Authors' Closure

The authors wish to thank the contributors for their valuable comments and criticisms. Perhaps the queries raised by Professor Jonas may be considered as being answered in part by the third and sixth of Professor Hillier's comments. In this case the lack of rate sensitivity would be a consequence of the mathematical form of the equations and of the properties of the material. In any case the effect is somethat obscured by the experimental errors associated with the drop-weight technique. Although exact numerical results are important it is considered that the main bearing of the paper has to do with the philosophical interpretation of plastic data.

In reply to Professor Wagner the authors agree that the effects of wave propagation were neglected. However, by a suitable choice of impact velocity, specimen size, and anvil dimension these effects may be minimized. Inhomogeneous deformations associated with frictional effects usually cause some errors when compressing plastic specimens. However, more recent and exact analyses of the barrelling tendency suggest that it may be possible in the future, to introduce a correction term to take care of this source of error. In hot-working operations little energy is lost in internal lattice distortion or specimen hardening. The more common energy loss is usually attributable to thermal conduction. This is a serious matter when straining at slow speeds or when using hot specimens and cold platens.

The existence of an equation of state is frequently refused credence on the ground that the instantaneous flow stress is dependent on the past history of the specimen (and thus on its structure). It is suggested that this past history of straining or heat-treatment itself may have been subject to the rules of an equation of state. However the position as outlined in the paper regarding the existence of an all-embracing equation of state is clear. No such law was postulated and no support was given to any particular mathematical form. It is agreed that mathematical models are merely based on empirical results which have been plotted on graph paper. However, engineers are usually optimists and would prefer to think that practical problems may be based on the basis of well-established physical laws. Where these do not exist it is natural to clutch at straws and to use or seek to extend the range of usefulness of whatever data may be available.

The authors wish to thank Professor Hillier for his perceptive comments regarding the plastic flow of metals which comments have contributed significantly to the value of the paper. Presumably a number of the conclusions stated are based on the results of experimental work performed by Professor Hillier and it is to be hoped that these results may be published in the near future. It would seem from comment number four that the extension of an empirical isothermal equation to take account of adiabatic heating may be fundamentally unjustified even though it would appear to be practically convenient. Presumably the much larger temperature sensitivity during plastic flow is to be attributed to the very localized nature of dislocation movement and to the accumulation of slip in submicroscopic lamellas. It is to be hoped that this discussion may lead to a critical reappraisal of existing plasticity concepts.

Low-Speed Slip Flow Over a Wedge[1]

Y. C. L. WU.[2] The authors presented a very useful method in treating the incompressible slip flow over a wedge. The agreement with numerical calculations certainly would inspire workers to seek solutions of boundary-layer flows by means other than purely numerical.

As pointed out by the authors the study of rarefied gas flow is motivated from the high-speed flight in the upper atmosphere, therefore, it would be of great interest to extend the present work to treat the compressible boundary layer. That the slip velocity agrees so well with experiments at high Mach number ($M = 10$) is somewhat surprising. Does this mean that the compressibility effect and variation in transport properties in hypersonic flow are not important in determining the slip velocity? Is this also true for skin friction?

Authors' Closure

The authors wish to thank Professor Wu for her interest and generous comment in regard to our work in low-speed slip flow over a wedge [1].[3]

It should be pointed out that the experimental data on slip velocity are available only for cases of flat plate flow within the hypersonic regime. Comparison with the low-speed theoretical analysis is possible only after these results are reinterpreted through compressibility transformations and this has been carried out in an early study [2]. A linear viscosity-temperature relationship has been introdued for the expedience of decoupling the momentum and energy equations. However, in another study of hypersonic flow past a flat plate [3], where a different viscosity-temperature relationship ($\mu \sim \sqrt{T}$) and a slightly different velocity-slip coefficient have been employed, the analytical results of velocity slip are not much different and they all compare favorably with the same set of experimental data. Unfortunately, experimental data of velocity slip under a variety of flow conditions have not been produced and a thorough evaluation of the theoretical results is not possible at the present. The authors fully appreciate Professor Wu's concern and cannot answer her question in a satisfactory manner although the limited amount of available experimental data does indicate that a simple account of transport properties for hypersonic flows, as suggested in these early studies, might be adequate under these flow conditions. For the same reason, the authors cannot also answer adequately the question concerning the skin friction, since the experimental information on skin friction within this flow regime is practically nil.

Perhaps it is pertinent to point out that analyses of slip-flow problems by the present scheme would yield a slip-velocity larger than 50 percent of the free stream close to the leading edge of the plate. This is possible only when specular reflection occurs there. Recent experimental data on heat transfer [4, 5] indicated that as the leading edge is approached, the rate of heat transfer does not tend to that of the diffused free molecular flow. The theoretical results produced from reference [3][4] now gives a better agreement with these new data. Again, additional abundant experimental data are needed to clairfy the flow conditions close to the tip of the plate.

References

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[2] Associate Professor, The University of Tennessee Space Institute, Tullahoma, Tenn.

[3] Numbers in brackets designate References at end of Closure.

[4] These results are misquoted reference [4]. Also the merged layer analysis has been further extended so that the results from reference [3] now tend asymptotically to strong interactions limit.

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Distribution of Mass, Velocity, and Intensity of Turbulence in a Two-Phase Turbulent Jet[1]

VICTOR W. GOLDSCHMIDT.[2] The authors have quantified the effects of aerosol droplets on a turbulent jet. Their results have suggested a decrease in the widening rate and turbulent intensities of the jet due to the presence of the aerosol droplets. Their careful and conscientious work is, however, subject to criticism. The specific points in question are as follows.

Spreading Coefficient

The use of Reichardt's solution (which does violate Galilean invariance as per footnote following equation (5.24) of reference [14]) is as unacceptable, but yet as useful, as any of the other available phenomenological theories. The authors use Reichardt's model and estimate a spreading coefficient of the jet based on equations (23) and (24). This procedure is acceptable only as long as the virtual origins of the jet remain unchanged. All evidence suggests, however, that any disturbance (such as the injection of the aerosol) could easily cause a considerable shift in the virtual origins. This was reported in reference [1][3] of this discussion for the case of plane jets.

The jet half width can be defined from equations (23) and (24) as that value of r for which $\bar{u}/\bar{u}_m = 0.5$. It becomes then

$$\frac{b}{D} = 1.182\, C_m \cdot \frac{X}{D} \tag{1}$$

The C_m values in Table 1 of the paper may be used (as these were seemingly computed at a given X/D station) to compute b. Alternatively the half widths may be read directly from the authors' Figs. 7 and 8. Table 1 of this Discussion, then results.

It is simple to obtain (from the authors' Table 1 and the discusser's equation (1) the corresponding values of b/D at the other X/D stations and confirm the linear relationship

$$\frac{b}{D} = K_1\left(\frac{X}{D} - C_1\right). \tag{2}$$

Table 1

Q_l/Q_a	b/D @ $X/D = 20$	b/D @ $X/D = 35$
0	1.68	2.93
2.16×10^{-6}	1.42	2.72
2.56×10^{-6}	1.38	2.60
3.08×10^{-6}	1.33	2.56

In the foregoing K_1 is the widening rate of the jet (in the order of 0.06 to 0.09) and C_1 is the location of the geometric origin of similarity. It is measured from the mouth of the jet, and made dimensionless by D.

From Table 1 and equation (2) of this Discussion, the values found in Table 2 result.

The tabulated values are based on the authors' Figs. 7 and 8, and the discusser's equation (2). They strongly suggest that the aerosols did *not* change the spreading of the jet, but rather simply moved the origin of similarity further downstream. The authors' conclusion 2 would then be incorrect.

Center-Line Velocity Decay Rate

The authors suggest, following Fig. 9, that the dependence of $\bar{u}_m/\bar{u}_0$ on X/D varies from the expected (single-phase) inverse relationship. If this were the case the expected similarity in the concentration flux profiles would not be present. It is hard to tell from Fig. 5 whether this is the case as the data points are not coded. However, earlier results (such as authors' reference [11]) have confirmed the similarity of the concentration flux profiles, and hence the relationship

$$\left(\frac{\bar{u}_m}{\bar{u}_0}\right)^{-1} = K_2\left(\frac{X}{D} - C_2\right) \tag{32}$$

should hold.

The coefficients K_2 and C_2 are, respectively, the center-line velocity decay rate and the dimensionless location of the kinematic origin of similarity. The values plotted in Fig. 9 are not easy to read, but from them Table 3 could be inferred.

Table 3 is not as conclusive as Table 2. It does, however, suggest that the main effect of the aerosol is simply a shift of the origins of similarity. Fig. 9 should be interpreted accordingly.

Aerosol Description

Inasmuch as the main objective of the work is that of quantifying the effect of the liquid droplet phase on the carrier stream a complete description of the aerosol is desirable. The authors have done this in part. They indicate using six DeVilbiss Type 841 nebulizers and generating cottonseed oil droplets calculated to be 13 microns in diameter. They estimated a corresponding standard deviation of 2 microns.

Unfortunately, the actual aerosol size was not measured. The calculated values are in disagreement with values expected from previous experimental evidence. Using dibutyl phthalate as a liquid phase (with properties not too different from cottonseed oil) mass mean diameters ranging from 4.3 to 7.5 microns were measured in the output of a Type 841 DeVilbiss nebulizer. This

Table 2

Q_l/Q_a	K_1	C_1
0	0.083	1.6
2.16×10^{-6}	0.087	3.7
2.56×10^{-6}	0.081	3.1
3.08×10^{-6}	0.082	3.8

Table 3

	$(\bar{u}_m/\bar{u}_0)^{-1}$				
Q_l/Q_a	@ $X/D = 20$	@ $X/D = 30$	@ $X/D = 35$	K_2	C_2
0	2.59	4.08	4.70	0.144	1.67
2.16×10^{-6}	2.43	3.85	4.65	0.148	3.99
2.56×10^{-6}	2.32	3.70	4.44	0.141	3.76
3.08×10^{-6}	2.27	3.51	4.13	0.124	1.78

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[1] By G. Hetsroni, and M. Sokolov, published in the June, 1971, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 93, Series E, pp. 315–327.

[2] School of Mechanical Engineering, Purdue University, Lafayette, Ind.

[3] Numbers in brackets designate References at end of Discussion.