mathbf{X}'}$$

$$\times P_r(\mathbf{X}) P(\mathbf{X}, t|\mathbf{X}', t')$$

$$/ P_s(\mathbf{X})$$

$$= -(\alpha) \frac{\partial}{\partial \mathbf{X}} \ln P_s(\mathbf{X}).$$  \hspace{1cm} (9)

These equations $(5)'$ and $(9)'$ coincides with those which were derived formally by the method of path integral without referring to the origin of the asymmetry. Equation (9) describes a backward evolution of a finally conditioned ensemble, while Eq. (5) describes a forward evolution of an initially conditioned ensemble. Comparing Eq. (5) with Eq. (9), we note that the backward equation (9) has an asymmetrical term $-(D) \frac{\partial}{\partial \mathbf{X}} \ln P_s(\mathbf{X})$ for time-inversion. This asymmetrical term indicates the drift motion of the ensemble.
caused by the entropy force which is originated from the priori information about the probability distribution of initial state. Taking this into account together with the fact that the information about the initial state is inevitable for retrodictive process, we can state that the origin of temporal asymmetry in steady state far from thermal equilibrium is explained as an asymmetry in the way of prediction and retrodiction.

3) Quite Recently, a systematic way of the construction of a time-reversed markoff process has been given by H. Hasegawa, Prog. Theor. Phys. 55 (1976), 90.

where \( \phi(r) \) is the pair potential, which may be written as \( \phi(r) = \epsilon \phi(r/r_0) \), and \( R_j \) denotes the j-th lattice point. Hereafter \( r_0 \) and \( \epsilon \) are taken as the units of length and energy respectively and we confine ourselves to the fcc, bcc, sc and diamond structures. On expansion, we have

\[
U(r) = U(0) + (1/2) K r^2 + \cdots
\]

The static lattice energy per atom and the force constant of the Einstein oscillator are given by

\[
E = (1/2) U(0) = (1/2) \sum_i z_i \phi(d_i),
\]

\[
K = (1/3) \sum_i z_i [\phi''(d_i) + 2\phi'(d_i)/d_i],
\]

respectively, where \( z_i \) is the number of i-th neighbor lattice points and \( d_i \) is its distance. If we put \( d_i = c d_i \) and \( v = c^3 d_i \), where \( v \) is the specific volume, then \( z_i, c_i \) and \( c \) are constants depending on the geometry of the lattice assumed. At high pressures, repulsive forces will act effectively. Thus we assume that \( \phi'(r) < 0 \). We also assume that \( \phi(r) \) is convex: \( \phi''(r) > 0 \). Then both the pressure \( P = -\partial E/\partial v \) and the compressibility \( \chi = - (1/v) \partial E/\partial P \) are always positive. Hence the thermodynamic stability conditions are satisfied.