Extended Particle and Renormalization of Fluctuation

Masataka HOSODA, Hiroshi KOZAKAI and Tadayoshi SHIMIZU*

Department of Physics, Tokyo Metropolitan University, Setagaya, Tokyo
*Department of Radiation, Komazawa University, Setagaya, Tokyo

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The perturbation theory around the classical solution of two dimensional non-linear field equation is developed by using the collective coordinate method of Gervais, Jevicki and Sakita (GJS). In contrast to GJS, the separation of the soliton degree of freedom is made Lorentz invariant. As an example of such calculation the soliton energy for a fixed momentum is computed explicitly up to one loop in the $\phi^4$-field theory. It is shown that the fluctuation around the kink solution makes a finite contribution to the quantum correction for the mass of soliton since all ultraviolet divergences can be removed by renormalization. Discussions about the formulation and its results are given, compared with those of GJS.

§ 1. Introduction

Recently there has been renewed interest in making quite a different connection between hadronic physics and field theory by attempting to relate the exact solutions of classical nonlinear field equation to physical hadrons. The exact solutions are now believed to represent new states in the spectrum of the corresponding quantum field and possess some of the features of extended particles. This attempt is different from the one in the bag theoretical point of view. In particular, the work of Gervais, Jevicki and Sakita (hereafter abbreviated by GJS) attracts our interest. They have proposed a generalization of the collective coordinate method of many body theory to two dimensional quantum field theory of $\phi^4$ interaction within the framework of the path integral formalism. As an example to such formulation they have showed by calculating the soliton energy for a fixed momentum $p$ that the Lorentz covariant form for it is established explicitly although the formulation is not manifestly Lorentz-covariant and the same mass counter term for the non-soliton sector does appear also for one soliton sector.

Most important point of their approach is to separate one degree of freedom, the center of mass motion, among many degrees of freedom of the system, by going to a moving frame where extended particle is at rest and to do then perturbation over the rest of the degrees of freedom, i.e. fluctuation around the static solution. Introduction of the new coordinate (collective coordinate) requires a constraint among the dynamical variables in order to restore the original dynamics and then makes the theory become a gauge theory. Thus, the feature of the collective motion in a moving frame is determined by giving explicitly a gauge-fixing-condition.
which is yet arbitrarily chosen. However, the soliton energy is independent of the choice of gauge-fixing-condition, so that one can calculate the soliton energy in any gauge.

It is better to say in the trial that a gauge is fixed by what kind of solution of the nonlinear field equation one uses for the static solution in a moving frame. GJS used the zero momentum classical solution as the static solution, so that the soliton energy is equal to $M_0 + \frac{p^2}{2M_0}$ in the lowest order but the relativistic form for it is shown to be restored if the higher order contributions are included. Thus their perturbation expansion is a non-relativistic one such as

$$E = \sqrt{p^2 + (M_0 + \Delta M)^2}$$

$$= \sqrt{p^2 + M_0^2 + \frac{M_0 \Delta M}{\sqrt{p^2 + M_0^2}} + \cdots}$$

$$= \left\{ M_0 + \frac{p^2}{2M_0} - \frac{p^4}{8M_0^3} + \cdots \right\} + \left\{ \Delta M - \frac{p^2}{2M_0^2} \Delta M + \cdots \right\} + \cdots, \quad (1.1)$$

where $M_0$ and $\Delta M$ are mass of soliton and the quantum correction to the soliton mass respectively. The terms in second parenthesis of Eq. (1.1) indicate the first two quantum corrections to the soliton energy.

In contrast with a nonrelativistic expansion of soliton energy, there is other type of expansion for it in terms of $p/E_0$ such as

$$E = E_0 + \frac{\sqrt{E_0^2 - p^2}}{E_0} \Delta M + \cdots$$

$$= E_0 + \left( 1 - \frac{p^2}{2E_0^2} + \cdots \right) \Delta M + O((\Delta M)^2), \quad (1.2)$$

where $E_0 = \sqrt{p^2 + M_0^2}$. It must be noted here that the relativistic form of soliton energy $E_0$ is obtained in lowest order and the quantum corrections are included only in the self mass of soliton. Motivated by this developments, we shall re-examine the same problem, Lorentz covariance and quantum correction for soliton energy, by developing a different perturbation expansion for the one soliton sector from that of GJS. We use the non-zero momentum classical solution as the static one in the moving frame and do perturbation around it. As a result the relativistic form for the soliton energy is obtained in the first approximation in contrast with that of GJS, the relativistic form being recovered by summing up all tree graph contributions. This suggests that our gauge is more appropriate for the separation of the collective mode from the fluctuation (or meson cloud) than GJS' gauge. In our gauge the fluctuation around the static solution in a moving frame gives only contributions for quantum correction of self mass of soliton.

In § 2 we mention briefly the gauge constraints associated with the separation of the center of mass motion and give the expression of transition matrix element within the framework of functional quantization approach. Furthermore, our per-
turbative expansion around the classical solution is presented. The meson propagators and vertices are also presented. In particular, the interaction Hamiltonian used for our perturbative expansion is given explicitly and compared with the corresponding one of GJS.

In § 3 we perform computation for the soliton energy up to one loop diagrams in our gauge. The last section is devoted to show that the quantum correction for soliton mass is made finite by the same mass counter term as that for non-soliton sector, and to discuss our results comparing with those of GJS.

§ 2. Transition matrix element and perturbation expansion

We consider the following Lagrangian in two-dimensional space-time,

$$\mathcal{L} = \frac{\hbar}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi; g),$$

(2·1)

where the potential is assumed to take the form $V(\phi; g) = (1/g^2) V(g\phi, 1)$. This form suggests the exploitation of the close connection between $g$ and Planck's constant $\hbar$, i.e. perturbation expansion in terms of $g$ has correspondence to the one in terms of $\hbar$ (W.K.B expansion). Discussion about it was done by Dashen et al. in detail, but we do not mention it further here.

Equation (2·1) possesses exact space-time dependent solution of the following form,

$$\phi_c(x, t) = \phi_0 \left( \frac{x - vt - x_0}{\sqrt{1 - v^2}} \right)$$

(2·2)

with $v = p/E$ for the one soliton sector and $\phi_0(x - X)$ is solution of $-\phi_0'' + \delta V/\delta \phi|_{\phi_0} = 0$, $X$ being a parameter and $\phi_0'' = (d^2/dp^2) \phi_0$.

If one regards the center of mass motion as a dynamical variable (collective coordinate) denoted by $X(t)$, one must introduce a gauge condition as is mentioned before. Because of the introduction of a new coordinate, one has a new system which is described by collective coordinate $X(t)$ and field $\phi(x, t) = \tilde{\phi}(\rho, t)$ in the moving frame ($\rho = x - X(t)$) and is thus different from the old one. The old system is described only by field $\phi(x, t)$ in the original coordinate system. One can show that this new system is equivalent to the old one provided that a gauge condition is imposed in order to cease to constrain the extra degree of freedom, just like the gauge condition in the quantum electrodynamics. This idea was developed by Hosoya and Kikkawa and further generalized by GJS, using Dirac's generalized Hamiltonian formalism.

The quantization of the system with constraints can be done by Faddeev's

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* As is well known, when we develop the perturbative expansion without the separation for the center of mass motion of the system, there is a infrared problem and thus such perturbative expansion can not be developed, so that the center-of-mass motion will be separated. Then the system moves collectively through the time development of the center of mass motion and has quantum fluctuations around the classical solution $\phi_c(x - X(t))$. 
functional method as follows:

\[
\int [dX] [dP] [d\tilde{\phi}] [d\tilde{\pi}] \delta(\psi_1) \delta(\psi_2) \det\{\psi_1, \psi_2\} \\
\times \exp\left[ i \int dt \left\{ P\dot{X} + \int d\rho \tilde{\phi} \dot{\tilde{\pi}} - H(\tilde{\phi}, \tilde{\pi}) \right\} \right] (2.3)
\]

for the transition matrix element. In the above equation \( P \) and \( \tilde{\pi} \) are canonical conjugate variables of \( X \) and \( \tilde{\phi} \), respectively and \( \psi_1 \) and \( \psi_2 \) are the imposed gauge and the gauge-fixing-conditions, respectively,

\[
\psi_1 = P + \int \tilde{\pi} \tilde{\phi}' d\rho = \int f(\rho) \pi(\rho, t) d\rho, \tag{2.4}
\]

\[
\psi_2 = \int f(\rho) \tilde{\phi}' (\rho, t) d\rho. \tag{2.5}
\]

Latter equality in Eq. (2.4) is due to the change of variable such as\(^a\)

\[
\tilde{\pi}(\rho, t) = \pi(\rho, t) - f(\rho) \frac{\int \pi(\rho, t) \tilde{\phi}'(\rho, t) - f(\rho)}{\int f(\rho) \tilde{\phi}'(\rho, t) d\rho} d\rho, \tag{2.6}\]

where \( f(\rho) \) is still an arbitrary function with \( \int f(\rho) d\rho = 1 \). GJS\(^b\) chooses \( f(\rho) = (1/\sqrt{M_0}) \phi_0'(\rho) \) in order to eliminate the zero frequency mode from the functional integral. On the other hand, we choose \( f(\rho) = (\sqrt{1/E_0}) \phi_{cl}(\rho) \) for the same purpose where \( \phi_{cl}(\rho) = \phi_0(\rho/\sqrt{1-v^2}) \), and \( \rho = x - X(t) \). This change of variable means that separation of the center of mass motion is done in a moving frame associated with the coordinate transformation,

\[
\rho' = \frac{x - X(t)}{\sqrt{1-v^2}}, \tag{2.7}\]

Then, the quantum fluctuations around the classical solution are treated by the perturbative method in the moving frame where the extended particle is at rest.

By performing the canonical transformation such as\(^c\)

\(^a\) The Poisson bracket of Eq. (2.3) is cancelled out by the Jacobian for the change of variable

\[
\frac{\partial \pi}{\partial \pi'(\rho, t)} = \frac{1}{\int f(\rho) \tilde{\phi}'(\rho, t) d\rho}.
\]

\(^b\) The Hamiltonian in this frame is given by \( H = H(\tilde{\phi}, \tilde{\pi}) + \lambda \psi_1 \), \( \lambda \) being generally the function of the canonical variables. The consistency condition for the gauge-fixed condition to be realized at any time is given by \( \dot{\pi} = \{X, H\} = 0 \). From this, \( \lambda \) is fixed to be \( (P + \int \pi \tilde{\phi}'(\rho, t) d\rho) / (\int f(\rho) \tilde{\phi}'(\rho, t) d\rho) \). The velocity of the center of mass is obtained from \( \dot{X} = \{X, H\} \)

\[
\dot{X} = +\frac{P}{E_0} + \ldots
\]

for \( f(\rho) = \sqrt{1/E_0} \phi_{cl}(\rho) \).
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\[ \phi(\rho, t) = \phi_{\text{ct}}(\rho) + \chi(\rho, t), \quad (2.8) \]

the Hamiltonian becomes as follows:

\[ H = \frac{(p + \int \pi \gamma' d\rho)^2}{2E_0(1 + \zeta/E_0)^2} + \int \left[ \frac{\pi^2}{2} + \frac{1}{2} (\phi_{\text{ct}}' + \chi')^2 + V(\phi_{\text{ct}} + \chi) \right] d\rho + H_{\text{mz}}, \quad (2.9) \]

\[ = H_t + H_{\text{f}} + H_{\text{mz}}, \quad (2.10) \]

where

\[ H_0 = \int \left[ \frac{1}{2} \gamma^2 + \frac{1}{2} (1 - \nu^2) \phi_{\text{ct}}'' + \phi_{\text{ct}}'' V^{(c)}(\phi_{\text{ct}}) \right] d\rho, \quad (2.11) \]

\[ H_f = E_0 - \frac{1}{2} \nu^2 E_0 + \nu^2 \zeta + \frac{1}{2} (p + \int \pi \gamma' d\rho)^2 + \frac{1}{2} \nu^2 \int \chi' d\rho \]

\[ + \sum_{n \neq 0} \int \frac{1}{H_1} \phi_{\text{ct}}'' V^{(c)}(\phi_{\text{ct}}) d\rho, \quad (2.12) \]

\[ H_{\text{mz}} = -\frac{1}{2} \phi' \int m^2 \left( \phi_{\text{ct}}' + \chi' \right) - \frac{1}{2} \right] d\rho, \]

and

\[ \zeta = \int \phi_{\text{ct}}'(\rho) \chi'(\rho, t) d\rho. \]

We add the mass renormalization counter term \( H_{\text{mz}} \) to the r.h.s. of Eq. (2.9) in order to carry out the renormalization afterward. It is remarked that a velocity dependent term, \( (1/2) \nu^2 \phi_{\text{ct}}'' d\rho \), is subtracted from the free-part of Hamiltonian \( H_0 \) while the same term is added to the interaction part \( H_f \) in order to expand the fields \( \chi \) and \( \pi \) in terms of eigenfunctions \( \Phi_n(\rho) \) such as

\[ \chi(\rho, t) = \sum_n \chi_n(t) \Phi_n(\rho) \quad (2.13) \]

and

\[ \pi(\rho, t) = \sum_n \pi_n(t) \Phi_n(\rho). \quad (2.14) \]

The \( \Phi_n(\rho) \)'s are solution of the eigenequation,

\[ \mathcal{L} \Phi_n(\rho) = \left\{ -(1 - \nu^2) \frac{d^2}{d\rho^2} + V^{(c)}(\phi_{\text{ct}}) \right\} \Phi_n(\rho) = \omega_n^2 \Phi_n(\rho), \quad (2.15) \]

explicit form of normalized eigenfunction being, for example, for the potential

\[ V(\phi) = -\frac{1}{2} \phi^2 + \frac{g^2}{4} \phi^4, \quad (2.16) \]
\( \Phi_n(\rho) = \sqrt{\frac{E_0}{M_0}} \left\{ 1 + 2(k_n^2 + 2) (2k_n^2 + 1) L \right\} \left( \frac{\sqrt{E_0^2 - M_0^2}}{M_0} \right)^{-1/2} \Psi_n \left( \frac{\theta}{\sqrt{1 - v^2}} \right) \), (2.17)

where \( \Psi_n(\rho) \) are the normalized solution of the eigenequation with \( v=0 \) and the explicit form of them are given by GJS.\(^{2}\)

Substituting Eqs. (2.13) and (2.14) into the Hamiltonian of free generating functional,\(^{2}\) the propagators are obtained as follows:

(i) \( G(t-t'; \rho, \rho') = \sum_n \Phi_n(\rho) \int \frac{d\omega}{2\pi} \frac{i e^{i\omega(t-t')}}{\omega^2 - \omega_n^2 - i\epsilon} \Phi_n^*(\rho') \) \hspace{1cm} (2.18)

for \( \chi(\rho, t) - \chi(\rho', t') \),

(ii) \( \partial_t G(t-t'; \rho, \rho') \) for \( \chi(\rho, t) - \pi(\rho', t') \) \hspace{1cm} (2.19)

and

(iii) \( \partial_t \partial_t G(t-t'; \rho, \rho') + \Delta(t-t'; \rho, \rho') \)

for \( \pi(\rho, t) - \pi(\rho', t') \), \hspace{1cm} (2.20)

where \( \Delta(t-t'; \rho, \rho') = -i \delta(t-t') \sum_n \Phi_n(\rho) \Phi_n^*(\rho') \).

The omission of the zero frequency mode is denoted by \( \Sigma_0 \) and thus the propagators are free of infrared divergences. The graphical representations are given in Fig. 1.

We have an infinite series of vertices coming from the interaction part \( H_I \) as follows:

\[
H_I = E_0 + \frac{1}{2} \frac{\rho^2}{E_0^2} - \frac{3}{2} \frac{\rho^2}{E_0} \left\{ \int \phi_\rho'(\rho) \chi'(\rho, t) d\rho \right\}^2
\]

\[
+ \frac{\rho}{E_0} \int \pi(\rho, t) \chi'(\rho, t) d\rho - \frac{2\rho}{E_0} \left\{ \int \phi_\rho(\rho) \chi'(\rho, t) d\rho \right\}
\]

\(^{2}\) This is also expressed by

\( \Phi_n(\rho) = \frac{1}{N_n} \exp(i k_n r \cdot \rho) \left( \frac{3 \tanh \frac{M_0 \rho}{\sqrt{2}} - 3 \sqrt{2} i \tan h \frac{M_0 \rho}{\sqrt{2}} - 1 - 2k_n^2} \right) \)

and

\( N_n = 2L(1+2k_n^2)(2+k_n^2) - 12\sqrt{2} \frac{1}{r} (1+k_n^2) \).
\[ \times \left\{ \int \pi(\rho, t) \chi'(\rho, t) \, d\rho \right\} + \cdots \]
\[ + \sum_{n=1}^{\infty} \int \frac{1}{n!} \chi'(\rho, t) V^{(1)}(\phi_c(\rho)) \, d\rho \]
\[ + \frac{1}{2} \partial m^2 \int \left\{ (\phi_c(\rho) + \chi(\rho, t))^2 - \frac{1}{g^2} \right\} \, d\rho . \quad (2.22) \]

In order to compare our interaction Hamiltonian with that of GJS, \( \tilde{H}_I \), we write it here explicitly

\[ \tilde{H}_I = M_0 + \frac{\rho'}{2M_0} - \frac{\rho'}{M_0} \xi + \frac{3}{2} \frac{\rho'}{M_0} \xi^2 + \cdots \]
\[ + \frac{\rho}{M_0} \int \pi' \, d\rho - \frac{2\rho}{M_0} \xi \int \pi' \, d\rho + \cdots \]
\[ + \sum_{n=1}^{\infty} \int d\rho \, \frac{1}{n!} \chi' V^{(1)}(\phi_c) + H_{\text{sm}}, \quad (2.23) \]

where

\[ \xi = \int \phi'_c(\rho) \chi'(\rho, t) \, d\rho \quad (2.24) \]

and

\[ H_{\text{sm}} = -\frac{1}{2} \partial m^2 \int d\rho \left\{ (\phi_s + \chi)^2 - \frac{1}{g^2} \right\} . \quad (2.25) \]

It should be noted that our Hamiltonian \( \tilde{H}_I \) has not the terms corresponding to the second and third terms of \( \tilde{H}_I \) but has an extra term proportional to \( \nu^2 \) except for replacing \( M_0 \) and \( \phi_s \) in \( \tilde{H}_I \) by \( E_0 \) and \( \phi_c t \). Especially, third term of \( \tilde{H}_I \) should be represented by the tad-pole graph such as Fig. 2. Absence of this term in \( \tilde{H}_I \) means that the relativistic form of the soliton energy is obtained in the first approximation in our case. Certainly we have such a term, \( E_0 = \sqrt{\nu^2 + M_0^2} \), in \( \tilde{H}_I \). On the other hand, GJS needs Feynman’s graphs connected to the tad-pole graph for calculating the one soliton energy. Summing up all tree graphs connected to the tad-pole graph, soliton energy takes relativistic form.\(^5\)

\[ \text{Fig. 2. Tad-pole diagram.} \]

In Fig. 3, we will explicitly represent the Feynman graphs corresponding to the vertices of \( \tilde{H}_I \).

By using the Feynman rule we shall explicitly calculate one loop contributions to the soliton energy. Then one has to perform mass renormalization in the one-soliton sector because of the appearance of logarithmic divergences in a loop contribution.
§ 3. Calculation and results

Let us calculate the one loop contribution to the soliton energy. We confine ourselves to the following potential:

$$V(\phi) = -\frac{1}{2}\phi^2 + \frac{g^2}{4}\phi^4$$

(2.16)

in order to compare the results with those of GJS in the next section.

Since the interaction Hamiltonian is obtained in Eq. (2.12), we can calculate the one loop contribution to the soliton energy by using the Feynman rules in § 2.

It should be remembered that our separation of soliton degree of freedom is Lorentz covariant, thus the expansion of perturbation theory will be done with respect to $p$ and $E_0$, $E_0$ being order of $g^{-2}$, while the expansion in GJS's case is done with respect to $p$ and $M_0$, $M_0$ being order of $g^{-2}$.

First let us write the order $g^0$ correction to the soliton energy. As was evaluated in detail by Dasen, Hasslacher and Neveu, it is equivalent to the sum of zero point energies

$$\sum_n \frac{1}{2}\omega_n,$$

(3.1)

where $\omega_n^2 = k_n^2 + 2$ and $\gamma k_n + (1/L)\delta(k_n) = (1/L)2\pi n$. $\gamma$ is a Lorentz-contraction-like factor $(1 - v^2)^{-1/2}$ and $\delta(k_n)$ is the scattering phase shift of wave equation in Eq. (2.15). Appearance of $\gamma$ is due to using $\phi_0(\rho)$ as a static solution. Therefore the sum of zero point energies include the higher order contributions with respect to $p$ and $g^2$ through the Lorentz contraction-like factor $(1 -(p/E_0)^2)^{-1/2}$.

Subtracting out the infinite vacuum energy and mass counter term, total expression of quantum correction to the soliton mass up to the order of $g^4$ and $p^2$ is
\[ \Delta M = \sum_n' \frac{1}{2} \omega_n - \sum_n' \frac{1}{2} \omega_n' - \frac{1}{2} m^2 \int_{-L/2}^{L/2} \left( \phi_0 (\rho) - \frac{1}{\rho^2} \right) d\rho \]  
\[ \equiv \Delta M - \frac{p^4}{2E_0^2} \Delta M' + O\left( \frac{p^4}{E_0^4} \right), \]  
where

\[ \omega_n' = \sqrt{k_n^2 + \frac{3}{\sqrt{2}}}, \quad k_n' = \frac{2\pi n}{L}, \]  
\[ \Delta M = \frac{1}{2} \omega_1 - \frac{3}{\pi \sqrt{2}} + \int \frac{dk}{2\pi} \frac{d\delta(k)}{dk} \frac{1}{2} \omega(k) - \frac{1}{2} \delta m^2 \int_{-L/2}^{L/2} \left( \phi_0 (\rho) - \frac{1}{\rho^2} \right) d\rho \]  
and

\[ \Delta M' = \sum_n \frac{\bar{k}_n^2}{2\omega_n} \frac{L \cdot R_n^z}{N_n^z} - \frac{1}{2} \delta m^2 \int_{-L/2}^{L/2} \left( \phi_0 (\rho) - \frac{1}{\rho^2} \right) d\rho. \]  

In order to compare our result with that of GJS afterward, the right-hand side of Eq. (3.3) is rewritten in terms of \( \bar{k}_n \) and \( L(R_n^z/N_n^z) = LF_n(L/2)T_n^*(L/2) = 1 - (1/L)(d\delta(\bar{k}_n)/d\bar{k}_n) + O(L^{-2}), \) \( \bar{k}_n \) being wave number for \( \gamma = 1. \)

\( \Delta M \) is finite because the logarithmic divergence of third term is cancelled out by that of fourth term. First and second terms in \( \Delta M' \) are linearly and logarithmically divergent, respectively. As is seen below, the linearly divergent term is cancelled out by the term coming from the one loop diagram which is proportional to \( p^2 \) and \( g^4. \)

Let us calculate the one loop diagrams in order to evaluate explicitly the correction to the soliton energy which is proportional to \( p^2 \) and \( g^4, \) i.e., the loop diagrams shown in Figs. 4 (a), (b) and (c). These contributions are of the form \(- (p^2/2E_0) \cdot (\Delta M_2/E_0)\). Calculation is straightforward done by using the Feynman rules. One gets the following expression corresponding to the graphs (a), (b) and (c), respectively:

\[ \Delta M_{2a} = - \sum_n' \frac{\langle \phi_n, \phi_n \rangle}{2\omega_n}, \]  
\[ \Delta M_{2b} = \sum_n' \frac{\langle \phi_n, \phi_n' \rangle}{2\omega_n} \]  
\[ + \sum_n' \frac{\langle \theta_n, \phi_n' \rangle}{2\omega_n} \]  
\[ \quad \Delta M_{2c} = \sum_n' \frac{\langle \phi_n, \phi_n' \rangle}{2\omega_n} \]  
\[ + \sum_n' \frac{\langle \theta_n, \phi_n' \rangle}{2\omega_n}. \]

Fig. 4. One loop graphs proportional to \( p^2. \)

*) \( \Delta M \) is explicitly calculable and reads

\[ \Delta M = \frac{1}{2} \left( \sqrt{\frac{1}{6} - \frac{3\sqrt{2}}{\pi}} \right). \]
\[ + \sum_{n,m} (\Phi_n, \Phi_m') (\Phi_n, \Phi_m') \quad (3.8) \]
and
\[ \Delta M_{2e} = 3 \sum_{n} (\Phi_n', \Phi_n) (\Phi_n, \Phi_n') \quad (3.9) \]
where
\[ (A, B) = \int_{-L/2}^{L/2} A(\rho) B(\rho) \, d\rho \quad (3.10) \]
\( \Delta M_{2i} (i = a, b, c) \) have similar expressions as those of GJS except for replacing \( \Psi_n(\rho) \) by \( \Phi_n(\rho) \) and the absence of a linearly divergent term in Eq. (3.7). By using the explicit form of the eigenfunction \( \Phi_n(\rho) \): Eq. (2.16), one can calculate the above three expressions \( \Delta M_{2i} (i = a, b, c) \). However we do not perform further calculation. These expressions include the higher order contributions with respect to \( p' \) and \( g' \). In the next section we shall pick up the zeroth order contributions with respect to \( p' \) and \( g' \) after evaluating some part explicitly.

\[ \frac{\overline{N}_{\text{free}}}{2} \quad (4.1) \]
\[ \frac{\overline{R}_{\text{free}}}{2} \quad (4.2) \]
\[ \Delta M_{2e} = 3 \sum_{n} (\Psi_n, \Psi_n') (\Psi_n, \Psi_n') \quad (4.3) \]

Thus the total expression is
\[ \Delta M = \Delta M_{2a} + \Delta M_{2b} + \Delta M_{2c} \]
\[ = \frac{1}{2} \omega_1 - \frac{3}{2 \sqrt{2}} \int \frac{dk}{2\pi} \frac{d\delta(k)}{dk} \frac{1}{2} \omega(k) - \sum_{n} \frac{\overline{R}_n}{2N_n} \text{N}^2 \quad (4.4) \]

Our result for one loop correction is different from that of GJS in the following two points:
(1) $\Delta M_{2\alpha}$, Eq. (4.1), is different from $\Delta M_{2\alpha}$ of GJS by the following term:
\[ \sum^n \frac{\bar{k}_n^2 L \cdot R_n^2}{2\omega_n N_n^2}. \] (4.5)

$\Delta M_{2\alpha}$ (Eq. (4.1)) has not such a term although other corrections, $\Delta M_{2\beta}$ and $\Delta M_{2\gamma}$, have the same expressions as those of GJS.

(2) There is not such a term as
\[ \Delta M = -\frac{1}{2} \delta m^2 \int \left( \phi^2 - \frac{1}{\delta^2} \right) d\rho \] (4.6)
in our case. GJS gets $\Delta M_{2\alpha}$ and $\Delta M_{2\gamma}$ from the Feynman graphs such as Figs. 5(a) and (b), respectively. On the other hand, our $\Delta M_{2\alpha}$ comes from the Feynman graph such as Fig. 4(a). Comparing these graphs, it is clear that the differences (1) and (2) come from the fact that the tadpole diagram is absent in our Hamiltonian. However it must be remembered that such contributions, Eqs. (4.5) and (4.6), are already included in the zero point energy, Eq. (3.6). Thus total contributions of the order of $\delta^3$ and $\delta^4$ to the quantum correction of soliton energy is expressed by
\[ \Delta M' + \Delta M = \frac{1}{2} \delta m^2 \int \left( \phi^2 - \frac{1}{\delta^2} \right) d\rho. \] (4.7)

This is equivalent to $\Delta M$ for non-soliton sector, thus we find that the first two quantum corrections to the soliton energy are summed up to be
\[ \Delta M = \frac{p^2}{2E_0} \frac{\Delta M}{E_0}, \] (4.8)

which is the first two terms of the expansion of $(\sqrt{E_0^2 - p^2})/E_0 \cdot \Delta M$ in terms of $p/E_0$. If one perform a similar calculation for higher order correction with respect to $p^2$ and $\delta^4$ in the one loop approximation, one can obtain the results $(M_0 \Delta M/\sqrt{p^2 + M_0^2})$. Finally summing up all contributions with many loops, total energy of the soliton is expected to be
\[ E = \sqrt{p^2 + (M_0 + \Delta M)^2}. \] (4.9)

This is relativistic form of a soliton energy with quantum corrections.

Fig. 5. One loop and mass counter term graphs proportional to $p^2$. 

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M. Hosoda, H. Kozakai and T. Shimizu
In this note we could develop the systematic perturbation theory for the extended particle such as the isobar and compute the correction to the energy for it, removing all ultraviolet divergences by renormalization. The infrared divergences are also removed by the separation of the center of mass motion or translational mode.

The perturbation theory developed in this note are also used for the quantum system around the particle extended in three dimensional space although one has to face with the problem of renormalization which is more complicated than the one in two dimensional space-time.

Finally, we believe that the method in this theory may be powerful and easily understandable one to know the dynamical structure of elementary particle.

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References

   K. Ishikawa, Tohoku Univ. Preprint TU/75/129.