DISCUSSION/AUTHORS CLOSURE

\[2W_f = -(1 - e^2)\left[2\mu B_2 P_x^2 + (B_2 - \mu B_1) P_y^2\right] - 2e_0^2 \mu B_2 P_y^2, \quad (k)\]

Thus, if \(e_0\) is independent of friction, Poisson's hypothesis exhibits a loss of kinetic energy due to internal irreversible deformation in addition to the energy absorbed by friction. On the other hand, for Newton's kinematic law of restitution, the normal component of impulse does work,

\[2W_f = -(1 - e^2)\left[2\mu B_2 P_x^2 + (B_2 - \mu B_1) P_y^2\right]\]

\[+ 2e_0^2 \mu B_2 P_y^2 \left(\frac{2P_y}{P_y^2 + \frac{2\mu B_1}{B_2 - \mu B_1} - 1}\right) \quad (f)\]

Here, if \(e\) is independent of friction, there is more energy recovered during restitution than was absorbed during compression. Consequently, with friction and slip reversal, \(e = e_0 = 1\) do not represent elastic collisions! A similar analysis shows that this conclusion applies also to eccentric collisions which slip stick.

References


Authors' Closure

Stronge (1990) develops a new definition of restitution, which is recapitulated in his discussion above. Stronge's definition of restitution is based on an underlying model of the material behavior. This construction provides a means of relating the coefficient of restitution of the elasticity of the materials.

The main issue raised in the discussion is whether Poisson's definition of restitution must be rejected. Stronge takes the position that Poisson's definition violates basic principles, and is untenable. We do not find the argument compelling. If we accept Stronge's modeling assumptions, then Poisson's definition does not work. But might there be another model of the underlying material interactions, departing from Stronge's assumptions, that supports Poisson's definition? The existence of a satisfactory model seems unlikely, but at this time the question remains open. The terms "fallacious" and "defective" should be reserved for ideas that are proven to be inconsistent with generally accepted principles.

A second issue is whether Poisson's coefficient of restitution is a material constant. Stronge suggest that it is actually a function of initial conditions, coefficient of friction, etc. We think the resolution of this issue is quite simple. One should choose the model one prefers, and then stick with it. If one chooses Poisson's definition, then \(e\) is a material constant, and an elastic collision is defined to be \(e = 1\). If instead one chooses Stronge's definition of an elastic collision, it would be senseless to use Poisson's coefficient of restitution.

The primary issue should be which definition to prefer. Since Stronge's definition of restitution is better founded than Poisson's or Newton's, and in the absence of any apparent drawbacks, the obvious conclusion is that everybody should adopt Stronge's model. We would like to observe that most of our paper is virtually unaffected by this choice. In particular, (1) the use of Routh's method is trivially extended to handle Stronge's restitution; (2) the concept of tangential impact is unaffected; (3) the taxonomy of rigid body impact is unaffected; and (4) the comparison of Poisson's definition to Newton's is unaffected, though less interesting.

The Complementary Potentials of Elasticity, Extremal Properties, and Associated Functionals

R. T. Shield and S. J. Lee. The paper by Wempner (1992) gives several references related to variational principles for finite elastic deformations but omits references to the work of Lee and Shield (1980a, 1980b). In the first paper (1980a) we derive a complementary energy principle which uses trial functions for the actual deformation gradient. This approach avoids difficulties associated with inversion of the constitutive relation which in general involves rotations as well as strains. However, the trial functions used in the complementary energy principle satisfy nonlinear equilibrium equations in general, and this causes varying degrees of difficulty in applying the principle depending on the form of the strain energy function and the particular problem.

In the second paper (1980b), the complementary energy principle and the principle of stationary potential energy were applied to obtain lower and upper bounds on the total strain energy in two problems. For the one-dimensional problem of the all-around extension of a plane sheet with a circular hole, close bounds on the total strain energy were obtained for two forms of the strain energy function, and accurate estimates for the stress resultant at the edge of the sheet were obtained. This problem was treated previously by Rivlin and Thomas (1950) by a numerical approach. The problem of the large extension and torsion of a long elastic cylinder which is bonded at the ends to rigid plates was discussed next, and an approach was described for estimating the resultant end loads from estimates of the total strain energy derived from the variational principles. As an illustration, accurate estimates for the twisting moment and the axial force were obtained for elliptical cylinders with axes in the ratios of 2:1 and 4:1 for a wide range of extension and twist, the neo-Hookean form of the strain energy being assumed for the material of the cylinders.

It may be noted that in Lee and Shield (1980b), the leading term on the right-hand side of Eq. (2.10) should be \(\mu^2\).

References


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