Initial Post-buckling Behavior of Thick Rings Under Uniform External Hydrostatic Pressure[7]

Robert Schmidt[8]. The third and sixth columns of Table 1 display values that seem unreasonable. After all, it is well known that consideration of the shear deformation in the analysis of thin structural elements does not appreciably alter the characteristic values obtained through purely flexural analysis. Moreover, the formula

$$q_{cr} = q_0 = (3EI/R^3)/[1 + 8(1 + \nu)kI/AR^2]$$
$$= (2Et^3/D^3)/[1 + 8(1 + \nu)kt^2/3D^2],$$

obtained by Schmidt and DaDeppo (1971), yields (for $k = 1.2$) the values $q_{cr} = 39.037$, 92.483, and 82803 for $D/t = 80$, 60, and 6, respectively. These values are noticeably greater than the authors' corresponding values 38.086, 90.239, and 82125 in column 3 of Table 1 (also see Smith and Simitses, 1969).

Furthermore, the authors' more precise 2-D analyses (columns 2 and 5 of Table 1) support the writer's contention stated in his first sentence.

The authors do not present a derivation of Eq. (7) in their paper, nor do they refer to a source. For that reason, it is difficult to verify its correctness. Nevertheless there is a reason or two to doubt its dependability. For instance, the pressure, q, is acting on the outer surface, while other quantities seem to be associated with the middle of the cross section. Moreover, this equation seems insufficiently similar to the corresponding equations presented by Budiansky (1974) and Chwalla and Kollbrunner (1938).

Since confidence in analyses is a necessary prerequisite for the use of the results, the writer would welcome the authors' comments on the matter discussed above. (Schmidt's 1979a and 1983 results confirm El Naschie's in columns 4 and 7 of Table 1.)

It is interesting to observe that Schmidt's (1979b) equations yield $q_{cr} = 23.363$, instead of 39.037, for $D/t = 80$, and also a negative eigenvalue, if the thin ring is fixed (clamped) at a point, i.e., if the two ends of hingeless arch coincide.

[7] By Lei Fu and A. M. Waas and published in the June 1995 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 62, pp. 338–345.

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Author's Closure[9]

We are happy to note of Professor Schmidt's interest in our work, and would like to apologize for the oversight in our part of leaving out some key previous work by Professor Schmidt. Equation (7) of our paper was derived as given in the text (see reference below) by Brush and Almroth. Since this equation applies to the derivation corresponding to the development of a one-dimensional first-order shear deformation model, the location (with respect to the ring's thickness or radial direction) at which the external pressure acts is not significant just as in the case corresponding to Kirchoff-Love kinematics. Of course, this is not the case for the two-dimensional analysis as is shown in our paper. Our one-dimensional treatment is not "purely" flexural since it does not restrict the ring deformation to be inextensional. This is why our Eq. (7) is insufficiently similar to previous other work mentioned by Professor Schmidt. Also, note that we used a "shear-correction factor" of 1.0 when evaluating the shearing energy contribution in the process of deriving the governing equations. We have constructed the table below for critical buckling pressures, using the numbers provided by Professor Schmidt and our calculations. The last two columns provide the percentage deviation of the one-dimensional shear deformation theories against the two-dimensional elasticity solution.

D/t	Schmidt and DaDeppo (1971)	Fu and Waas (1D shear)	Fu and Waas (2D)	Relative Error of Schmidt and DaDeppo (1971)	Relative Error of Fu and Waas (1D shear)
80	39.037	38.086	39.035	0.005%	−2.43%
60	92.483	90.239	92.476	0.007%	−2.42%
6	82803	82125	81232	1.93%	−1.10%

From this table, the solution by Schmidt and DaDeppo (1971) is seen to diverge as the ring becomes thicker (D/t tending to zero), whereas the Fu and Waas (1995) solution is seen to approach the two-dimensional solution, albeit from below. For thicker rings, ring centerline extensionality is just as important as incorporating shear deformation.

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