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Discussion

Rotatability Considerations for Spherical Four-Bar Linkages with Applications to Robot Wrist Design (86-DET-18)[1]

C. H. Chiang.[2] Professor Gupta is to be congratulated for having derived additional criteria of the rotatability of the input link of a spherical four-bar linkage. These criteria have been applied to check the rotatability of a given spherical four-bar, as well as in designing new spherical four-bars. The author has tried to differentiate those criteria not using supplementary transformations from those using supplementary transformations, and hence he claimed that his criteria could be applied without requiring a sequence of supplementary transformations of spherical link lengths. If this writer has understood the paper properly, the primary linkage and secondary linkage of the author are just two mutually supplementary linkages, and in the example of a spherical four-bar with $\alpha_{n-2} = \alpha_{n-1} = \Delta$, the linkage with a sum of two link lengths of $2\pi - 2\Delta$ is just a supplementary linkage of the original linkage with the sum of two link lengths 2Δ. It seems therefore to this writer that the concept of supplementary linkages cannot always be circumvented.

This writer believes that the procedure suggested by him in [2] can easily be applied to all examples given in the present paper. This procedure may be stated briefly as follows:

(*1*) Reduce the lengths of the links of a given spherical four-bar until the sum of all four links is a minimum. The reduction is carried out, by replacing each time two longest link lengths by their respective supplementary arc lengths.

(*2*) Check if the reduced four-bar satisfies Grashof's rule. A Grashof linkage is a crank-rocker, a rocker-crank, a drag-link, or a double-rocker according to whether the input link, the output link, the fixed link, or the coupler is the shortest link.

In order to show clearly the application of this procedure, the nine examples given in the present paper are listed in Tables 1, 2, and 3, in the same sequence as they were given by the author.

It can be seen from Table 1, the analysis of a given spherical four-bar linkage, that the final reduced four-bar loop is unique, unless the sum of the two longest link lengths is just 180 deg, as examples 3, 4, and 5 show. In such cases, however, the rotatability will not be altered if an alternative four-bar loop is taken for checking the rotatability of the linkage. Thus, for instance, in example 5, if the reduced four-bar loop is taken as $(\alpha_f, \alpha_i, \alpha_c, \alpha_o)$ = (50 deg, 20 deg, 95 deg, 85 deg), the linkage remains a crank-rocker. In any case the final reduced sum is always a minimum.

In Table 2, in the design of new spherical four-bar linkages, the ranges of α_f are divided into subranges for the purpose of type determination of the linkage. Although this process looks

Table 1 Analysis

		α_f	α_i	α_c	α_o	Result
Example 1	Given:	100°	160°	120°	105°	
	Reduced to:	80°	20°	60°	75°	Crank-Rocker
Example 2	Given:	155°	60°	110°	100°	
	Reduced to:	25°	60°	70°	100°	Drag-Link
Example 3	Given:	150°	60°	120°	130°	
	Reduced to:	30°	120°	60°	50°	Non-Grashof Double-Rocker
Example 4	Given:	160°	50°	85°	85°	
	Reduced to:	20°	50°	85°	95°	Drag-Link
Example 5	Given:	50°	160°	85°	85°	
	Reduced to:	50°	20°	85°	95°	Crank-Rocker

Table 2 Design

		Range of α_f	$\alpha_n = \alpha_i$	$\alpha_{n-1} = \alpha_c$	$\alpha_{n-2} = \alpha_o$	Result
Example 6		0°-- 25°	30°	80°	75°	Drag-Link
		25°-- 35°	"	"	"	Non-Grashof Double Rocker
		35°-- 125°	"	"	"	Crank-Rocker
		125°-- 175°	"	"	"	Non-Grashof Double-Rocker
		175°-- 180°	"	"	"	Crank-Rocker
Example 7 Shin Meiwa Wrist	Given:	135°-- 180°	62.5°	95°	70°	
	Reduced to:	45°-- 0°	62.5°	85°	70°	Drag-Link
	Given:	135°-- 180°	75°	105°	50°	
	Reduced to:	45°-- 0°	75°	75°	50°	Drag-Link

Table 3 Design, examples 8 and 9

Range of $\alpha_n (=\alpha_i)$	Range of α_f	α_{n-1} $(= \alpha_c)$	α_{n-2} $(= \alpha_o)$	Result
0° -- Δ	0°-- Δ	Δ (with Δ ≤	Δ 90°)	Drag-Link if $\alpha_f < \alpha_n$ Crank-Rocker if $\alpha_f > \alpha_n$
	Δ -- 2Δ	"	"	Crank-Rocker if $\alpha_n + \alpha_f < 2\Delta$ Drag-Link if $\alpha_n^s + \alpha_f^s < 2\Delta$ otherwise Non-Grashof Linkage
	2Δ -- 180°	"	"	Drag-Link if $\alpha_n^s + \alpha_f^s < 2\Delta$ otherwise Non-Grashof Linkage
Δ -- 90° (or Δ--2Δ if 2Δ<90°)	0° -- Δ	"	"	Drag-Link if $\alpha_n + \alpha_f < 2\Delta$ otherwise Non-Grashof Linkage
	Δ -- 180°	"	"	Drag-Link if $\alpha_n^s + \alpha_f^s < 2\Delta$ otherwise Non-Grashof Linkage
2Δ -- 90° if 2Δ<90°	0° -- 180°	"	"	Non-Grashof Linkage

Note: $\alpha_n^s = 180° - \alpha_n$, $\alpha_f^s = 180° - \alpha_f$.

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[1] By K. C. Gupta, and published in the September, 1986, issue of the JOURNAL OF MECHANISMS, TRANSMISSIONS, AND AUTOMATION IN DESIGN, Vol. 108, pp. 387–391.

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somewhat cumbersome, it provides a clear picture of the variation of the type of the linkage.

In Table 3, the case considered is: $\alpha_{n-2} = \alpha_{n-1} = \Delta$, $\Delta \leq 90$ deg. The tabulation covers only one-half of the whole range of α_n, namely $0 \leq \alpha_n \leq 90$ deg. The other half would be identical with this table if α_n, α_f, α_n^s, α_f^s are replaced respectively by α_n^s, α_f^s, α_n, α_f, where $\alpha_n^s = 180$ deg. $- \alpha_n$, $\alpha_f^s = 180$ deg. $- \alpha_f$. In this case the zones of the types of linkages can best be conceived by means of a diagram as shown in Fig. 6. In the square area, only in the two right triangular zones in the upper right and lower left corners, complete gripper spin is possible.

Example 9 is a special case of example 8, in which $\Delta = 90$ deg. The two triangular zones fill up the whole square, hence the gripper can always spin freely.

Note that the condition $\alpha_n^s + \alpha_f^s < 2\Delta$ is equivalent to the condition $2\pi - 2\Delta - \alpha_n < \alpha_f$, as given by the author in (33).

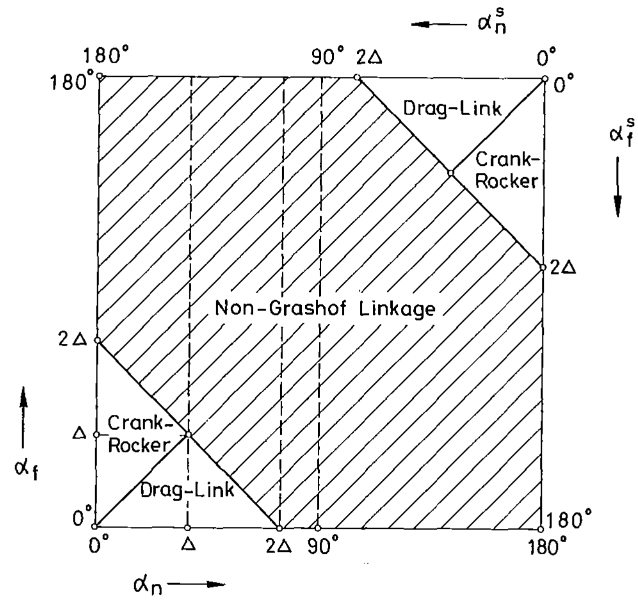

Fig. 6 Examples 8 and 9

Author's Closure

The author thanks Professor Chiang for his interest in this paper. In view of his comments in the beginning paragraph of his discussion, the following clarification may be helpful to the readers.

When the link dimensions are considered to be in the range $[0, \pi]$, there is a group of eight supplementary spherical four-bar linkages, including the source linkage. There is no doubt that the primary and secondary linkages, as defined by the author, belong to this group. However, both of these linkages are identified at the outset and the source linkage is either primary or secondary according to conditions (12). In fact, the primary and secondary linkages are particular supplementary linkages which have the same coupler- and output-link dimensions as the source linkage. Thus the group of eight supplementary linkages has been reduced a priori to a definite pair of linkages which contains the source linkage. As shown in the paper, this definiteness in the identification of the particular linkage at the outset, to which the stated criterion can be applied directly, leads to some useful explicit forms of rotatability conditions which may be manipulated further; for examples, see conditions (15, 22, 23). It may also be verified easily that all of the alternate statements of rotatability which have been referred to in the paper do not always apply to the primary linkage itself.

The observation that the results of all examples in the paper may be verified by using the criterion in reference [2], or those in references [3–7, 13–15] as well—with varying amounts of effort—is obviously indisputable, but this is not the main issue; recently, Professors J. Angeles and C. R. Barker have also communicated to the author that they have verified numerical examples in the paper by using new criteria which they have developed. Going beyond specific examples, however, tabular representations of general explicit conditions such as (22, 23) tend to become cumbersome; even a simple condition like (33) is only partially represented in discussor's Table 3, notwithstanding the remark that "... other half would be identical with this table if ... " Depicting gross motion characteristics in type charts [7], which the discussor has done in Fig. 6, is certainly useful for visualization.