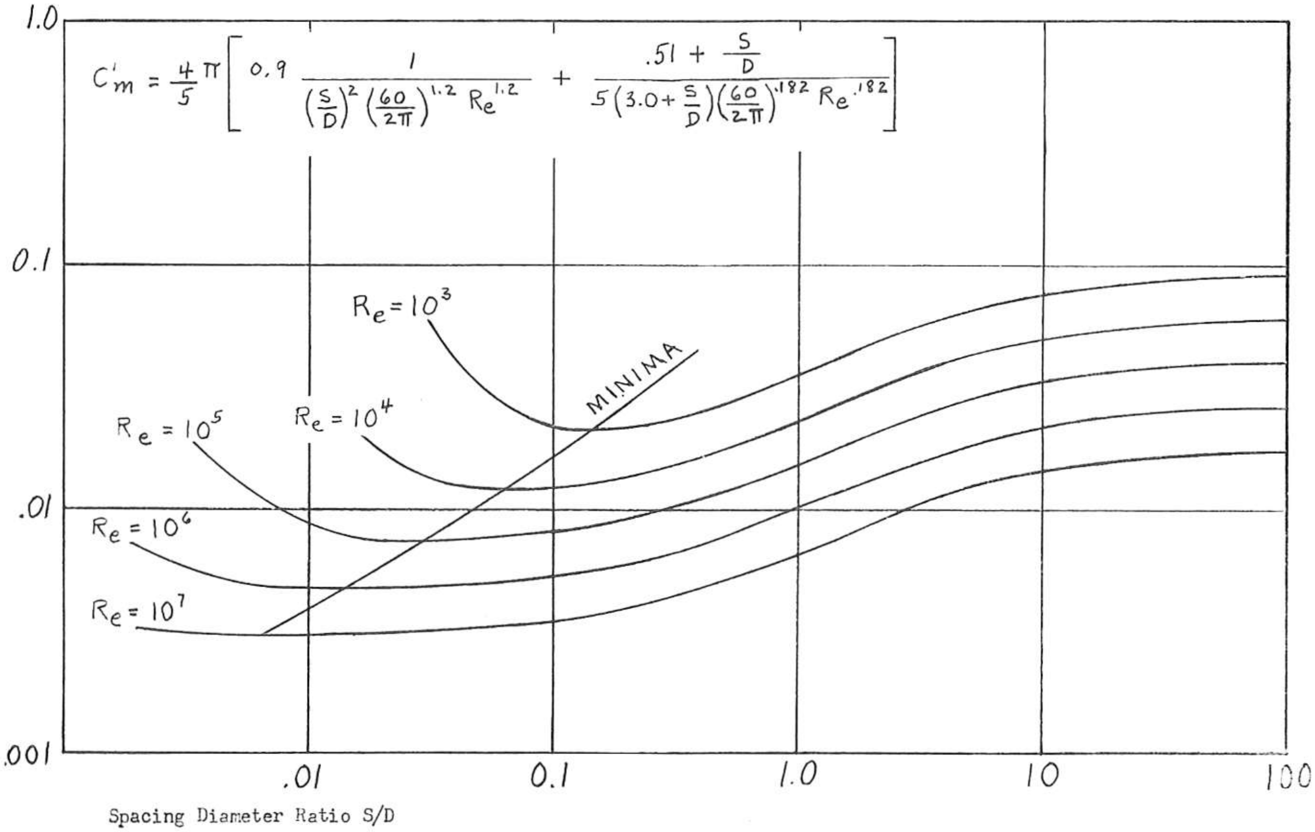

Fig. 9 Friction factor on rotating disk

DISCUSSION

H. P. Eichenberger[3]

The authors must be commended for their work both on smooth and rough disks since the engineers of this country had to get along for a long time with approximate formulas based on inadequate data. In 1950 test results were published (the authors' reference [3]) which have been used at this Company extensively. They are plotted in Fig. 9 and show especially at low Re a very pronounced minimum as a function of the spacing diameter ratio. While the authors' data at $s/a = 0.0224$ ($s/D = 0.0112$) fall exactly on the curves given by Pantell, the authors do not indicate as low or pronounced a minimum as Pantell's equation rewritten in the authors' notation on Fig. 9. This reviewer wonders whether the authors did not measure this region or whether a discrepancy exists. From the standpoint of the designer, exact location of this minimum appears to be of interest, if a design with minimum disk friction is desired

Consideration of the reasons for this minimum might lead to a disk design which reduces the friction work below the values indicated by the minima of Fig. 9 as follows: The disk friction increases for spacing s smaller than s_{opt} leading to the minima in Fig. 9 because of the steeper velocity gradient which is essentially a viscous effect. For a spacing larger than s_{opt} the momentum exchange is increased because of the increased outflow near the disk and inflow near the wall. Now this can be prevented to some extent by a wall design as shown in Fig. 10 where s_2 would be much larger than s_{opt} indicated in Fig. 9. Analysis and tests establishing the magnitude of the reduction of disk friction might be useful.

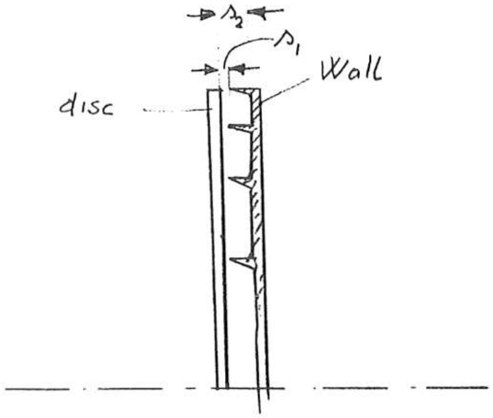

Fig. 10 Design for minimum disk friction

Authors' Closure

Dr. Eichenberger has raised two questions aimed at the very practical objective of minimizing disk friction torque for a particular Reynolds number.

The first of these is concerned with the determination of an optimum axial spacing ratio s/a to achieve minimum torque at a given Reynolds number. All discussion in regard to this point will be confined here to the case of a smooth disk in a smooth casing. While the results of smooth disk tests performed by the authors have been presented more fully in [1], the smooth disk torque curves included in Figs. 3, 4, and 5 of this paper may be used for comparison with those plotted in Fig. 9. The curves in this latter figure are plots of empirical relationships obtained by

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[3] Chief, Advanced Systems Engineering, Thompson-Ramo-Wooldridge, Inc., Cleveland, Ohio.

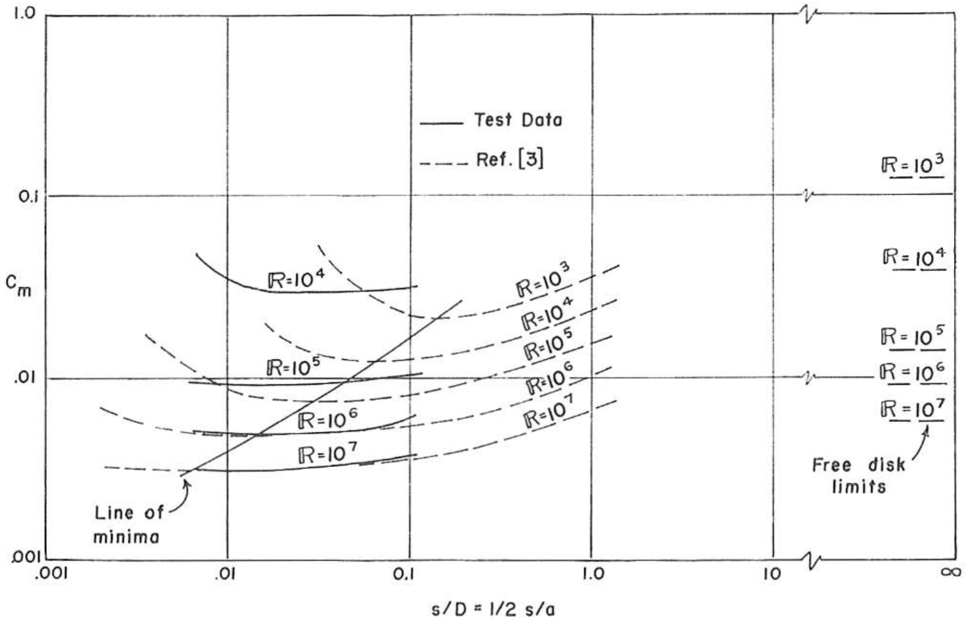

Fig. 11

Pantell [3] from smooth disk torque tests in water, within the approximate Reynolds number range of 10^6 to 2×10^7, and within the approximate geometry limits s/a ($= 2\ s/D$) of 0.01 to 0.3.

For purposes of the following discussion, Fig. 9 has been replotted as Fig. 11; the Pantell empirical extrapolation curves are drawn as dashed lines, while curves representing smooth disk data from the present study are drawn as solid lines over the test ranges covered. The extrapolated curves are not extended as far in Fig. 11 as in Fig. 9. It might be expected that if the curves of C_m versus s/D, for R constant, are extended to very large s/D ratios they should each become asymptotic to the value given by the appropriate free disk equation for the corresponding Reynolds number. These limiting values for "free disks" (infinite s/D) are indicated in Fig. 11. For R values of 10^3 and 10^4, the C_m values were calculated from the theoretical equation of Cochran for laminar flow over a free disk; for R values of 10^5, 10^6, and 10^7, Karman's turbulent flow equation was used. These relations are listed in earlier reference [1].

Of the dashed curves in Fig. 11 only those curves of $\mathsf{R} = 10^6$ and $\mathsf{R} = 10^7$ may be considered as representing actual test data, and then only to the upper limit of s/a cited. These two curves agree well with the test results of the present study for corresponding Reynolds numbers; such agreement is not obtained for lower values of R. Test points obtained in the present program in the range $10^6 < \mathsf{R} < 10^7$ were entirely in Regime IV for $s/a = 0.115$ and 0.217 ($s/D = 0.0575$ and 0.1085, respectively), and in Regimes III and IV for $s/a = 0.0127$ and 0.0255 ($s/D =$ 0.00635 and 0.01275, respectively). Over this range of s/a tested the turbulent flow range never extended below $\mathsf{R} = 10^5$ to any appreciable extent. For the smaller s/a values listed, the apparent transition from laminar to turbulent flow was at Reynolds numbers slightly below 10^5; for the wider clearances, laminar flow Regime II persisted to Reynolds numbers above 10^5.

As the extrapolated curves in Figs. 9 and 11 are based upon turbulent flow data, it cannot be expected that these curves should be valid in the regions where the boundary layer flow on the disk is laminar. The indicated minimum C_m values are all lower than the measured values for Reynolds numbers of 10^5 and lower. At $\mathsf{R} = 10^4$, for example, the minimum C_m indicated by the Pantell extrapolation is approximately 0.013, at s/D ($= {}^1/_2\ s/a$) of 0.065; for this value of R, the minimum C_m found in the present tests was nearly 0.030, and the magnitude of C_m had only a 10 per cent variation over the tenfold range of s/a from 0.0225 to 0.217, in all of which cases laminar Regime II prevailed. Dr. Eichenberger's observation that no pronounced minimum torque condition exists is correct, so that from a standpoint of design choice there does not appear to be a clearly defined optimum spacing.

The authors doubt that the suggested geometry shown in Fig. 10 would lead to a decrease of the frictional torque to a value less than the minimum obtained for a disk rotating in a plane-walled chamber. In such casings, increasing the axial clearance between disk and stator increases the frictional torque. The greater length of the cylindrical wall offers a greater stationary surface area to the fluid set in motion by the disk, and for equilibrium between the driving torque on the fluid by the disk and the resisting torque of the stationary surfaces the tangential velocity gradients across the boundary layers on the stator may thus be less steep than those across the disk boundary layer. Therefore with increased axial clearance (keeping R constant) the fluid "core" velocity decreases, with resultant steepening of the disk boundary layer velocity gradient and increase of the frictional torque. Subdividing the gap between disk and end wall into a number of concentric cylindrical annuli would produce the same relative effect in each chamber section and would therefore seem to lead to an increased, not decreased, resistance.

The authors express their thanks that these questions have been raised, since they have helped to bring some of the results of this study into a clearer focus from a design standpoint.