Mathematics of pattern growth in condensed matter

Interfacial Wave Theory of Pattern Formation in Solidification: Dendrites, Fingers, Cells and Free Boundaries. , Jian-JunXu,
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Domenico Truzzolillo

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16 February 1943, Heisenberg wrote the following to Dirk Coster, a physicist in the occupied Netherlands known for helping Jews hide from the Nazi regime:

> I am happy to write that I consider Dr. Goudsmit to be one of the best researchers in the area of theoretical physics. His name has become known in the entire scientific world through the development of the magnetic properties of the electron. . . . Since Goudsmit has always been very hospitable towards us Germans, also in America, even during a period in which one could already sense currents hostile to Germany (I was his guest in the summer of 1939), I would be very sorry if, for reasons unknown to me, difficulties would arise for his parents in Holland. (author's translation)

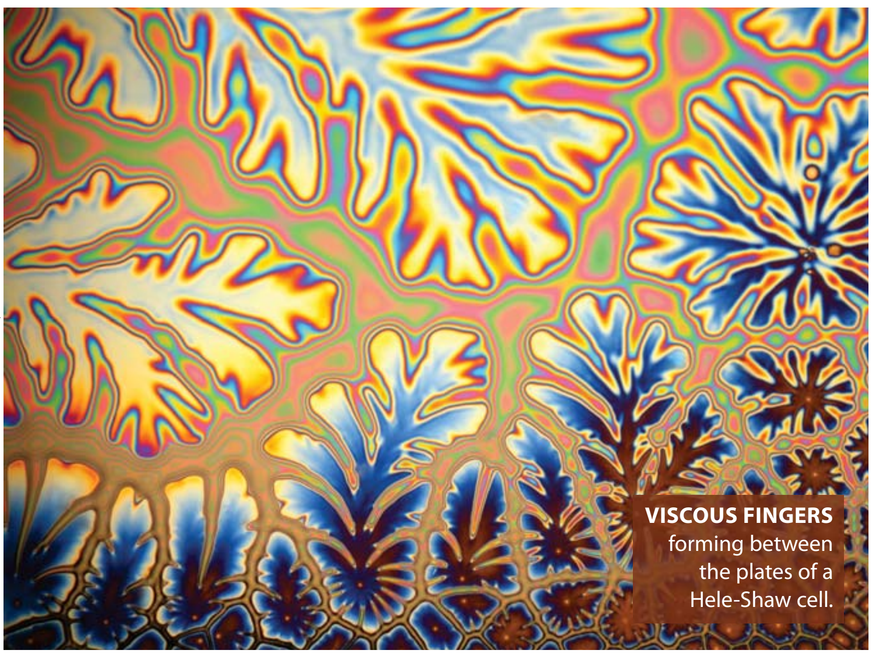
On 7 June of the same year, Coster replied to Max von Laue: "Thank you very much for your friendly letter of February 15th. Unfortunately just like Heisenberg's response, it came into my possession too late in order to try to do something with it, and the inevitable had already happened."

Anyone well versed in the history of the Third Reich and especially the Holocaust, as Cassidy certainly is, knows that there was little if anything that Heisenberg or anyone else could have done to stop the deportation of Goudsmit's parents. Even cautious expressions of support like the one Heisenberg sent could well have had negative consequences—something that both Heisenberg and von Laue undoubtedly understood very well. Although it is certainly possible that Goudsmit blamed Heisenberg after the war for his parents' deaths, documents from the immediate postwar period suggest instead that he appreciated what Heisenberg had done.

Ultimately, however, Cassidy has made Farm Hall come alive, with all the important contradictions and conflicts it embodies. His book should be of special interest to physicists and physics students and is a valuable addition to our understanding of this ambiguous chapter in the history of modern physics.

Mark Walker
Union College
Schenectady, New York

CLAIRE TREASE, KINGSTON UNIVERSITY (WIKIMEDIA COMMONS)



VISCOUS FINGERS forming between the plates of a Hele-Shaw cell.

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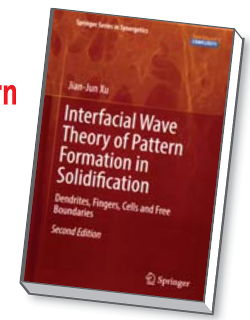

Interfacial Wave Theory of Pattern Formation in Solidification
Dendrites, Fingers, Cells and Free Boundaries
Jian-Jun Xu
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Jian-Jun Xu's book *Interfacial Wave Theory of Pattern Formation in Solidification: Dendrites, Fingers, Cells and Free Boundaries* covers a large and well-chosen collection of fundamental results in the field of interfacial stability and pattern formation. The author is an accomplished theoretical materials scientist who has made valuable contributions to the fields of asymptotic analysis and nonlinear partial differential equations, which are relevant in materials science, solidification physics, fluid dynamics, interfacial wave theory, pattern formation, and crystal growth.

Throughout the book, Xu starts from real-world problems and details analytical methods to describe pattern formation. Because he avoids numerical methods, his book will be a useful reference for those who are teaching courses focused on mathematical modeling or applied mathematics. The author does not hide his preference for asymptotic methods, even though they are often limited in their applications. Xu's approach contrasts with that of other textbooks, such as *Pattern Formation and Dynamics in Nonequilibrium Systems* (2009) by Michael Cross and Henry Greenside, which place more emphasis on the physics of fractals and the nature of the periodicity in fractal patterns.

Xu is able to draw analogies between phenomena appearing in completely different contexts—for example, solidification of pure melts, hydrodynamic instabilities in porous media, and the eutectic growth of mixtures. However, *Interfacial Wave Theory of Pattern Formation in Solidification* may not stand alone as a graduate textbook, since it focuses on a wide range of mathematical methods that are sometimes quite sophisticated and omits much of the basic introductory material that one would hope for. By textbook standards, the book also lacks high-quality diagrams, photographs, and sketches. All of that should be taken into account by professors considering this book for courses.

As a physicist working on interfacial phenomena and pattern formation in liquids, I spend much of my time trying to identify experimentally the key features of both spontaneous and artificial pattern growth and to link them to the thermodynamics of interfaces. For that reason I was particularly pleased with Xu's coverage of material that is often missing from conventional texts on pattern growth and nonequilibrium phenomena in condensed matter. For example, Xu discusses the unidirectional solidification and spatially periodic cellular growth in the thin region between the bounding plates of a Hele-Shaw cell.

Xu begins with a short introduction on the theory of free dendridic growth and macroscopic continuum models, in which he surveys the classical hypotheses and equations governing Hele-Shaw flows and crystal growth. He then details the mathematical theory behind the Mullins–Sekerka instability that drives directional solidification. Those topics are quite rare for a textbook and extremely valuable. I much appreciated the clear distinction between cases of pattern growth in the absence of surface tension and those in the presence of interfacial stresses, which is particularly useful when Xu addresses dendridic growth and Chuoke-Saffman-Taylor instability. The text concludes with an in-depth discussion of cellular and eutectic growth.

Many cross-references throughout the book connect material in different chapters. The author also provides extensive and moderately up-to-date references to the literature, in which the reader may find more general overviews of specific topics.

The danger with a book of this size and ambition is that the central principles of pattern formation can get lost in the quantity of material presented. In a work that discusses so many topics, it is impossible to analyze and present everything in detail without losing some clarity. In many cases the author assumes that the reader has an advanced background in solidification processes. Consequently, many problems, equations, and other information may be hard for nonspecialists to understand.

For all those reasons, I cannot unreservedly recommend *Interfacial Wave Theory of Pattern Formation in Solidification* as a stand-alone course textbook, but I do think it would be a good guide to the literature and a possibly inspiring source for lectures focusing on the mathematical aspects of pattern formation. Moreover, such an ambitious, advanced text will surely be of great interest to mathematical physicists and researchers in chemical physics, engineering, and materials science.

Domenico Truzzolillo
CNRS–University of Montpellier
Montpellier, France