A Multi-Regge Model with the Threshold Factor for Cluster Production

Two-Component Analysis of Multiplicity Distributions in p-p Collisions at 50-400 GeV/c

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The threshold factor in rapidity space $J$ for the production of one additional cluster is shown to have the smaller value 0.1-0.2 than that predicted by Chew and Koplik. The negative correlation among the clusters is brought in through $J$. The weaker correlation, i.e. $J=0.1$, is more provable. The cluster size of 1.3-1.4 negative particles per cluster is predicted as most provable. But the larger value up to 1.7 cannot be excluded. The diffractive cross section is found to be about 9-10mb and the diffractive multiplicity distribution is shown to be consistent with the Poisson distribution with the mean 1.4-1.7.

§ 1. Introduction

It has widely been believed that the short-range-correlation part of multiple productions at high energies is governed by the multi-peripheral dynamics. In the practical applications only the simplest version of it, that is a multi-Regge model in the strong-ordering limit, has been mostly considered. Chew and Koplik introduced a threshold factor in it for the purpose of the simple realization of the oscillatory cross section.13

In short, the threshold factor for the production of clusters (or H-quanta19) is simply related to the size of the cluster in the rapidity space, that is, the strong-ordering condition for the adjacent clusters in the peripheral chain becomes $\gamma_c^i + J < \gamma_c^{i+1}$, where $\gamma_c^i$ is the rapidity of the $i$-th cluster and $J$ is the minimum rapidity separation of the end particles in a cluster (see Fig. 1), and the threshold $Y_{th}$ of the production of $n$-clusters is given by

$$Y_{th} = J_{th} + (n-1)\, J,$$

where the threshold of the multi-peripheral production itself $J_{th}$ is additionally introduced. By assuming the multi-peripheral kinematics, i.e., small transferred momentum squared $t$, Chew and Koplik estimated $J$ to be about $\log(s'/-t)$, where $s'$ is the squared mass of the cluster. But this estimation of $J$ which amounts
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Fig. 1. Multi-Regge diagram in the rapidity space for the production of clusters.

Perhaps to 1~3 seems to be too large as is shown in the following.

Suppose that the particles in a cluster distribute uniformly within the rapidity gap \( \mathcal{J} \) on the average, then the mass of the cluster becomes

\[
m_c = \mu_t [\cosh (\mathcal{J} / (l-1)) - 1]^{1/2} / [\cosh (\mathcal{J} / (l-1)) - 1]^{1/2},
\]

where \( l \) is the particle multiplicity of the cluster and \( \mu_t \) is the mean transverse mass of the decaying particles. If the strong ordering is assumed even between particles belonging to one cluster, that is if \( \mathcal{J} / (l-1) \) is sufficiently large, Eq. (1·2) becomes

\[
\mathcal{J} \sim \log (m_c / \mu_t) \sim \log (s' / s)\tag{1·3}
\]

However, the strong ordering in a cluster is incompatible with the concept of the cluster. In the present paper we consider \( \mathcal{J} \) as a free parameter having no regard to Eq. (1·3).

The average multiplicity of negative particles of a cluster (H-quantum) was found to be about 2.0 from the analysis of the cosmic-ray data. \(^4\) On the other hand, it was shown that the cluster size of about 1.25 negative particles was consistent with the inclusive two-particle distributions at FNAL and ISR energies if the independent emission of clusters was assumed. \(^5\) And it has recently been pointed out that \( l_\ell \) (the average multiplicity of negative particles) = 2.0 is also consistent with the inclusive data if the negative correlation between clusters is taken into account. \(^6\) It seems, however, that the possibility other than \( l_\ell = 2.0 \) or 1.25 must also be considered.

The threshold factor \( \mathcal{J} \) brings about the negative correlation in our model. And the value of \( l_\ell \) depends on the parameter \( \mathcal{J} \). We will study what combinations of \( \mathcal{J} \) and \( l_\ell \) are allowed by testing our model with the multiplicity distribution at \( p_{lab} = 50 \sim 400 \text{ GeV}/c \).

The two-component model was firstly introduced in order to interpret the increasing second correlation moment \( f_2 \) of the multiplicity distributions in the \( p-p \) collision at high energies. Although many papers along this idea have been published, \(^7\) they remain on the level of the empirical rule. We intend, in this paper, to build up the more realistic model which includes the production threshold and the size parameter of the cluster in the short-range correlation part of the cross section. It will be seen most probable that the cluster contains 1.3~1.4 negative particles on the average and its mass amounts to about 1.6 GeV which corresponds
to $\Delta = 0.1$.

The diffractive part is also determined as the energy-independent term of the cross section. The multiplicity distribution of it is found to be consistent with the Poisson distribution.

§ 2. Model and its parameters

The inelastic cross section $\sigma_n$ of the production of $n$ negative particles is divided into the energy-independent diffractive part which has relatively low multiplicities, and the multiperipheral part,

$$\sigma_n = D_n + M_n. \quad (2.1)$$

After specifying the model of the multi-peripheral cross section $M_n$ completely, we subtract $M_n$ from the experimental cross section and determine the diffractive cross section $D_n$ as the average of the differences on the six energies from 50 GeV/c to 405 GeV/c. We have observed the approximate energy-independence of the differences for the desirable model of the multi-peripheral part.

As the multi-peripheral part $M_n$ we employ the cluster model, the production mechanism of which is provided with the multi-Regge model with the threshold factor $(1.1)$. The cluster introduced here is only of the single kind and its decaying mechanism is not specified in detail but is represented by the average multiplicity $l_-$ of negative particles in the decaying products. For a given negative-particle multiplicity $n$ the multiplicity of clusters $n_c (= n / l_-)$ may not be integral. We assume that the multi-Regge formula for the cluster production can be continued analytically to the non-integral value of $n_c$ and is given by

$$\bar{\sigma}_{n_c} = \beta \exp \{ (2\alpha - 2) Y \} g^{n_c} \{ Y - 4\alpha - (n_c - 1) \Delta \}^{n_c} \Gamma (n_c + 1), \quad (2.2)$$

where $Y = \log s - \log \{ 2m_p^2 + 2m_p (p_{lab})^2 + m_p^2 \}^{1/2}$ and $\alpha$ and $g^2$ are the intercept and coupling strength to the cluster respectively of the exchanged Regge trajectory. In deducing Eq. (2.2) the strong ordering is assumed with the threshold factor $(1.1)$.

The multi-peripheral cross section $M_n$ is given by

$$M_n = l_{-1}^{-1} \bar{\sigma}_{n_c}, \quad (n_c = n / l_-) \quad (2.2)'$$

The $\alpha$ is fixed to 0.5 for definiteness.*\(^*)\) The $g^2$ is uniquely determined for a

*\(^*)\) $M_n$ is generally given by $M_n = \sum_{n_c} \omega(n_c, n) \bar{\sigma}_{n_c}$, where $\omega(n_c, n)$ is the probability with which $n_c$ clusters decay into the total of $n$ final particles. If the width of the probability distribution is sufficiently narrow we can approximate this by the Eq. (2.2)' where the factor $l_{-1}^{-1}$ is necessary so as to certify the relation $M = \sum_{n_c} \bar{\sigma}_{n_c} (= l_{-1}^{-1} \sum \bar{\sigma}_{n/l_-})$. However this approximation becomes perhaps wrong for small values of $n$. The improvement of the model by introducing another parameter is a problem of the future. The present authors thank Dr. M. Hama for his comment on this point.

*\(^*)\) If we take the larger value for $\alpha$, $g^2$ becomes small for a given value of $\Delta$. Then, we must give the larger value for $l_- in order to restore the energy dependence of the average multiplicity.
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given value of $\mathcal{A}$ by requiring that the multi-peripheral cross section becomes independent of energy at the sufficiently high energy,

$$M = \sum_n M_n \to \text{const.}, \quad Y \to \infty.$$  \hspace{1cm} (2.3)

For example, $g' = 1.105$ satisfies this condition for $\mathcal{A} = 0.1$. The beginning of the asymptotic region is affected mainly by another threshold factor $\mathcal{A}_{\text{th}}$. If $\mathcal{A}_{\text{th}}$ is taken to be 3.0 or so, $M$ increases by about 0.5mb between $p_{\text{lab}} = 50 \text{GeV}/c$ and 400 GeV/c. It must be noted that no oscillatory behaviour of the cross section $M$ is observed if we put the above restriction on the coupling strength $g'$.

The normalization factor $\beta$ is so chosen that the sum $M + D$ agrees with the experimental cross section $\sigma_{\text{in}}$ at $p_{\text{lab}} = 200 \text{GeV}/c$, where

$$D = \sum_n D_n$$  \hspace{1cm} (2.4)

is the total diffractive cross section and is regarded as an adjustable parameter.

We have after all four free parameters $\mathcal{A}$, $\mathcal{A}_{\text{th}}$, $D$ and $L$. In § 3 four cases $\mathcal{A} = 0.0$, 0.1, 0.2 and 0.3 are investigated separately. It will be found that the fitness to the experimental data is definitely wrong for the case $\mathcal{A} \geq 0.3$ and $\mathcal{A} = 0.0$.

Before proceeding to the comparison with experiment we add a mention of a feature of our multi-peripheral model. From Eq. (2.2) the second correlation moment of the cluster distribution is calculated as follows,

$$f_{2c} = \sum_n n(n-1) \bar{\sigma}_{n_c}/M - \langle n_c \rangle^2,$$  \hspace{1cm} (2.5)

where $\langle n_c \rangle$ is the average multiplicity of clusters. The $f_{2c}$ is negative by virtue of the threshold factor $\mathcal{A}$. The $f_{2c}/\langle n_c \rangle$ is $-0.17$ for $\mathcal{A} = 0.1$ and $-0.28 \sim -0.30$ for $\mathcal{A} = 0.2$ in the energy range $p_{\text{lab}} = 50 \sim 400 \text{GeV}/c$. On the other hand, the clustering tends to push the correlation between produced particles up to the positive one. If the multiplicity $L_-$ increases above from 1.0, the correlation moment

$$f_{2M} = \sum_n n(n-1)M_n/M - (\sum_n nM_n/M)^2$$  \hspace{1cm} (2.6)

of the distribution of the multi-peripherally-produced negative particles becomes large and turns out to be positive above some value of $L_-$. The points of $L_-$ for which $f_{2M} = 0$ are about 1.2 and 1.5 for $\mathcal{A} = 0.1$ and 0.2 respectively.

§ 3. Comparison with experiments

For the four cases $\mathcal{A} = 0.0$, 0.1, 0.2 and 0.3 the better solutions are looked for in the subspace $2.0 < \mathcal{A}_{\text{th}} < 4.0$, $5.0 \text{mb} < D < 15.0 \text{mb}$ and $0.8 < L_- < 2.1$ of the parameter space by using the $\chi^2$-test. The experimental data on $\sigma_{\text{in}}$ at $p_{\text{lab}} = 50, 69, 102, 205, 303$ and 405 GeV/c are taken from Refs. 8) and 9). There are two independent analyses of the independent data at $p_{\text{lab}} = 303 \text{GeV}/c$. We have arbitrarily chosen the old one$^8$ in doing the $\chi^2$-analysis and tested the fitness to the new data$^9$ afterwards. Our model can be applied to those energies because the inelastic...
cross section appears independent of energy within the error of about 0.6 mb in this energy range. There are total of 67 data points. The minimum value of $\chi^2$ in our parameter subspace is found to be about 1.33 per degree of freedom. So we accept the solutions with $\chi^2/F$ (per degree of freedom) less than 1.5 as the good ones taking the error in the data into account (see Appendix). The criterion must perhaps be relaxed furthermore because the large values of $\chi^2$ result sometimes from the irregular points of the experimental data as is shown in Appendix. In addition to the overall estimation of the $\chi^2$ we regard the high-$n$ tail of the distribution as important where the multi-peripheral cross section is the dominant component.

The correlation moments are complementary to the multiplicity distribution itself, so we select the solutions for which the energy dependence, especially around $p_{lab}=50$ GeV/c, of the second moment

$$f_2 = \frac{\sum n(n-1)\sigma_n/\sigma_{in} - (\sum n\sigma_n/\sigma_{in})^2}{(n-1)}$$

agrees well with the experimental one among the good solutions mentioned above. The behaviours of the higher correlation moments are not suited for the criteria in the present simple model of the cluster, because they are sensitive to the decaying mechanism of the cluster. We will only give the third moments for the selected solutions in Fig. 6. The dependence of the average multiplicity on $Y(=\log s)$ is approximately linear. Its gradients for the better solutions are consistent with the data as is shown in Fig. 2.

i) **Model with $\Delta=0.1$**

$\chi^2/F$ takes its minimum value 1.33 at $L_1=1.4, D=9.2$ mb and $\Delta_{th}=3.4$. But the energy dependence of $f_2$ is wrong for this solution as is shown in Fig. 2. Therefore we can not take this solution as the better one. The agreement of $f_2$ with the experimental values is made better if $\Delta_{th}$ is taken to be 3.3 and $D$ is assumed to have the larger value. The curve named (b) in Fig. 2 is for this case. The allowed range of $D$ with which $\chi^2/F$ becomes less than 1.50 is from 9.2 mb to 10.8 mb. The fitness of this solution is given in Table I and Figs. 3 for each energy points.

![Fig. 2. The average multiplicity and the second correlation moment of the negative-particles distribution. (a) $\Delta=0.1, L=13$, $D=9.0$ mb and $\Delta_{th}=3.1$, (b) $\Delta=0.1, L=1.4$, $D=10.0$ mb and $\Delta_{th}=3.3$, (c) $\Delta=0.1, L=1.4$, $D=9.2$ mb and $\Delta_{th}=3.4$. The encircled point is the new datum from Ref. 10).](https://academic.oup.com/ptp/article-abstract/56/4/1258/1897903)
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There is another set of solutions which satisfies our criteria. It is named (a) in Table I, Fig. 2 and Figs. 3.

If we employ the new data at 303 GeV/c the fitness of the solution (b) becomes slightly wrong while that of the solution (a) is not affected (see Table I). The both data are given in Fig. 3(e) for comparison. The \( \chi^2 \) per data point is fairly large at \( p_{\text{lab}}=102 \) GeV/c for both cases. It is mainly due to the fact that the predicted values are larger than the experimental ones by about twice the experimental error at \( n=5, 6 \) and 7. It seems almost impossible to improve the fitness leaving the good behavior of \( f_2 \) around \( p_{\text{lab}}=50 \) GeV/c as it is.

Table I. \( \chi^2 \)'s of the typical models

<table>
<thead>
<tr>
<th>Name of model</th>
<th>( A )</th>
<th>( L )</th>
<th>( D ) (mb)</th>
<th>( A_{\text{th}} )</th>
<th>( \chi^2/F ) (the number of data points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.1</td>
<td>1.3</td>
<td>9.0±0.7</td>
<td>3.1</td>
<td>1.46</td>
</tr>
<tr>
<td>(b)</td>
<td>0.1</td>
<td>1.4</td>
<td>10.0±0.8</td>
<td>3.3</td>
<td>1.42</td>
</tr>
<tr>
<td>(c)</td>
<td>0.1</td>
<td>1.4</td>
<td>9.2</td>
<td>3.4</td>
<td>1.33</td>
</tr>
<tr>
<td>(d)</td>
<td>0.2</td>
<td>1.7</td>
<td>11.4</td>
<td>3.6</td>
<td>1.55</td>
</tr>
<tr>
<td>(e)</td>
<td>0.3</td>
<td>2.0</td>
<td>13.3</td>
<td>3.8</td>
<td>2.08</td>
</tr>
<tr>
<td>(f)</td>
<td>0.5</td>
<td>1.1</td>
<td>7.1</td>
<td>3.0</td>
<td>1.78</td>
</tr>
</tbody>
</table>

\( \chi^2/F \) (the number of data points) at \( p_{\text{lab}} \) (GeV/c)

<table>
<thead>
<tr>
<th>( p_{\text{lab}} ) (GeV/c)</th>
<th>50</th>
<th>69</th>
<th>102</th>
<th>205</th>
<th>303</th>
<th>405</th>
<th>303*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.96</td>
<td>2.02</td>
<td>0.84</td>
<td>1.65</td>
<td>1.79</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>0.65</td>
<td>2.19</td>
<td>0.90</td>
<td>1.50</td>
<td>1.59</td>
<td>1.77</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>1.07</td>
<td>1.32</td>
<td>0.84</td>
<td>1.51</td>
<td>1.44</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>1.90</td>
<td>1.76</td>
<td>0.78</td>
<td>1.42</td>
<td>1.86</td>
<td>1.73</td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td>3.68</td>
<td>2.43</td>
<td>0.86</td>
<td>1.68</td>
<td>2.28</td>
<td>2.36</td>
<td></td>
</tr>
<tr>
<td>(f)</td>
<td>2.13</td>
<td>2.16</td>
<td>1.12</td>
<td>1.88</td>
<td>1.51</td>
<td>1.31</td>
<td></td>
</tr>
</tbody>
</table>

* The experimental data are taken from Ref. 10 which are called "new data" in the text.

Fig. 3. (Figure captions are printed on the next page below.)
Fig. 3. The topological cross sections of negative particles. The solid and the dashed lines are the predictions of the model (a) and (b), respectively. In Fig. 3(c) both lines coincide with each other. The encircled points in Fig. 3(e) are the new data from Ref. 10.
The diffractive cross sections are shown in Fig. 4 for two models (a) \(D=9.0\) mb and \(\langle n\rangle_a=1.45\) and (b) \(D=10.0\) mb and \(\langle n\rangle_a=1.67\). They are consistent with the Poisson distributions with the given mean values \(\langle n\rangle_a\).

\[\chi^2/F\] is minimum when \(l_1=1.7, D=11.4\) mb and \(J_{th}=3.6\). Although the minimum of 1.55 does not fulfill our criterion for good solution, we can not exclude this solution because the large value of \(\chi^2\) results mainly from the isolated irregular points of the experimental data and does not necessarily represent the systematic deviation from the data (see Appendix). The fitness is wrong for \(n\geq5\) at \(p_{lab} = 69\) GeV/c in contrast to the high \(n\) behavior at \(p_{lab} = 405\) GeV/c where the agreement with the data is very well for \(n\geq8\). This is reflected in the behavior of the second correlation moment \(f_2\) which is shown in Fig. 5. Although the model may not be adopted for lower energies, the fitness to the higher energies is rather well.

\[\text{Fig. 4. The topological cross section of negative particles produced diffractively. The solid line is for the model (a) } D=9.0\text{ mb and } \langle n\rangle_a=1.45 \text{ and the dashed line is for the model (b) } D=10.0\text{ mb and } \langle n\rangle_a=1.67. \text{ Their Poisson analogues are shown by the circles and the black dots.}\]

\[\text{Fig. 5. The second correlation moment for the model (d) } J=0.2, (e) J=0.3 \text{ and (f) } J=0.0.\]

\[\text{Fig. 6. The third correlation moment for the model (a), (b) and (d). The data are from CERN report TH-1570 (1972).}\]
iii) Model with $\Delta = 0.0$ and 0.3

The fitness is also shown in Table I. For the case $\Delta = 0.3$, $\chi^2/F$ exceeds 2.0 and the systematic deviations from the data are seen at $p_{lab} = 69$ and 102 GeV/c. The breakdown of the model presents itself in the behavior of $f_2$ which has the minimum at $p_{lab} = 69$ GeV/c and is hardly continued to the negative values below 50 GeV/c (Fig. 5).

For $\Delta = 0.0$, $f_2$ tends to deviate from the data at higher energies (Fig. 5). Inadequateness of the model is seen from the high-$n$ behavior of the distributions. The fitness becomes wrong for the high-$n$ tails of the lower-energy distributions if we adjust the parameters to fit the high-$n$ tails at $p_{lab} = 303$ and 405 GeV/c.

The third correlation moment $f_3$ is shown in Fig. 6 for the models (a), (b) and (d). They are roughly consistent with the data in the energy range of our consideration.

§ 4. Summary and discussion

(1) We have verified the validity of the two component model, the short-range-correlation part of which is represented by the multi-Regge model with the threshold factor for the cluster production. The most probable value of the threshold factor is about 0.1. It can be imagined that the clusters with the smaller multiplicities, that is 1.3~1.4 negative particles per cluster, are procuced with the weak negative correlation (repulsion) in the rapidity space. If $l = 3l_c$, the average mass of the cluster is predicted to be about 1.6 GeV from Eq. (1.2).

The diffractive cross section is predicted to be 9~10mb which is rather large compared to the previously published analyses. Its multiplicity distribution is like a Poisson distribution with the mean 1.4~1.7. The energy dependence of the diffractive cross section is, if any, small below 400 GeV/c.

(2) The case with the stronger negative correlation among clusters, that is $\Delta = 0.2$, can not be excluded. The cluster size of average 1.7 negative particles in this case is perhaps consistent with the value 2.0 which was obtained from the productions in the central region of the rapidity space.$^6$

(3) The independent production model, that is $\Delta = 0.0$, is excluded.

(4) The model with $\Delta$ greater than 0.3 is excluded, so our results do not agree with the prediction of Chew and Koplik.$^7$ No oscillatory behavior is seen in the inelastic total cross section in our model.

(5) The model gives the constant total inelastic cross section above 400 GeV/c. This is not consistent with the experimental evidence at the ISR energy. We can nevertheless predict the rough feature of the multiplicity distribution at the top energy of the ISR [1400 GeV/c] so far as the rise of the cross section amounts to 10% or so. It is shown that the distribution has a shape like a plateau because the diffractive cross section amounts to the rather large part and has the large mean of multiplicities in our model. The plateau begins at $n = 1$
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and reaches to \( n = 6 \). The shallow dip appears around \( n = 3 \) in the case of the model (a).

(6) The energy (or \( Y \)) dependence of the third correlation moment is not so good (see Fig. 6). If the average decay properties of the cluster are independent of \( Y \), we may not expect to improve distinctly the \( Y \) dependence of \( f_3 \) even by introducing another parameter into the model of the cluster (see footnote of § 2). The \( Y \) dependence of \( f_3 \) of the experimental data appears to be approximated by a cubic equation of \( Y \) having zeros at \( Y \approx 6.6 \) and 4.3. The model (b), for example, gives the zero at \( Y \approx 6.6 \) if we put \( l_\pi \approx 1.5 \sim 1.6 \) leaving the other parameters unchanged. It is suggested from this estimation that the average size of the cluster varies with the incident energy. The lower zero is not reproduced by a slight change in \( l_\pi \). Therefore, the more detailed treatment of the decay distribution of the cluster seems necessary. The close investigations on these points are now in progress.

Appendix

The experimental data used in this work lack unity of significant figures. Some of the errors have only one significant figure and bring the large error in the value of \( \chi^2 \). We have estimated the gross error of \( \chi^2 \) resulting from those data points for the model (c) which has the minimum of \( \chi^2 \). The obtained error amounts to about 12% or so. Therefore, we can say that the solutions with \( \chi^2/F \) less than 1.50 are qualified for good ones.

By inspecting the topological cross sections with the same energy, we find that the theoretical value is largely different from the experimental one at the only one point in a certain case. \( \chi^2 \) may become large on account of such an isolated irregular point. For example, \( n = 7 \) at 405 GeV/c is such a case for the model (d). It adds about 15 to the sum of \( \chi^2 \) over the remaining points. Although \( \chi^2 \) is 1.9 per data point for this energy, the fitness of the solution to the remaining points is very well. That is, if we neglect this irregular point \( \chi^2 \) is reduced by about 0.9 per data point. Therefore we must carefully examine if the large value of \( \chi^2 \) represents the systematic deviation from the experimental data or is merely the product of the influence of the irregular data.

References