A Dynamical Approach to the Scaling Violations

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The nucleon structure functions are evaluated by making use of the approximate solution of the B-S equation derived previously by one of the authors (R.H.) on the basis of a quark-gluon model. It can be seen that the Bjorken scaling is violated in this scheme and $Q^2$-dependence of the scaling violation terms obeys a power-law, while the Callan-Gross relation holds to the scaling part. The results of the analyses strongly suggest that each constituent of the hadron must have a non-point-like substructure.

§ 1. Introduction

Recently, the experiments by the SLAC-MIT collaboration have given us new information on the validity of the Bjorken scaling in the electroproduction phenomena. As it has been shown by many authors, the Bjorken scaling can be derived from the conventional composite model of hadrons, i.e., the quark-parton model and some field theoretic models. However, the experiments reveal that the structure functions, $W_1$ and $2MW_1$, have a characteristic pattern of $Q^2$-dependence.

Prior to the experiments, on the basis of a compositeness of constituents specified by the size and the excited states of quarks, Matumoto has suggested a model of field theory at the sub-constituent level and has pointed out a possibility of scaling violation. There he introduced heuristically the form factors characterized by the size and the excited states. After the new experiments were reported, several authors have tried to understand the characteristic features of the new experiments.

Needless to say, the dynamical structure at subhadronic level must play an important role in understanding the mechanism of scaling properties. Indeed, to disclose properties of hadron we have to examine the effects of the subhadronic interactions among constituents. So far, however, there has been no direct approach to the structure of quark-antiquark scattering amplitude. In order to calculate the scattering amplitude, one of the authors (R.H.) has derived, on several assumptions, an approximate solution of the Bethe-Salpeter (B-S) equation which involves exchange of neutral vector gluons between quarks.

Let us estimate the nucleon structure functions to investigate the effect of the constituent interaction on the hadrons. In connection with the wave function of nucleon, we introduce an interaction vertex between the nucleon and $i$-th quark shown in Fig. 1. The vertex caused by this interaction is related to an amplitude...
of the $i$-th quark emitted from the nucleon, which corresponds to a distribution function in quark-parton model. Thus, the vertex function can be expressed as

$$V_i(P; p_1, p_2, p_3, p') = f_i(P; p_1, p_2, p_3) u_i(p_i), \quad (1\cdot 1)^\ast$$

where $P, p_i$ and $p'$ stand for the 4-momenta of nucleon, quark and gluon, respectively. These 4-momenta satisfy the condition,

$$P^2 = M^2, \quad P = p' + \sum_{i=1}^3 p_i \quad (1\cdot 2)$$

and in Eq. (1·1) $u_i(p_i)$ represents a spinor of the $i$-th quark. Furthermore, as to the constituents we make the following two postulates:

(i) The masses of constituents ($m_i$ and $\mu$ stand for quark mass and gluon mass, respectively) are small.

(ii) $p_i^2$ lies in a small interval such that $(p_i^2)^{1/2} \ll M$ (nucleon mass).

In the present paper, on the basis of our previous formulation\(^\ast\) to concrete processes, we shall clarify the mechanism of scaling violation. In § 2, we shall express the nucleon structure functions in terms of the quark structure functions by using the B-S scattering amplitude. Section 3 is devoted to an analysis of $\nu W_2$ and $2MW_1$ and comparison with the experimental data.\(^9\) The result is that a $(Q^2)\gamma$-term appears in the scale breaking part of $\nu W_2$ and $2MW_1$, and the scaling part has a simple form like $\omega - 1/\omega$. The nucleon structure functions calculated in our scheme are in good agreement with experiments. In § 4, we shall give concluding remarks.

§ 2. Constituent structure function

In this section, we shall calculate the structure functions of $i$-th quark and then show that the quark has a composite substructure.

The nucleon ST, $W_{\nu \sigma}(P, q)$ can be expressed in terms of the imaginary part of the virtual Compton scattering amplitude. With the help of the B-S scattering amplitude the relation between the quark ST, $W_{\nu \sigma}(p_i, q)$ and $W_{\nu \sigma}(P, q)$ in our scheme (Fig. 2) is given as follows:

$$W_{\nu \sigma}(P, q) = \int \sum_i |f_i(P; p_1, p_2, p_3)|^2 W_{\nu \sigma}(p_i, q)$$

$$\times \delta^4(P - p' - \sum_{i=1}^3 p_i) \prod_{j=1}^3 d^4p_j d^4p', \quad (2\cdot 1)$$

$$W_{\nu \sigma}(p_i, q) = \frac{e_i^2}{(2\pi)^4} \int d^4k \left[ \frac{(k + m_i) \gamma^\nu (k + q + m_i) \gamma^\sigma (k + m_i)}{(k^2 - m_i^2)^2} \right].$$

\(^*\) Note that $f_i(P; p_1, p_2, p_3, p')$ represents a spinor. The quark spinors other than that of $i$-th quark may be included in $f_i$, if necessary.
Fig. 2. (a) Dominant diagram for virtual Compton scattering when quarks scatter each other exchanging gluons between them. (b) A diagram of virtual Compton scattering between $i$-th quark $p_i$ and virtual photon $q$.

$$\times \delta (m_i^2 - (k + q)^2) \text{Im} (T_{abcd}(p_t, k) \bar{u}_{i,a}(p_t) u_{i,b}(p_t)),$$

where $e_i$ denotes the charge of the $i$-th quark. In the above equation quark-antiquark scattering amplitude$^*$ is represented by $T_{abcd}(p_t, k)$, which can be expressed in terms of 16 independent $\gamma$-matrices, i.e., in the Fierz expansion, as follows:

$$T_{abcd}(p_t, k) = \frac{1}{16} \sum_{A,B=1}^{16} (\gamma^A)_{a,c} (\gamma^B)_{d,b} (\gamma_A)_{c,a'} (\gamma_B)_{b',a'} T_{a'c'c'c'}(p_t, k).$$

Then Eq. (2.2) is rewritten as

$$W^{\nu\nu}(p_t, q) = \frac{e^2}{16} \sum_{A,B=1}^{16} \bar{u}_i(p_t) \gamma^B u_i(p_t) \int \frac{d^4k}{(2\pi)^4} \frac{\delta (m_i^2 - (k + q)^2)}{(k^2 - m_i^2)^2} \times \text{Im} \left[ (\gamma_A)_{a,c'} (\gamma_B)_{b',a'} T_{a'c'c'c'}(p_t, k) \right] \text{Tr} [\gamma^A(k + m_i) \gamma^B(k + m_i)] \times \gamma^\nu (k + q + m_i) \gamma^\nu (k + m_i).$$

The amplitude $T_{abcd}(p_t, k)$ satisfies an integral equation,

$$(\gamma^A)_{ea} (\gamma^B)_{bd} T_{abcd}(p_t, k) = ig^2 \text{Tr} (\gamma^A_{i'a'} \gamma^B_{i'b'}) \left( p_t - k \right)^2 - \mu^2$$

$$- ig^2 \int \frac{d^4l}{(2\pi)^4} (\gamma^B)_{bd} T_{b'c'd'}(p_t, l) \left[ (l + m_i) \gamma^A_{i'a'} (l + m_i) \right] \gamma^\nu (l + m_i) \gamma^\nu (l + m_i).$$

We assume that the form of the interaction Hamiltonian be usual, i.e.,

$$H_i \sim g \bar{\psi} i \gamma^\mu \tau^\mu \psi.$$
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1.7

1.1

tensor \( T \) and pseudoscalar \( P \) give rise to only small contributions of order \( O(m_t^2) \) to the ST due to the postulates (i) and (ii). Similarly, \( S \) and \( V \) should be chosen for \( \gamma^a \) corresponding to \( S \) and \( V \) for \( \gamma^A \), respectively.\(^1\) Thus the relevant cases are the following two:

(I) \( \gamma^A = I \), \( \gamma^B = I \) (S-S part)

(II) \( \gamma^A = \gamma^g \), \( \gamma^B = \gamma^p \) (V-V part).

Since it can be shown that the V-V part gives a more important contribution than the S-S part to \( W^{\mu \nu}(p_i, q) \), we may consider only the case (II). (The S-S part contribution can be easily estimated. The result is shown in Fig. 5.)

The approximate solution for the V-V part of the B-S equation (2·5) obtained in the previous paper\(^5\) is as follows:

\[
\langle \gamma^A \rangle_{\gamma^B} = -8i g^4 (k^2 - m_t^2) f^\mu_\nu (p_i, k),
\]

\[
f^\mu_\nu (p_i, k) = \left[ a(\lambda_0, \lambda) g^\mu_\nu + b(\lambda_0, \lambda) \frac{k^\mu k^\nu}{k^2} \right] f(p_i, k; \sqrt{\lambda}),
\]

\[
f(p_i, k; \sqrt{\lambda}) = \frac{1}{\pi^{3/2} \lambda^{1/4} (\mu^2)^{3/4}} \int_0^\lambda dy \int_0^\infty d\gamma \frac{\gamma^{1/2} y^{-2/3} (\ln(1/y))^{-1/2}}{[\gamma + (1-y)(m_t^2 - k^2) + y (\mu^2 - (p_i - k)^2)]^{3/2}},
\]

where \( a(\lambda_0, \lambda) \) and \( b(\lambda_0, \lambda) \) are parameters depending on \( \lambda_0 (= g^4/4\pi^2) \) and \( \lambda \) is the effective coupling constant to be determined as the function of \( \lambda_0 \). As shown in the previous paper,\(^5\) \( \sqrt{\lambda} \approx 0.5 \) leads to the following values of the ratio \( a/b \),

\[
(1) \quad b \approx -2.26a \quad \text{for} \quad \lambda_0 \approx 1.23,
\]

\[
(2) \quad b \approx 0.30a \quad \text{for} \quad \lambda_0 \approx 1.0. \quad (2·10)
\]

It may be plausible to take the deep inelastic region as

\[
y = p \cdot q \to \infty, \quad Q^2 \to \infty, \quad \frac{2y}{Q^2} = \omega \quad \text{(fixed)} \quad (2·11)
\]

because they correspond to the Bjorken limit

\[
y = p \cdot q \to \infty, \quad Q^2 \to \infty, \quad \frac{2y}{Q^2} = \omega \quad \text{(fixed)}. \quad (2·12)
\]

Since it is not so easy to perform the \( k \)-integration in Eq. (2·4) due to the factor \( \delta(m_t^2 - (k+q)^2) \), we rewrite the variable \( k \), following the method of Landshoff et al.\(^6\) in the form,

\[
k = Xp_i + Yq + \kappa, \quad (2·13)
\]

where \( \kappa \) satisfies \( \kappa p_i = \kappa q = 0 \) and represents effectively a two-dimensional space-like

\(^1\) See the remark given by Ref. 8.)
vector. Then, in the deep inelastic region shown in Eq. (2·11), \( \delta^i k \) is expressed by

\[
d^i k = v_i dXdY d^i \kappa.
\] (2·14)

By making use of Eqs. (2·7), (2·8), (2·9), (2·13) and (2·14) we can evaluate \( W_i^{\mu\nu}(p_\mu, q) \).

\[
W_i^{\mu\nu}(p_\mu, q) = -g^{\mu\nu} W_{i,1}(\omega_i, Q^2) + p_\mu^* p_\nu^* W_{i,2}(\omega_i, Q^2) + \cdots,
\] (2·15)

where

\[
W_{i,1}(\omega_i, Q^2) = 8 \alpha_i \lambda_0 \int dXdY d^2 \kappa \frac{\nu_i \delta (m_i^2 - (k + q)^2)}{(k^2 - m_i^2)} f(p_\mu, k; \sqrt{\lambda})
\times \left[ a(XY^2 \nu_i^2 - Y^2(Y + 1)Q\nu_i) - b(XY(2Y + 1)\nu_i^2 - Y^2(Y + 1)Q\nu_i) \right],
\] (2·16)

\[
W_{i,2}(\omega_i, Q^2) = 8 \alpha_i \lambda_0 \int dXdY d^2 \kappa \frac{\nu_i \delta (m_i^2 - (k + q)^2)}{(k^2 - m_i^2)} f(p_\mu, k; \sqrt{\lambda})
\times \left[ aX(X^2 p_i^2 + Y^4 Q^2 - \kappa^2) - bX^2(Xp_i^2 + Y\nu_i) \right]
\] (2·17)

and

\[
\alpha_i = e_i^2 / 4\pi. \hspace{1cm} (2·18)
\]

It is convenient to divide the \( k \)-integration in Eqs. (2·16) and (2·17) into the following two regions of \( k \)-variable:

One is the region where \((-k^2)\) becomes finite as \( Q^2 \to \infty \) and the other is the region where \((-k^2)\) tends to of order \( O(Q^2) \) for the deep inelastic region in Eq. (2·11). For the sake of convenience, we also express the ST, \( W_{i,1}(\omega_i, Q^2) \) and \( \nu_i W_{i,2}(\omega_i, Q^2) \) as:

\[
W_{i,1}(\omega_i, Q^2) = F_{i,1}(\omega_i) + G_{i,1}(\omega_i, Q^2),
\] (2·19)

\[
\nu_i W_{i,2}(\omega_i, Q^2) = F_{i,2}(\omega_i) + G_{i,2}(\omega_i, Q^2),
\] (2·20)

where \( F(\omega_i) \) and \( G(\omega_i, Q^2) \) represent the scaling term and the scaling violation, respectively.

Let us estimate \( W_{i,1}(\omega_i, Q^2) \) and \( \nu_i W_{i,2}(\omega_i, Q^2) \) in the first region of \( k \)-variable where the scaling will be found to hold. Owing to the condition satisfied by \( \kappa \) we obtain from Eq. (2·13)

\[
k^2 = X^2 p_i^2 + 2Y \left( X - \frac{1}{\omega_i} Y \right) \nu_i + \kappa^2. \hspace{1cm} (2·21)
\]

Introducing \( \bar{Y} \) by

\[
2 \nu_i Y = \bar{Y} - X p_i^2,
\] (2·22)

we can express \( F_{i,1}(\omega_i) \) and \( F_{i,2}(\omega_i) \) as follows:

\[
F_{i,1}(\omega_i) = 2 \alpha_i \lambda_0 \bar{F}(\omega_i),
\]
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\[ F_{i,2}(\omega_i) = 2\omega_i^{-1} F_{i,1}(\omega_i), \]  

(2.23)

\[ \tilde{F}(\omega_i) = \int_D d\tilde{Y} d^2 k f(p_i, k; \sqrt{\lambda}) \frac{\delta(p_i^2 - \omega_i^2 \cdot k^2) - b/2 (p_i^2 + \omega_i \tilde{Y})}{\omega_i^2 (m_i^2 - k^2 - \omega_i^{-1} \tilde{Y})}, \]  

(2.24)

where \( D \) stands for the integration region of \( \tilde{Y}, k^2 \) plane (Fig. 3(a)) and

\[ f(p_i, k; \sqrt{\lambda}) = \frac{B(1 + \sqrt{\lambda}, 2 - \sqrt{\lambda})}{\pi^{\frac{3}{2}} \lambda^{\frac{1}{4}} (\mu^2)^{\frac{3}{2}}} \frac{I(p_i, k; \sqrt{\lambda})}{(m_i^2 - k^2 - \omega_i^{-1} \tilde{Y})^{2 - \sqrt{\lambda}}}, \]  

(2.25)

\[ I(p_i, k; \sqrt{\lambda}) = \int_0^1 dy \frac{y^{-\nu / 2} (\ln (1/y))^{\nu - 1 / 2}}{(1 + 2p_i k y / (m_i^2 - k^2))^{2 - \sqrt{\lambda}}} \]  

(2.26)

Carrying out the integration, we obtain the explicit forms of \( \omega_i \) dependence,

\[ F_{i,1}(\omega_i) = (-a) \phi_i \left( \eta \omega_i - \xi \omega_i^{-1} \right), \]  

(2.27)

\[ F_{i,2}(\omega_i) = 2\omega_i^{-1} F_{i,1}(\omega_i), \]  

(2.28)

\[ b = (-a) r, \]  

(2.29)

where

\[ \phi_i = \alpha_i \lambda_0 \frac{B(1 + \sqrt{\lambda}, 2 - \sqrt{\lambda})}{\pi^{\frac{3}{2}} \lambda^{\frac{1}{4}} (\mu^2)^{\frac{3}{2}}} \frac{I(1 - 2\sqrt{\lambda})}{2(1 - \sqrt{\lambda}) (2 - \sqrt{\lambda})} \]  

(2.30)

\[ \eta = 2(2 - \sqrt{\lambda}) r \left[ 1 - (1 + A)^{\nu / 2 - 1} \right] - 2(1 - \sqrt{\lambda}) \]  

(2.31)

\[ \times \left( r + t + \frac{1}{2} rt \right) \left[ 1 - (1 + A)^{\nu / 2 - 2} \right] \]  

(2.31)

\[ \xi = (1 - \sqrt{\lambda}) \left( 1 + \frac{1}{2} r \right) \left[ 1 - (1 + A)^{\nu / 2 - 2} \right] 4 \cdot \frac{p_i^2}{m_i^2} \]  

(2.32)

and \( A(t) \) represents a cutoff for the square of 4-momenta (the transverse momenta) of scattered quark.

We shall consider the second region where the scaling violation appears. In this case, \( \tilde{Y} \) is introduced as

\[ 2y_i (Y + 1) = \tilde{Y} - Xp_i^2. \]  

(2.33)

Then the scale breaking terms \( G_{i,1}(\omega_i, Q^2) \) and \( G_{i,2}(\omega_i, Q^2) \) are expressed as

\[ G_{i,1}(\omega_i, Q^2) = 2\pi \alpha_i \lambda_0 \int_D dX d\tilde{Y} f(p_i, k; \sqrt{\lambda}) \frac{(Q^2)^2 X \omega_i^2 (a - (1/2) b)}{X \tilde{Y} + (1 + \omega_i X) Q^2}, \]  

(2.34)

\[ G_{i,2}(\omega_i, Q^2) = 4\pi \alpha_i \lambda_0 \int_D dX d\tilde{Y} f(p_i, k; \sqrt{\lambda}) \frac{(Q^2)^2 X \omega_i (a + (1/2) \omega_i X b)}{X \tilde{Y} + (1 + \omega_i X) Q^2}, \]  

(2.35)

\[^*) This approximation is found to be fairly good by the numerical integration.\]
Fig. 3. (a) The integration region of $\bar{Y}$-$x^2$ plane where the scaling parts are given. (b) The integration region of $X$-$\bar{Y}$ plane where the breaking parts are given.

where

$$f(p_t,k;\sqrt{\lambda}) \approx \frac{B(1+\sqrt{\lambda},2-\sqrt{\lambda})}{\pi^{3/2} \lambda^{1/4} (\mu^2)^{\sqrt{\lambda}}} \frac{I(1-2\sqrt{\lambda})}{[X\bar{Y}+(1+\omega_t X)Q^2]^{2-\sqrt{\lambda}}}$$  \hspace{1cm} (2.36)

and $\bar{D}$ is the integration of $X$-$\bar{Y}$ plane (Fig. 3 (b)). Carrying out the $X$-$\bar{Y}$ integrations, we obtain

$$G_{i,1}(\omega_t, Q^2) = -4(-a) \frac{\phi_t}{(m_t^2)^{\sqrt{\lambda}}} \omega_t \left(1 + \frac{1}{2} r\right) (Q^2)^{\sqrt{\lambda}} \left[1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda} - 1}\right],$$  \hspace{1cm} (2.37)

$$G_{i,2}(\omega_t, Q^2) = 8(-a) \frac{\phi_t}{(m_t^2)^{\sqrt{\lambda}}} (Q^2)^{\sqrt{\lambda}} \left\{ [1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda} - 1}] - \frac{1}{2} r \left(\frac{\sqrt{\lambda} - 1}{\sqrt{\lambda}} (1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda}}) - (1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda} - 1}) \right) \right\},$$  \hspace{1cm} (2.38)

where the new parameter $\tilde{A}$ represents a cutoff for the square of 4-momenta of $i$-th scattered quark, which is proportional to $Q^2$. Consequently the explicit forms of the ST of $i$-th scattered quark are written as follows:

$$W_{i,1}(\omega_t, Q^2) = (-a) \phi_t \left[t(\bar{\eta}\omega_t - \omega_t^{-1} \bar{\xi}) - 4 \left(1 + \frac{1}{2} r\right) \frac{\omega_t}{(m_t^2)^{\sqrt{\lambda}}} (Q^2)^{\sqrt{\lambda}} \left[1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda} - 1}\right]\right],$$  \hspace{1cm} (2.39)

$$W_{i,2}(\omega_t, Q^2) = 2(-a) \phi_t \left[t \left(\bar{\eta} - \frac{\bar{\xi}}{\omega_t}\right) - 4 \left(\frac{4}{(m_t^2)^{\sqrt{\lambda}}} (Q^2)^{\sqrt{\lambda}} \left[1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda} - 1}\right]\right) \right] - \frac{1}{2} r \left(\frac{\sqrt{\lambda} - 1}{\sqrt{\lambda}} (1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda}}) - (1 - (1 + \omega_t \tilde{A})^{\sqrt{\lambda} - 1}) \right).$$  \hspace{1cm} (2.40)
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The structure functions (2.39) and (2.40) manifest $Q^2$-dependence, so that we may introduce a form factor $\Gamma_i(\omega_i, Q^2)$. The situation may be illustrated in Fig. 4. By making use of this form factor, we may express the structure function $W_{i,1}(\omega_i, Q^2)$ as

$$W_{i,1}(\omega_i, Q^2) \sim \Gamma_i(\omega_i, Q^2)^2 \times \delta(x_i - \omega_i^{-1}). \quad (2.41)$$

Comparing (2.41) with $W_{i,1}(\omega_i, Q^2)$ given by (2.39), we may rewrite $\Gamma_i(\omega_i, Q^2)^2$ as

$$\Gamma_i(\omega_i, Q^2)^2 = f_i(\omega_i) + g(Q^2)f_i^*(\omega_i), \quad (2.42)$$

where

$$f_i(\omega_i) \delta(x_i - \omega_i^{-1}) \sim (-a) \phi_i t(\eta \omega_i - \xi \omega_i^{-1}), \quad (2.43)$$

$$f_i^*(\omega_i) \delta(x_i - \omega_i^{-1}) \sim -4(-a) \left(1 + \frac{r}{2}\right) \phi_i t \left[m_i^2 \left[1 - (1 + \omega_i A) \right] \right]$$

$$\quad \times \delta(x_i - \omega_i^{-1}). \quad (2.44)$$

and

$$g(Q^2) \sim (Q^2)^{\nu_4}. \quad (2.45)$$

To conclude this section, we would like to make the following remark. The Callan-Gross relation,

$$\omega_i F_{i,2}(\omega_i) = 2 F_{i,1}(\omega_i) \quad (2.46)$$

holds in the constituent level and the scaling violation is induced by the $(Q^2)^{\nu_4}$-term in $G_i(\omega_i, Q^2)$.

§ 3. Nucleon structure functions and parametrization

In this section, we shall give explicit forms of the nucleon ST by using Eqs. (2.39) and (2.40), and analyse them by comparing them with the experimental data. To obtain these expressions, we need to know the relation between $\omega_i$ and $\omega$.

We adopt throughout this paper the following values of $m_i$:

$$m_p = m_n = M/3 \ (\approx 0.3) \text{ GeV}, \quad (3.1)$$

although rather arbitrary values of the quark mass have been predicted by various theoretical works. Three quarks and gluon are presumed to pick up fractions,

* As for charm quark mass, $m_c \approx 2.0 \text{ GeV}$ is used.
\( p_i = x_i P \) and \( p' = z P \) of the nucleon momentum, respectively. Their transverse momenta are neglected, for the sake of simplicity. So \( \omega_i \) can be related to \( \omega \) as

\[
\omega_i = x_i \omega .
\]  
(3.2)

The nucleon ST, \( 2MW_1 \) and \( \nu W_2 \) are expressed in terms of those of constituents as follows:

\[
2MW_1(\omega, Q^2) = 2M \int \sum_{i=1}^{3} h_i(x_i) W_{i,1}(\omega_i, Q^2) \, dx_i, \tag{3.3}
\]

\[
\nu W_2(\omega, Q^2) = M^2 \int \sum_{i=1}^{3} h_i(x_i) x_i \nu_i W_{i,2}(\omega_i, Q^2) \, dx_i, \tag{3.4}
\]

where \( h_i(x_i) \) is defined by

\[
h_i(x_i) = \int |f_i(P; p_i, p_{i'}, p_j, p_{j'})|^2 \delta^4(P - p' - \sum_{i=1}^{3} p_i) \, dp_i \, dp_{i'} \, dp_j \, dp_{j'}. \tag{3.5}
\]

Here the subscripts \( i, j \) and \( k \) run from 1 to 3, denoting one of three quarks.

According to the analysis of the quark-parton model, the quarks carry approximately half the nucleon momentum. Assuming the longitudinal momentum distribution of each quark to be identical with one another, we can get \( x_i = 1/6 \). Then we shall take the following simple form for the distribution function \( h_i(x_i) \):

\[
h_i(x_i) = f_i(x_i) \delta \left( x_i - \frac{1}{6} \right). \tag{3.6}
\]

Owing to such a simplification one can reduce Eqs. (3.3) and (3.4) to

\[
2MW_1(\omega, Q^2) = 2M \sum_{i=1}^{3} f_i \left( \frac{1}{6} \right) W_{i,1} \left( \frac{\omega}{6}, Q^2 \right), \tag{3.7}
\]

\[
\nu W_2(\omega, Q^2) = M^2 \sum_{i=1}^{3} f_i \left( \frac{1}{6} \right) \nu_i W_{i,2} \left( \frac{\omega}{6}, Q^2 \right). \tag{3.8}
\]

Substituting Eqs. (2.39) and (2.40) into the above equations, we obtain the explicit forms of \( 2MW_1 \) and \( \nu W_2 \),

\[
2MW_1(\omega, Q^2) = F_1(\omega) + G_1(\omega, Q^2), \tag{3.9}
\]

\[
\nu W_2(\omega, Q^2) = F_2(\omega) + G_2(\omega, Q^2), \tag{3.10}
\]

where

\[
F_1(\omega) = \alpha \omega - \beta \omega^{-1}, \tag{3.11}
\]

\[
F_2(\omega) = \omega^{-1} F_1(\omega), \tag{3.12}
\]

\[
G_1(\omega, Q^2) = - A(\omega) \frac{1}{2} \left( 1 + \frac{1}{2} r \right) \left( 1 - \frac{1}{\sqrt{1 + \left( \frac{\omega}{3} \right)}} \right), \tag{3.13}
\]

\[
G_2(\omega, Q^2) = - A(\omega) \frac{1}{2} \left[ \left( 1 - \frac{1}{\sqrt{1 + \left( \frac{\omega}{3} \right)}} \right) - \frac{1}{2} r \left( \sqrt{1 + \left( \frac{\omega}{3} \right)} + \frac{1}{\sqrt{1 + \left( \frac{\omega}{3} \right)}} \right) \right]^{1/2}. \tag{3.14}
\]
and
\[ \alpha = \frac{1}{3} (-a) t \phi , \quad \beta = 12 (-a) t \phi , \]
\[ \phi = \sum_{i=1}^{3} \phi_i , \]
\[ A = 4 (-a) \phi . \]

In order to compare them with the experimental data, the values of \( \sqrt{\lambda} \) and \( \bar{\lambda} \) are assumed to be \( 1/2^{**} \) and \( 2,^{(* *)} \) respectively.

Furthermore, assuming
\[ \alpha = 0.283 , \quad \beta = 0.416 , \]
\[ A = 0.0287 , \quad Ar = 0.0075 , \]
we express \( F_1, F_2, G_1, G_2 \) by using two parameters \( \omega \) and \( Q^2 \) as
\[ F_1(\omega) = 0.283\omega - \frac{0.416}{\omega} , \]
\[ F_2(\omega) = \omega^{-1} F_1(\omega) , \]
\[ G_1(\omega, Q^2) = -0.0325\omega (Q^2)^{1/2} \left( 1 - \frac{1}{\sqrt{1 + (\omega/3)}} \right) , \]
\[ G_2(\omega, Q^2) \]
\[ = -0.0287 (Q^2)^{1/2} \left( 1 - \frac{1}{\sqrt{1 + (\omega/3)}} \right) + 0.0037 (Q^2)^{1/2} \left( \sqrt{1 + \frac{\omega}{3}} + \frac{1}{\sqrt{1 + (\omega/3)}} - 2 \right) . \]

In what follows, we shall see the scaling properties obtained from our analysis.

(a) **Scaling term**

\( F_1(\omega) \) is expressed by a simple function and is illustrated by a solid line curve in Fig. 5 and is compared with the experimental data. It seems satisfactory that such simple forms of \( F_1 \) and \( F_2 \) are in good agreement with the experimental data.

(b) **Breaking term**

Now we shall examine the gross properties of the scaling violation given

* The validity of the solution of the B-S equation has been discussed in Ref. 8) with \( \sqrt{\lambda} \sim 1/2. 

** This parametrization is insensitive to the value of \( \bar{\lambda} \).
Fig. 6. (a) The structure functions $\nu W_2$ and $2MW_1$, given by (3.17)–(3.20) and experimental data.\textsuperscript{11} Dashed lines indicate the contribution from both vector-vector and scalar-scalar parts. (b) $R_P$ which has been obtained from our calculation and experimental data.\textsuperscript{11}

by (3.19) and (3.20) by comparing them with the experimental results by SLAC-MIT collaboration.\textsuperscript{11} According to their data, $2MW_1$ has the falling $Q^2$-dependence up to $\omega=10$. On the other hand, $\nu W_2$ has the rising $Q^2$-dependence at large $\omega(\gtrsim10)$ and the falling $Q^2$-dependence at small $\omega(\lesssim6)$. Such gross features can be explained by our expression of the breaking part, Eq. (3.20), which has the same pattern at small $\omega$ and large $\omega$. However, the more detailed analysis shows that the rising $Q^2$-dependence begins at a larger value of $\omega$ than 10 in our parametrization. Equation (3.20) gives the gradually falling behaviour up to $\omega\approx10$, \hfill
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Fig. 7. (a) The production process of charm quarks, which contributes to the structure functions. (b) Its lowest order contribution to the structure functions.

which deviates slightly from the present experimental data. This situation is shown in Fig. 6.

The rising $Q^2$-dep. at $\omega \approx 10$ could be explained by the production effect of charm quarks in our scheme. The charm quarks are considered to be produced through the diffraction process as shown in Fig. 7(a). Taking into account only the lowest order (see Fig. 7(b)), we estimate the production effect on $G_1(\omega, Q^2)$ and $G_2(\omega, Q^2)$. The contribution of the production effect is calculated as

$$G_1^{\text{charm}}(\omega, Q^2) \propto \ln \left(1 + \frac{\omega Q^2 A^*}{2(Q^2 + m_c^2)}\right) - \ln \left(1 + \frac{\omega Q^2 A^*}{2(Q^2 + A_c^2)}\right),$$

$$G_2^{\text{charm}}(\omega, Q^2) \propto \frac{2}{\omega} \left[\ln \left(1 + \frac{\omega Q^2 A^*}{2(Q^2 + m_c^2)}\right) - \ln \left(1 + \frac{\omega Q^2 A^*}{2(Q^2 + A_c^2)}\right) - \frac{A_c^2 - m_c^2}{Q^2} \ln \left(\frac{2 + \omega A^*}{\omega}\right)\right],$$

where $z = 1/2$ is taken and $\omega Q^2 \gg A_c^2 > m_c^2$ is assumed. The new parameter $A_c(A^*)$ is the same meaning as $t(A)$ in Eq. (2.31) (Eqs. (2.37) and (2.38)). The variable $\omega Q^2$ is related to the threshold of charm quarks through

$$W_{th}^2 = (\omega - 1)Q^2 \gg (2m_c)^2.$$

Equation (3.21) has a less rapid falling $Q^2$-dependence. This contribution gives a little change of the behaviour to $2MW_1(\omega, Q^2)$. As for $G_2^{\text{charm}}(\omega, Q^2)$, however, the minus $(Q^2)^{-1}$-term which results in the rising-$Q^2$ behaviour at $\omega \approx 10$ appears.

§ 4. Concluding remarks

We have investigated in this paper a dynamical approach to the scaling and its violation in the framework of quark-gluon model. To understand the scaling properties in our scheme, the effect of the strong interactions on the constituents must be taken into consideration. This suggests that the quark may not be a point-like object but behaves as a composite particle in the nucleons. The above
mentioned dynamics is embodied in our quark-antiquark scattering amplitudes which have been evaluated by the use of the approximate solution to the B-S equation in the ladder approximation.

As a result, the following characteristics of the nucleon structure functions have been obtained in our scheme:

1. The scaling part of $2MW_1$ takes the form of $0.283\omega - 0.416/\omega$ and the Callan-Gross relation\textsuperscript{13} $F_1(\omega) = \omega^{-1} F_2(\omega)$ holds.

2. The scale breaking part has $(Q^2)^{1/2}$-dependence, which is rather different from a familiar logarithmic $Q^2$-dependence.

It should be emphasized that the scaling term is found to take a very simple form in spite of the intricate structure of the B-S amplitudes. This implies that we have extracted one meaningful aspect of the dynamical properties in the quark-gluon system.

We should like to add a comment on $(Q^2)^{1/2}$-dependence in the nucleon ST. The $(Q^2)^{1/2}$-dependence arises from a condition $\sqrt{\lambda} \approx 0.5$. We take notice that $\sqrt{\lambda}$ is a function of the coupling constant $\lambda_0$ between quark and gluon, and there is some ambiguity in the choice of $\sqrt{\lambda}$ for the lack of a dynamical condition in determining the value of $\lambda_0$. If we interpret that the quark mass is to be generated from dynamical spontaneous symmetry breakdown mechanism,\textsuperscript{13} then $\lambda_0$ is necessarily larger than that of the present case (hence $\sqrt{\lambda}$ becomes larger than 0.5). The value of $\sqrt{\lambda}$ adopted in this paper is only an example in order to see the explicit $Q^2$-behaviour of $\nu W_2(\omega, Q^2)$ and $2MW_1(\omega, Q^2)$.

Almost the same consideration as electro-production can be made for neutrino-induced reactions. We will discuss $x$, $y$-distribution and the so-called $y$-anomaly\textsuperscript{13} in (anti)neutrino reactions and also the relation between the ratio $W_{1n}/W_{1p}$ and the elastic form factor in a forthcoming paper.

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