The pulse-probe method of conductivity measurement

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Summary. An alternative to the steady heating of a cylindrical probe, in the ‘needle-probe’ method of conductivity measurement, is the observation of the thermal decay from a short, calibrated, heat pulse. The theoretical solution is the time-differential of that for the former method, and requires only the measurement of point temperatures rather than the determination of a gradient. A careful analysis of the theoretical decay function shows that it should be possible to make accurate conductivity measurements in as little as three probe ‘time constants’ if external information is available on the heat capacity of the medium. A self-contained method, using two temperature determinations from a record about six time-constants long, can be used where such information is not available. The theory was tested by measurements on the ocean-floor, and the data correspond to the theory when a correction was applied for some internal probe conduction problems.

Introduction

The measurement of thermal conductivity in soft solids is most conveniently accomplished by the insertion of a cylindrical probe, and the application of some form of transient heating pulse to the medium through the instrument. An important use of the method is the determination of sediment conductivity in the ocean-floor in situ, since this bypasses the difficult task of retrieving, storing and measuring a representative sample of the material. The first such transient method to receive widespread practical application was that of Von Herzen & Maxwell (1959), where the cylindrical probe is heated continuously by a uniform resistance wire. The temperature rise against time approaches a logarithmic asymptote whose slope is proportional to the thermal resistivity of the material surrounding the probe. The asymptotic slope is approached rather slowly, however, and this introduces difficulties both in the laboratory and in the field. Local temperature drifts become more significant at long times in the laboratory, and, at sea, ship drift becomes progressively harder to control, leading to an increased incidence of mechanical disturbances. Additional drawbacks to the method are the sensitivity of the results to the stability of the heater.
power source, due to the proximity between heater and sensor, and the effect of contact resistance between the probe and the medium on the shape of the temperature curve.

The continuous-heating method has, nonetheless, been adapted for \textit{in situ} measurements on the ocean-floor by Sclater \textit{et al.} (1969), Christoffel & Calhaem (1969) and Lister (1970), and also for downhole measurements in continental rocks (e.g. Beck 1965). The problem of the long measuring time has been reduced by various means of utilizing the exact tables of the cylindrical heating function tabulated by Sass (1965) and Huppert & Sclater (1968). However, results in the early-time region are affected by parameters of the medium that are both unknown and of little interest (primarily the thermal capacity), and they are also quite sensitive to the probe-medium contact resistance (Jaeger 1959; Beck 1965).

Therefore it is worth finding out if a related transient thermal solution for cylindrical space could not reduce some of the drawbacks of the continuous-heating method. The time-differential of the steady-heating function is simply the decay of a brief heat pulse applied to the cylindrical probe, and the theoretical decay function is already used routinely to eliminate frictional heating effects from thermal gradient measurements in the ocean-floor. Excellent correspondence is observed between the theoretical decay and the temperature measurements, even at very early times after entry (e.g. Lister 1970). The decay measurements are much less sensitive to mechanical disturbance than the heating data, and they seem to be less affected by unwanted parameters in the early-time region. One would also expect the thermal decay to be less sensitive to contact resistance between the probe and the medium, since the only heat crossing the boundary is that required to cool the heat capacity of the probe.

Practical application of the method in the field has been restricted by the short but intense calibrated heat pulse needed to approach the delta function heat source assumed by the theory. The battery contained in a small recording instrument is unable to supply the required power level, but the new generation of multistation telemetering heat-flow instruments must inherently contain a much larger battery to power the telemetry and operate for long periods. Therefore it has been possible to make a field test of the suggestion by Bullard (1954) that the decay of a calibrated heat pulse be used to measure the thermal resistivity of ocean sediment. This paper presents the experimental results, and, in a detailed analysis, compares them to the theoretical decay curve for an infinite cylinder. The intention is to develop the properties of the decay curve into correction factors that can be applied to practical measurements on a routine and perhaps automated basis. For the multipenetration approach to heat-flow measurement places a premium on the reduction of in-bottom time during each penetration, and raises the possibility of really large numbers of measurements being made on a single cruise. Thus, not only must good measurements be made as soon as possible after applying the heat pulse, but the computation of results should be as simple as possible and not require extraneous information, such as the need for a nearby core sample to estimate sediment composition.

\section*{Theory of the method}

The decay of a transient heat pulse in an infinite perfectly conducting cylinder immersed in a medium is described by the function $F(\alpha, \tau)$ introduced by Bullard (1954). Here $\alpha$ is twice the ratio of the thermal capacity of the medium displaced by the cylinder to that of the cylinder itself. It is expressed as

$$\alpha = \frac{2\pi a^2 pc}{m}$$  \hspace{1cm} (1)
where \( a = \) cylinder radius, \( \rho = \) medium density, \( c = \) medium specific heat and \( m = \) the heat capacity of the cylinder. If the conductivity of the medium is \( k \) and its diffusivity \( \kappa = k/\rho c \), then the dimensionless time symbol \( \tau \) is given by

\[
\tau = \frac{kt}{a^2} \tag{2}
\]

where \( t \) is the time elapsed after the application of the heat pulse.

It is not immediately obvious why a function of \( \alpha \) and \( \tau \) can be used to measure the medium conductivity \( k \), rather than its diffusivity \( \kappa \). The physical reason is that the heat pulse is a calibrated quantity of heat, not an initial temperature rise: in the subsequent measurement of temperature, a heat capacity is being measured implicitly. Mathematically, it can be shown that the function \( F(\alpha, \tau) \) tends to \( 1/2\alpha \tau \) at long times after the heat pulse:

\[
F(\alpha, \tau) \rightarrow \frac{1}{2\alpha \tau} \quad \text{as} \quad \tau \rightarrow \infty. \tag{3}
\]

The initial temperature rise in the cylinder is \( T_0 = q/m \), where \( q \) is the heat input per unit length. Since \( F(\alpha, 0) = 1 \), the temperature at long times is given by

\[
T = \frac{T_0}{2\alpha \tau} = \frac{q}{2m} - \frac{m}{2\pi a^2 \rho c} \cdot \frac{a^2 \rho c}{kt} = \frac{q}{4\pi kt} \tag{4}
\]

simply a function of \( q, k \) and \( t \).

We are interested in estimating \( k \) in as short a time as possible, and should therefore concentrate on the deviations from the asymptotic function at early times. It is convenient to define a new function \( L(\alpha, \tau) = 2\alpha \tau F(\alpha, \tau) \) such that the measured temperature is given by

\[
T = \frac{q}{4\pi kt} L(\alpha, \tau) \tag{5}
\]

where \( L(\alpha, \tau) \) is a dimensionless correction factor. A careful examination of the properties of this function should indicate what can and cannot be done for the measurement of conductivity, to some specified accuracy, in a short time.

The first and most obvious problem is that \( \alpha \), the heat-capacity ratio, is not known. By considering the properties of typical ocean sediment, and comparing them to the heat capacity of a solidly constructed probe, it is easy to show that \( \alpha \) variations are unlikely to be a significant problem, at least in the ocean. The density of the solid fraction of ocean sediment was determined by density-curve matching to be 2.35 g cm\(^{-3} \) (Ratcliff 1960), and the specific heat estimated to be 0.182 cal/(g °C) (Bullard 1954). Representative values of the latter are 0.181 cal/(g °C) (dry red clay), 0.186 (calcium carbonate), 0.170 (silica) (Bullard 1954) confirming that this estimate is reasonable. The heat capacity of ocean sediment can be calculated from these figures as a function of water content by weight \( W \):

\[
\rho = \frac{W/100 \rho_L c_L + (1 - W/100)\rho_L c_S}{W/100 + (1 - W/100)\rho_L/\rho_S} = \frac{18.7 + 81.3 W}{43.8 + 56.2 W} \tag{6}
\]

assuming that \( \rho_L c_L \), the heat capacity of sea-water, is close enough to 1.000 cal/(g °C) for this estimation. A comparison table can now be constructed of sediment heat capacities and those of common materials that could be considered for probe construction.
Table 1.

<table>
<thead>
<tr>
<th>Sediment water content by weight</th>
<th>( \rho c_{\text{sed}} )</th>
<th>( \rho c_M )</th>
<th>Miscellaneous materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 (per cent)</td>
<td>1.000</td>
<td>0.95</td>
<td>Stainless steel</td>
</tr>
<tr>
<td>90</td>
<td>0.973</td>
<td>0.91</td>
<td>Carbon steel</td>
</tr>
<tr>
<td>80</td>
<td>0.943</td>
<td>0.81</td>
<td>Copper</td>
</tr>
<tr>
<td>70</td>
<td>0.909</td>
<td>0.81</td>
<td>Probe: 3/4 steel 1/4 oil, wires etc.</td>
</tr>
<tr>
<td>60</td>
<td>0.870</td>
<td>0.73</td>
<td>Brass</td>
</tr>
<tr>
<td>50</td>
<td>0.825</td>
<td>0.50</td>
<td>Mineral oil</td>
</tr>
<tr>
<td>40</td>
<td>0.773</td>
<td>0.47</td>
<td>Pyrex glass</td>
</tr>
<tr>
<td>30</td>
<td>0.710</td>
<td>0.42</td>
<td>Lucite</td>
</tr>
<tr>
<td>20</td>
<td>0.635</td>
<td>0.34</td>
<td>Polyvinyl chloride</td>
</tr>
<tr>
<td>10</td>
<td>0.543</td>
<td>0.32</td>
<td>Polystyrene</td>
</tr>
<tr>
<td>0</td>
<td>0.427</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The usual variation of water content in ocean sediment is between 40 and 60 per cent, so that \( \alpha \), for a probe of 75 per cent steel and 25 per cent oil, varies between 1.90 and 2.14. It now remains to examine the effect of this on the function \( L(\alpha, \tau) \).

The correction factor \( L(\alpha, \tau) \) is plotted against \( \tau \) for three values of \( \alpha \) in Fig. 1 (upper). Data are from Huppert & Sclater (1968). Although they are almost coincident at \( \tau = 1 \), the three curves demonstrate a consistent variation over the useful range of times \( 3 < \tau < 8 \). The variation is highlighted in Fig. 1 (lower), where percentage ratio deviations from the standard \( \alpha = 2 \) curve are plotted. Substantial changes in \( \alpha \) produce only small deviations of the \( L \)-function, and the effect of the expected range of sediment heat capacities should be

**Figure 1.** Upper. Plot of \( L(\alpha, \tau) = 2\alpha \tau F(\alpha, \tau) \) against dimensionless time \( \tau \) for three different values of the thermal-capacity ratio \( \alpha \). Data from Huppert & Sclater (1968). Lower. Plot of percentage deviations of the ratio \( L(\alpha, \tau)/L(2, \tau) \) against \( \tau \) for two different values of \( \alpha \).
limited to less than 1 per cent. Unless highly unusual sediments are encountered, the heat capacities need not be estimated, and analysis based on the \( a = 2 \) curve can be applied with confidence.

The effect of poorly known \( \tau \) is significant in the \( 3 < \tau < 8 \) range, since \( L(2, \tau) \) varies from 0.85 to 0.94 and \( \tau \) must be determined to an accuracy of about \( \pm 10 \) per cent if the conductivity is to be found to 1 per cent accuracy. There are two possible approaches to estimating \( \tau \): iteration of the conductivity determination using some curve or table of mean sediment properties, and direct use of the temperature versus time curve. The first approach requires only knowledge of the probe radius and a relationship between conductivity and heat capacity for typical ocean sediments. In the hope of making the method more general, rather than restricted to ocean sediments, it is worth examining whether there is sufficient information in the temperature curve itself to estimate \( \tau \) to the required accuracy. To analyse the \( F(2, \tau) \) function to this extent, it is convenient to have a table at much closer intervals than those published by Bullard (1954) and Huppert & Sclater (1968). Considerable effort would need to be expended to re-run the programs for direct calculation of the \( F \)-function, and therefore the table was obtained by interpolating the prior published data, Bullard's for the early times, and Hupper & Sclater's for the later times. The method used was Bessel's interpolation formula incorporating modified second differences (Hartree 1955). The table is presented in Appendix A; since we have found it extremely useful, it may also be of value to others working in this field.

The information in a measured temperature curve on the value of \( \tau \) is contained in the way the curve deviates from the asymptote. It is possible to express this quantitatively in a number of ways, but one very simple one should suffice. A measure of the curvature is contained in the function

\[
R(\theta, 2, \tau) = \frac{\theta F(2, \theta \tau)}{F(2, \tau)}
\]  

(7)

where the \( R \)-ratio tends downwards toward 1.000 as \( \tau \rightarrow \infty \). Convenient values of \( \theta \) are 2 and 3; the lower value allows estimation from a relatively short time series of data, while the higher value is better for curves that only become free of extraneous error at relatively long times (see later discussion). The principal interest in this exercise is to predict \( L \), not specifically to find \( \tau \). Hence, the results are presented in Fig. 2 in the form of a graph comparing \( R \) and \( L \), with \( \tau \) simply a parameter that varies non-linearly along the curves. The upper curve, for \( \theta = 3 \), allows \( \tau \) to be discriminated to reasonable accuracy at times as late as \( \tau = 3 \) to 4, and has almost a 45° slope against \( L \). An error in \( R \) would thus propagate directly to a similar fractional error in \( L \), but a relatively long time series is needed: to \( \tau = 9 \) even for a beginning time at \( \tau = 3 \). Overall, if temperature curves can be obtained that conform accurately to the \( F \)-function at least by \( \tau = 3 \), and extend beyond \( \tau = 6 \), then \( L \) can be determined to the desired 1 per cent accuracy.

Another possible source of error is the finite length of a practical heat pulse. How short does it have to be, not to introduce significant error at these relatively early times? A first approach is simply to compute the error in the first term of the power series expansion of \( F(\alpha, \tau) \), since this can be done analytically without difficulty:

\[
A_{AV} = \frac{1}{2\Delta} \int_{\tau_0}^{\tau_0 + \Delta} \frac{d\tau}{2\alpha} = \frac{1}{\alpha \tau_0} \left( \frac{\Delta^2}{2 \tau_0} + \cdots \right).
\]  

(8)

The magnitude of the error can be gauged by taking \( \Delta = 0.2 \), for a heater pulse 0.4 long in \( \tau \) units (Table 2).
Figure 2. Plot of $L(2, \tau)$ (ordinate) against $R(\theta, 2, \tau)$ with $\tau$ as the along-line parameter. Upper curve: $\theta = 3$; lower curve: $\theta = 2$. $R$ is a convenient function of $\tau$ that is readily obtainable from a thermal decay curve: $R = \theta F(2, \theta \tau)/F(2, \tau)$.

The results of the asymptotic correction are cause for a little concern, even at $\tau_0 = 2.8$, but, being a hyperbolic function, it may exaggerate the deviations at early times. It is possible to make a good estimate for the $F$-function, if that function is fitted by a parabola over the short range of time corresponding to the heater pulse:

$$F \approx F(2, \tau_0) - \alpha \Delta \tau + \beta \Delta \tau^2$$

$$F_{AV} = \frac{1}{2\Delta} \int_{\tau_0 - \Delta}^{\tau_0 + \Delta} F d\tau \approx \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} [F(2, \tau_0) - \alpha \Delta \tau + \beta \Delta \tau^2] d(\Delta \tau) = F(2, \tau_0) + \frac{\beta \Delta^2}{3}.$$  

These results are given also in Table 2, and show that the effect of the finite heater pulse can be neglected by the time $\tau = 3$, and would require only a modest correction when $\tau = 1$.

Finally, one should examine the effect of an origin-time error on the $R$-ratio method of determining the correction factor $L$: the effect of such an error on $k$ determination by the asymptotic curve is obvious. This has been done by direct computation, using an arbitrary time offset $\Delta \tau = \pm 0.1$ and the table in Appendix A. It should be noted that this would be an unacceptable time error for $k$ determination by the asymptotic curve before $\tau = 10$, at which point the direct error in $L$ would be negligible also (less than 0.1 per cent), but a large

<table>
<thead>
<tr>
<th>Centre time $\tau_0$</th>
<th>1</th>
<th>2</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviation of asymptote (per cent)</td>
<td>1.8</td>
<td>0.33</td>
<td>0.17</td>
</tr>
<tr>
<td>Deviation of $F(2, \tau)$ (per cent)</td>
<td>0.67</td>
<td>0.22</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Conductivity measurement

Table 3.

<table>
<thead>
<tr>
<th>Nominal τ</th>
<th>Change in L due to 0.1 time error* (per cent)</th>
<th>Direct effect on k calculation, same error (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 3 5</td>
<td>4½ 3 2</td>
</tr>
</tbody>
</table>

* Using R(2, 2, τ).

change is needed to reduce the effect of possible rounding errors in the table. The percentage errors in L and in the direct calculation are presented in Table 3.

The effect of an origin time error on the estimation of \( L(2, \tau) \) by the curvature method is thus the same as the simple error in \( t \); as they add rather than cancel, the application of the self-contained method of \( \tau \) estimation merely doubles the total conductivity error produced by an origin-time error.

The results of this detailed investigation of the theory of the pulse-probe method show that it is highly promising for fast and accurate conductivity measurements in soft solids. It is not particularly sensitive to variations in heat capacity of the medium, and it is possible to estimate diffusivity from the temperature curve itself to a sufficient accuracy for correcting the early-time deviation. The power of the method is that one is not attempting to measure the slope of a temperature curve, as in the needle-probe method. The calculation merely requires one or two measurements of temperature at known times. The differentiation has been done already by the application of a heat pulse instead of the steady source, and this inherently shortens and simplifies the determination.

Practical test

The pulse-probe method of conductivity measurement was tried out on a cruise of Endeavor in 1976, off the west coast of Canada. A new telemetering instrument was put together for that expedition, incorporating advances both in electronic and in mechanical design. Those aspects of the instrument are discussed briefly in Hyndman et al. (1978); of concern here are only the results of conductivity measurements and the thermal design of the sensor string. The station selected for analysis is the best of the two early conductivity penetrations in the hemi-pelagic sediment fan west of the Queen Charlotte Islands (Q5, P1: 51° 23.81'N, 134° 24.58'W 3256 m (uncorr.)). As it was an early test, the instrument was left undisturbed to obtain the longest record thought to be useful (through \( \tau = 8 \)), and the surface sediments in this relatively deep area should be the most uniform encountered on the cruise.

The sensor string contains three single-point thermistors for gradient determination and two linear arrays of nine thermistors to measure average thermal resistivity between the gradient sensors. Each array has three sets of three series sensors connected in parallel to be electrically equivalent to a single thermistor but responsive to average temperature. Data from the upper averaging sensor and the lowest point sensor were selected for analysis, to provide a test for conformance to the theory in the presence of possible end effects and longitudinal inhomogeneity of the sediment properties. Equilibrium temperatures were determined by fitting the temperature decay from the initial frictional rise to the \( F(2, \tau) \) function, as described by Lister (1970). Raw temperature data from the electrical heat pulse were then corrected by subtracting the expected residuals from the frictional heat decay. Since the calibrated heat pulse is applied when the frictional \( \tau \) is between 4.5 and 5, the corrections amount to a few thousandths of a degree only. The preliminary corrected data are plotted against time, scaled by \( F(2, \tau) \), as the heavy symbols in Fig. 3. Curvature in what should be straight lines is obvious, and the reductions were checked carefully against errors
in origin time. The curvature is unaffected by the change in assumed probe time-constant plotted for the upper sensor, and could not be corrected by a change to any reasonable time constant.

It is clear that some phenomenon other than probe-sediment thermal decay is affecting the results, and obvious possibilities are the thermal time constants associated with the internal structure of the sensor string itself. The internal diameter of the sensor tube is small (4 mm), and the pull-through length is substantial. In the practical string design that evolved over the years since Lister (1970), the thermistors, the wires to the thermistors, and the separately sleeved heater wires are all bundled together inside tough shrinkable insulation. The shrinkable tube is filled with epoxy gel, and the space between the string and the steel tube is filled with mineral oil after assembly. Thus, there is good thermal contact between the heater wire and the thermistors, but a substantial thickness of poorly conducting insulation between the metallic core and the outer steel tube, especially longitudinally between the thermistors where the wires made a very small bundle. Although there are many important thermal paths during and immediately after the calibrated heat pulse, the longest time constant is clearly that between the small wire bundle and the steel tube. Since the longitudinal conductance of the copper is high, and transfer to the steel wall will occur preferentially at a few places where the thermal resistance is low, there is a possibility that the longest thermal decay can be modelled as thermal flow from a metal reservoir (the wires) through a thermal resistance (the thinnest insulation). This would result in the decay being exponential over some range of time, with a time constant in the 10–100 s range.

When the best straight lines, tangential to the last part of the curves (τ > 4), had been hand-fitted to plots like Fig. 3, the residuals were calculated from the equation of the line and plotted on semi-logarithmic graph paper. A substantial portion of the curve thus obtained was straight, and a preliminary exponential correction was subtracted from the temperature data to improve the fitting of the straight line. The deviations from this second

Figure 3. The sensor temperature rise above the equilibrium temperature, corrected for the residual entry disturbance, plotted against the dimensionless \( F(2, \tau) \). The 'time constant' quoted is the conversion factor between real time and dimensionless time: \( \tau = \kappa t / a^2 \), where the probe radius \( a = 5 \text{ mm} \), and \( \kappa \) is the sediment diffusivity. Filled symbols: data as recorded; open symbols: data corrected by the best-fit exponential from Fig. 4. Heat impulse about 0.35 long in \( \tau \) units, value 4.57 cal/cm.
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Figure 4. Plot of the difference between observed temperature and the best straight line fitted to the longest-time data on a graph like Fig. 3. One iteration of replotting corrected temperature against $F(2, \tau)$ has been made to refine the equation of the line. A straight line in this semi-logarithmic format is equivalent to an exponential temperature decay: slopes are shown by the inset.

Iteration are plotted semi-logarithmically in Fig. 4. From 100 s until digital noise scatters the data, the points fall on straight lines, with a time constant of about 40 s for the lower sensor and 45 s for the upper sensor. The fit for the upper sensor is slightly better for the 80 s probe-sediment time constant than for 70 s, but the slopes are similar, confirming that the exponential decay is real. The apparent initial temperature for the decay, about 40°C above ambient, is consistent with earlier data from the steadily heated probe (Lister 1970) and the ratio of the power densities used by the two methods.

The final corrected temperature data is shown by the unfilled symbols in Fig. 3. The points fall on straight lines within the plotting accuracy except for a glitch at $\tau = 2.7$, probably due to a minor mechanical disturbance of the instrument, and thus conform to the $F$-function at times as early as $\tau \sim 2$. The internal thermal decay problems appear to be the limiting factor in the application of the method: estimates of $\tau$ by the $R$-ratio method are distorted by the exponential curvature, and a direct calculation of $k$ is also in error due to the extra temperature rise. Although steps would normally be taken to correct the error for measurements made with this instrument, and it is hoped to eliminate the problem in future instruments by redesign of the sensor string, it is instructive to examine how large an error would actually be introduced by ignoring the correction.

Using the temperature measurement at $\tau \sim 3$ to calculate $k$, produces a $-2$ per cent error because the temperature is 2 per cent too high. The $R(2, 2, 3)$ ratio is also 2 per cent low, giving an apparent $\tau$ value of 4 and an estimate of $L$ 4 per cent too high. Thus the final error in $k$, due to the application of the self-contained method of calculating both $k$ and $\tau$ from two temperature samples, is $+2$ per cent. It is true that a similar accuracy could be obtained by simply assuming $\tau = 6$ and using the temperature rise there alone to calculate $k$, since the time estimate for $\tau = 6$ would have to be off by 20 per cent to produce 2 per cent error (Fig. 2). However, the result actually confirms the power of the method, since the error...
introduced by obviously unacceptable deviations is smaller than that normally associated with the needle-probe, continuous-heating, method (Von Herzen & Maxwell 1959).

The temperature 'glitch' at $\tau \approx 2.7$, mentioned above, brings up another aspect of the problems in routine reduction of field measurements in a largely uncontrolled environment. Such glitches are typical of ocean-floor measurements, although normally absent in laboratory and borehole data. The simple method of direct reduction, by taking temperature measurements at two arbitrary times, is susceptible to serious error due to glitches before the nature of the results would call attention to that particular computed value. A possible scheme for improved computer reduction would be to pick several different pairs of times (or simply times if $\tau$ is estimated externally) and proceed with the calculation of $k$s for all of them. The results can then be binned into a histogram and tested for conformance to the known precision of the method. If the removal of values differing too much from the mean does not produce a satisfactory subset of the data (this is a form of 'majority voting'), the record can be flagged for manual interpretation.

Conclusions

The experimental test of the pulse-probe conductivity method has verified that data conforming to the $F(\alpha, \tau)$ function can be obtained, subject to adequate recovery of the internal probe structure from the high-temperature transient. If this problem can be resolved, viable measurements, to 1 per cent nominal accuracy, can be obtained in a total probe immersion time to $\tau = 9$, allowing to $\tau = 4$ to 5 for the decay of the initial entry transient. This is about half the total immersion time needed for an in situ measurement by the needle probe technique and the heating energy requirement is similar (Lister 1970). Accuracy of the results is potentially greater, with the same temperature measurement accuracy in the instrument. Early-time corrections can be made by a self-contained method that does not require some external estimate of diffusivity, and thus probe time constant, giving the method potential for applications other than on the ocean-floor.

A final consideration in the more general application of the self-contained method is its sensitivity to variations in $\alpha$. This has not been treated here because it was shown that significant $\alpha$ variations were not expected on the ocean-floor, and because the wide range of fundamental data needed to interpolate a good table are not available for $\alpha = 1.5$ or 2.5. Calculation of a few points, and plotting them on a graph similar to Fig. 2, shows that the $R$ versus $L$ lines diverge from those for $\alpha = 2$. A 25 per cent variation in $\alpha$ produces an error of about 3 per cent in $L$ at $R(2, \alpha, \tau) = 1.08$ ($\tau \sim 3$) and about 2 per cent at $R = 1.03$ ($\tau \sim 9$), showing that convergence is slow at longer measurement times. The relative insensitivity of the results to $\alpha$ would make the method acceptable when the highest $k$ accuracies are not required. Wherever there is a correlation between conductivity and heat capacity, a simple iteration of the results would produce sufficient convergence. It would probably be worthwhile to develop full tables of $F(\alpha, \tau)$ for other values of $\alpha$, and generate the plots corresponding to Fig. 2, but with $\alpha$ rather than $\theta$ as the variable parameter.

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References


Note

Appendix A has not been printed: copies of the \( F(\alpha, \tau) \) table are available from the author. Enclose US $1 with your request to cover postage and handling.