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discussion

On the Added-Mass of a Pulsating Cylinder[1]

M. C. JUNGER.[2] Professor McCormick's solution describes the pressure *inside* an axisymmetrically pulsating fluid-filled cylindrical shell,[3] rather than the pressure field generated by a cylinder immersed in an infinite fluid medium. For the latter configuration, the $r = 0$ axis does not lie within the fluid-filled region, however small the cylinder. The Y_0-function can therefore not be dropped from equation (8). Rather, the coefficient B multiplying this function is set equal to $-iA$, thus generating a solution which displays the anticipated propagating-wave phase dependence, exp $[i(\omega t - kr)]$, for large kr.[4] The resulting logarithmic singularity of the added-mass has been pointed out in the literature.[5,6] In summary, the singularity is not a result of the assumption of an incompressible fluid, but rather of the source geometry. Significantly, this difficulty is not encountered if the cylinder is of finite length.

[1] By M. E. McCormick, published in the September, 1970, issue of the Journal of Applied Mechanics, Vol. 37, Trans. ASME, Vol. 92, Series E, pp. 864–865.

[2] Chief Scientist, Cambridge Acoustical Associates, Inc., Cambridge, Mass.

[3] Robey, D. H., "On the Contribution of a Contained Viscous Liquid to the Acoustic Impedance of a Radially Vibrating Tube," *Journal of the Acoustical Society of America*, Vol. 22, No. 1, Jan. 1955, pp. 22–25.

Author's Closure

Intuitively, I did neglect the function $Y_0(ka)$ because of its behavior as $ka \rightarrow 0$. My mistake was in concentrating on the wave number k and not the radius a; i.e., in the limiting process, for a given radius, the Y_0-term is dominant for large values of wavelengths. My remarks in the section "Discussion" of my Brief Note were for the infinite cylinder only.

I am sincerely thankful to Dr. Junger for his comments.

[4] Morse, P. M., *Vibration and Sound*, 2nd ed., McGraw-Hill, New York, 1948, p. 298.

[5] Rschevkin, S. N., *The Theory of Sound*, Macmillan Co., New York, 1963, p. 402.

[6] Junger, M. C., "Radiation Loading on Cylindrical and Spherical Surfaces," *Journal of the Acoustical Society of America*, Vol. 24, No. 3, May 1952, pp. 288–289.

An Approximate Equivalent Linearization Technique for Nonlinear Oscillations[1]

R. E. JONCKHEERE.[2] In this interesting Note, Denman shows how an amplitude-dependent approximation to the angular frequency of an undamped nonlinear autonomous system with a single degree of freedom may be obtained by a first-order Chebyshev approximation to the function $f(x)$ in the interval $[-A, A]$. The differential equation of motion being

$$\ddot{x} + f(x) = 0 \qquad |x| \leq A \qquad f(x):\ \text{odd}$$

the result is

$$\omega^2 \cong \frac{2}{3A}\cdot\left[f(A) + f\left(\frac{A}{2}\right)\right] \tag{1}$$

However, this solution can also be obtained in an easier and less formal way by a direct application of the analytical formulation of the min-max principle of Denman's graphical method.[3]

[1] By H. H. Denman, published in the June, 1969, issue of the Journal of Applied Mechanics, Vol. 36, Trans. ASME, Vol. 91, Series E, pp. 358–360.

[2] Lecturer, Faculty of Applied Science, University of Brussels, Belgium.

[3] H. H. Denman, and Liu, Y. K., "A Graphical Procedure for the Approximation of the Period of Nonlinear Free Oscillations," Journal of Applied Mechanics, Vol. 31, Trans. ASME, Vol. 86, Series E, 1964, pp. 718–719.

If $f(x)$ is odd and df/dx is strictly increasing (or decreasing) on $[0, A]$ the approximating min-max line $P(x) = \omega^2 \cdot x$ will go through the origin and requires that two points be found with abcissas x_1 and x_2 on $[0, A]$ and such that $x_1 < x_2$, with the condition that the error

$$E(x) = f(x) - P(x) \tag{2}$$

attains equal extreme values with alternate signs. Obviously this locates x_1 between the origin and the intersection of $f(x)$ and $P(x)$, whereas x_2 will be situated at the interval's limit A.

The extremum requires that

$$\frac{dE(x)}{dx} = 0 \quad \text{at} \quad x = x_1$$

which yields

$$\omega^2 = \left(\frac{df}{dx}\right)_{x_1} \tag{3}$$

The equal-ripple property of the min-max approximation is expressed by

$$f(A) - P(A) = -[f(x_1) - P(x_1)]$$

Substitution of the values $P(A) = \omega^2 A$ and $P(x_1) = \omega^2 x_1$ leads to

$$\omega^2 = \frac{f(x_1) + f(A)}{x_1 + A} \tag{4}$$

Function $f(x)$ being odd and analytic can be represented by a power series expansion involving only odd-power terms:

$$f(x) = ax + bx^3 + cx^5 + \ldots$$

Applying (3) and (4) and setting $x_1 = k \cdot A$, $k < 1$, we find

$$k = \tfrac{1}{2} + \psi(A^2, \ldots)$$

For small amplitude oscillations the first-order approximation $k = \frac{1}{2}$ is adequate. Substitution of $x_1 = \frac{A}{2}$ into formula (4) yields (1).

It should be observed that due to a typographical error in Denman's paper [formula 10] the symbol A was omitted in the denominator of (1).

Author's Closure

Mr. Jonckheere's remarks reflect the fact, well known to applied mathematicians, that Chebyshev polynomial approximations have nearly minimum-maximum error. While the two methods give the same result as $A \to 0$, the graphical procedure is better when the nonlinear restoring function is known only graphically, and it is so simple it can be applied by a draftsman. The approximate equivalent linearization technique is preferable when the function is known analytically, and can be extended to higher-order approximations.

In reference to Mr. Jonckheere's last remark, since $T_1(x') = T_1(x/A) = x/A$, equation (10) in my paper is correct.

The Hydroelastic Stability of a Flat Plate[1]

E. H. Dowell.[2] Weaver and Unny have recently presented an interesting study of the title problem. It is the purpose of the present Discussion to point out that the problem has been studied in considerable greater generality by the present author in reference [1].[3] In particular, the effects of fluid compressibility, plate two-dimensionality, and nonlinear structural behavior have been taken into account. There is also an experimental literature [2]. To answer a major question asked by Weaver and Unny, the more comprehensive theory and the most recent as well as earlier experiments all indicate a simple static divergence with no flutter. For a recent survey of the aeroelastic stability of plates and shells (of which hydroelastic stability is a special case) the reader may refer to reference [3].

References

1 Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate II," *AIAA Journal*, Vol. 5, No. 10, 1967, pp. 1856–1862.

2 Gislason, T., Jr., "An Experimental Investigation of Panel Divergence at Subsonic Speeds," *Princeton University AMS Report* No. 921, 1970.

3 Dowell, E. H., "Panel Flutter: A Review of the Aeroelastic Stability of Plates and Shells," *AIAA Journal*, Vol. 8, No. 3, 1970, pp. 385–399.

Authors' Closure

The authors would like to thank Dr. Dowell for his interest in their work and for bringing to their attention the recent experimental report (Dowell's reference [2]).

It seems to us that the problem of the hydroelastic behavior of a plate is not entirely solved. While Dowell's reference [1] gives a very comprehensive analysis, its emphasis is on supersonic flow. Furthermore, if the fluid is a liquid, compressibility may well be neglected and it is to be hoped that reasonable results for the hydroelastic problem could be obtained with a computational effort short of that required in this reference.

There seems to be little doubt about the occurrence of static divergence. However, it is difficult to understand Dr. Dowell's remark about no flutter. Flutter has been predicted for higher velocities by our theory as well as, apparently, by Dowell (his reference [1, p. 1859], Dowell [4],[4] and Dugundji, et al. [5]. The latter, dealing with long panels on an elastic foundation, was confirmed experimentally. In addition, the early paper by Jordan [6] discussed the occurrence of flutter in his experiments and a survey of U. S. companies [7] reported a number of incidents of subsonic panel flutter experienced on flight vehicles. Perhaps experiments in a water tunnel with the consequent higher dynamic pressures, and hence lower critical flow velocities would shed more light on the question.

References

4 Dowell, E. H., "Flutter of Infinitely Long Plates and Shells—Part 1: Plate," *AIAA Journal*, Vol. 4, No. 8, 1966, pp. 1370–1377.

5 Dugundji, J., Dowell, E. H., and Perkin, B., "Subsonic Flutter of Panels on Continuous Elastic Foundation," *AIAA Journal*, Vol. 1, No. 5, 1963, pp. 1146–1154.

6 Jordan, P. F., "The Physical Nature of Panel Flutter," *Aero Digest*, Feb. 1956, pp. 34–38.

7 Mirowitz, L. I., et al., "Panel Flutter Survey and Design Criteria," Aerospace Industries Association of America, Report ARTC-32, Aug. 1962.

[1] By D. S. Weaver and T. E. Unny, published in the September, 1970, issue of the JOURNAL OF APPLIED MECHANICS, TRANS. ASME, Vol. 92, Series E, pp. 823–827.

[2] Associate Professor, Department of Aerospace and Mechanical Sciences, Princeton University, Princeton, N. J.

[3] Numbers in brackets designate References at end of Discussion.

[4] Numbers in brackets designate References at end of Closure.

The Free Plastic Compression of Pure Metals[1]

M. J. HILLIER.[2] The authors have set out to provide an extension of empirical laws of plastic behavior to a wider range of conditions. For practical purposes they appear to have succeeded and are to to congratulated. The following opinions, therefore, should be taken primarily as a criticism of our present state of fundamental knowledge.

In order to describe the plastic behavior of materials it appears necessary, and it may be sufficient, to recognize the validity of the following points of view:

1 The so-called quasi-static "strain-hardening" curve is unlikely to represent basic data for there is little evidence that it has ever been obtained under either isothermal or adiabatic conditions. At low rates of deformation the apparent rate of strain hardening is often controlled by the rate of heat conduction. At high rates of deformation a significant proportion of the apparent strain hardening may often be accounted for by material inertia.

2 The so-called "stress relaxation" has not been shown to be

[1] By V. S. Shankla and R. F. Scrutton, published in the December, 1970, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 92, Series E, pp. 1121–1133.

[2] Associate Professor, Faculty of Engineering and Applied Science, Memorial University of Newfoundland, St. John's, Newfoundland, Canada.