Unity of Weak, Electromagnetic and Strong Interactions in Superconductivity Model

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Our previous work for deriving the Weinberg-Salam model from leptonic system ($\nu$, and $e$) based on an analogy with superconductivity has been extended by Terazawa, Akama and Chikashige to a more realistic model including quarks. We present here a further unified model including $\nu$, $\mu$ and quarks with both flavor and color degrees of freedom. It is shown that a Lagrangian of self-interacting leptons and quarks generates three models for elementary-particle interactions, i.e., the Weinberg-Salam model for weak and electromagnetic interactions of leptons and quarks, the asymptotically free gauge model of Gross, Wilczek and Politzer for strong interaction of quarks, and also the vortex model of Nielsen and Olesen for dual strings. All elementary-particle forces are shown to be related with a single coupling strength, i.e., the fine-structure constant, and this coincides with the prediction of Terazawa et al. and hence with that of Georgi and Glashow based on the $SU(5)$ gauge model. Mass values of vector bosons and Higgs scalars will also be predicted.

§ 1. Introduction

Recently, starting with a Lagrangian of self-interacting leptons ($\nu$, $e$) only, we have constructed an effective Lagrangian of the Weinberg-Salam type and proposed that the photon, Higgs bosons and weak-vector bosons are all composites of lepton and antilepton.$^8$ We have found, among other things, the Weinberg angle to be $\sin^2 \theta_W = 1/4$. The idea is essentially the same as that of Nambu and Jona-Lasinio who gave a dynamical model of elementary particles based on an analogy with superconductivity.$^9$

Very lately, Terazawa, Akama and Chikashige$^9$ have extended the above lepton model to a more realistic one including quarks. They have considered a system of $SU(3)$-color triplets, $p$ and $n$ quarks, besides $\nu_e$ and $e$, and pointed out that the system generates the Weinberg-Salam model$^9$ for weak and electromagnetic interactions of leptons and quarks, and that the asymptotically free gauge model of Gross, Wilczek and Politzer for strong interaction of quarks.$^9$

In the present paper we propose a further unified model including $\nu_e$, $e$, $\nu_\mu$, $\mu$ and quarks with both flavor and color degrees of freedom. It is shown that a Lagrangian of self-interacting leptons and quarks generates three models for elementary-particle interactions, i.e., the Weinberg-Salam model, the asymptotically free gauge model of Gross, Wilczek and Politzer, and also the vortex model of Nielsen and Olesen for dual strings.$^9$ All elementary-particle forces are shown to be related with a single coupling strength, i.e., the fine-structure constant, and
this coincides with the prediction of Terazawa et al. and hence with that of Georgi and Glashow based on the SU(5) gauge model.\(\textsuperscript{7}\)

At the first glance it seems that unrenormalizability of our starting Lagrangian with four-fermi interactions makes the whole procedure highly ambiguous. However, this is not the case. In our work we never use such an unrenormalizable perturbation expansion in four-fermi coupling constants. But we introduce, following the path-integral technique of Kikkawa\(\textsuperscript{8}\) and Kugo,\(\textsuperscript{9}\) some auxiliary fields\(\textsuperscript{10}\) which correspond to bosonic collective variables. The effective Lagrangian for those bosonic collective variables is easily obtained after carrying out path-integrals over fermionic fields. Our expansion used here is an expansion in those bosonic variables, but not in the four-fermi coupling constants. Thus in our procedure there is no ambiguity arising from unrenormalizability.

In §2 we shall present our starting Lagrangian and derive from this an effective Lagrangian of the Higgs type. In §3 mass values of vector bosons and Higgs scalars arising from spontaneous-symmetry breaking will be predicted. The last section is devoted to concluding remarks.

### §2. Derivation of effective Lagrangian

We begin with the following Lagrangian:

\[
L = \sum_{j=1,2} \left\{ \bar{\psi}_j i \gamma^\mu (\partial - iY_j U - i\tau U^a) \psi_j \\
+ \bar{\psi}_j i \gamma^\tau (\partial - iY_j U) \psi_j \\
+ a_j (\bar{\psi}_j K_j \psi_j + \text{h.c.}) \\
+ \sum_{j=1,2} \left\{ \bar{L}_j i \gamma^\tau (\partial - iY_j U - i\tau U - i\lambda^a (U^a + V^a)) L_j \\
+ \bar{R}_j i \gamma^\tau (\partial - iY_j U - i\tau U - i\lambda^a (U^a + V^a)) R_j \\
+ \bar{R}_j \gamma^\tau (\partial - iY_j U) R_j \\
+ b_j (\bar{L}_j K_{2j} R_j + \text{h.c.}) + c_j (\bar{L}_j K_{3j} R_j + \text{h.c.}) \right\} \\
+ |K_1|^2 + k \text{Tr} |K_2|^2 + l \text{Tr} |K_3|^2 + m |U_s|^2 + n |U_a|^2 + p (U_s)^2 \\
+ q (V_s)^2, \tag{2.1}
\]

where

\[
L_1 = A L \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad r_1 = A_R e^-; \quad l_2 = A L \begin{pmatrix} \nu_e \\ \mu^- \end{pmatrix}, \quad r_2 = A_R \mu^-,
\]

\[
L_1 = A L \begin{pmatrix} p \\ n_\theta \end{pmatrix}, \quad R_1 = A_R n_\theta; \quad L_2 = A L \begin{pmatrix} p' \\ \lambda_\theta \end{pmatrix}, \quad R_2 = A_R \lambda_\theta,
\]

\[
R_1 = A_R p, \quad R_2 = A_R p'
\]

and
The quark fields $L_j$, $R_j$ and $R_j^a$ are all color triplets, while the leptonic fields $l_j$ and $r_j$ are color singlets. The $L_j$ and $l_j$ are weak iso-doublets, while $R_j$ and $r_j$ are iso-singlets. The vector $U_{s}^{a}$ and axial-vector $V_{s}^{a}$ are color octets, and $\text{Tr}$ denotes a trace on color. The $Y_j$'s are the weak hypercharges of corresponding leptons or quarks. The Lagrangian (2·1) is invariant under the global $SU(3)_{L}^{\text{color}} \otimes SU(3)_{R}^{\text{color}} \otimes SU(2)_{L} \otimes U(1)$ group.

The bosonic fields $K_1$, $K_2$, $K_3$, $U_{s}$, $U_{s}^{a}$ and $V_{s}^{a}$ play the role of auxiliary fields, because they do not have kinetic terms. The Lagrangian (2·1) is effectively equivalent to the purely self-interacting fermionic system. This can be easily seen by taking variations with respect to bosonic and fermionic fields independently. The above choice of auxiliary fields is the necessary and sufficient one for our purpose.

In order to obtain an effective Lagrangian for those bosonic fields, we make use of the path-integral technique.\[^{6,9}\] Define the effective Lagrangian $L_{\text{eff}}$ by

$$
\exp\left\{i \int d^4x L_{\text{eff}}\right\} = \int [d(\text{fermionic fields})] \exp\left\{i \int d^4x L\right\}.
$$

Carrying out the path-integrals over fermionic fields, we get

$$
\int d^4x L_{\text{eff}} = -i \sum_{j=1,2} \text{Tr} \log \left| 1 + \frac{1}{i \partial}\left(Y_{ij} \tau U + \tau \gamma \right) A_L + \frac{1}{i \partial} a_j K_1 A_R \right|
$$

$$
\frac{1}{i \partial} a_j K_1^+ A_L , \quad 1 + \frac{1}{i \partial} Y_{rj} \gamma U A_R
$$

$$
\frac{1}{i \partial} b_j K_2^+ A_L , \quad \frac{1}{i \partial} b_j K_3 A_R
$$

$$
\frac{1}{i \partial} c_j K_3^+ A_L , \quad 0, \quad 1 + \frac{1}{i \partial} (Y_{rj} \gamma U + \gamma (U^a - \gamma V^a)) A_R
$$

$$
\int d^4x \left\{ h |K_1|^2 + k \text{Tr} |K_2|^2 + l \text{Tr} |K_3|^2 + m U_{s}^{a} + n U_{s}^{a} + p (U_{s}^{a})^2 + q (V_{s}^{a})^2 \right\}. \quad (2·3)
$$

The logarithmic terms correspond to a series of fermion-loop diagrams if they are expanded into the Taylor series in bosonic fields. We retain only divergent
terms proportional to a cutoff parameter $A^2$ or $\log A^2$, but "effective potentials" of $K_i$'s will be considered exactly. We take a universal cutoff throughout this paper. Infrared divergences do not occur here when the symmetry is spontaneously broken.\(^{11,12}\) After some trace calculations we get an effective Lagrangian

\[
L_{\text{eff}} = -\frac{1}{3} \left( \beta_1 + \beta_2 + 3 \beta_3 + 3 \beta_4 \right) U_{\mu}^2 - \frac{1}{9} \left( 9 \beta_1 + 9 \beta_2 + 11 \beta_3 + 11 \beta_4 \right) U_{\mu}^2
\]

\[
- \left( \alpha_1 + \alpha_2 + 3 \alpha_3 + 3 \alpha_4 - m \right) U_{\mu}^2 - \left( 3 \alpha_1 + 3 \alpha_2 + \frac{11}{3} \alpha_3 + \frac{11}{3} \alpha_4 - m \right) U_{\mu}^2
\]

\[
-\frac{1}{3} \left( \beta_1 + \beta_2 \right) \text{Tr} \left( G_{\mu}^2 \right) - 4 \left( \alpha_1 + \alpha_2 \right) \left( U_{\mu} a_{2} + V_{\mu} a_{2} \right) + p U_{\mu} a_{2} + q V_{\mu} a_{2}
\]

\[
+ \left( a_{1}^2 \beta_1 + a_{2}^2 \beta_2 \right) \left( \partial - iU - i\tau U \right) K_{1}^2
\]

\[
+ \left( b_{1}^2 \beta_1 + b_{2}^2 \beta_2 \right) \text{Tr} \left[ \left( \partial - iU - i\tau U \right) K_{1}^2 - iK_{1} a_{2} \left( U_{a}^2 + V_{a}^2 \right) K_{1}^2 \right]
\]

\[
+ \left( c_{1}^2 \beta_1 + c_{2}^2 \beta_2 \right) \text{Tr} \left[ \left( \partial + iU + i\tau U \right) K_{1}^2 - iK_{1} a_{2} \left( U_{a}^2 + V_{a}^2 \right) K_{1}^2 \right]
\]

\[
+ h \left| K_{1} \right|^2 + k \text{Tr} \left[ K_{1}^2 \right] + l \text{Tr} \left[ K_{1}^2 \right]
\]

\[
+ I_1 + I_2,
\]

where

\[
I_1 = -\frac{2i}{(2\pi)^4} \int d^4p \log \left( 1 - \frac{a_{1}^2}{p^2} \left| K_{1} \right|^2 \right) - \frac{2i}{(2\pi)^4} \int d^4p \log \left( 1 - \frac{a_{2}^2}{p^2} \left| K_{1} \right|^2 \right)
\]

\[
I_2 = -\frac{2i}{(2\pi)^4} \text{Tr} \int d^4p \log \left( 1 - \frac{b_{1}^2}{p^2} \left| K_{1} \right|^2 + c_{1}^2 \left| K_{1} \right|^2 \right)
\]

\[
- \frac{2i}{(2\pi)^4} \text{Tr} \int d^4p \log \left( 1 - \frac{b_{2}^2}{p^2} \left| K_{1} \right|^2 + c_{2}^2 \left| K_{1} \right|^2 \right)
\]

Here we have used

\[
Y_{1i} = Y_{i1} = -1, \quad Y_{L1} = Y_{L2} = -\frac{1}{3}, \quad Y_{r1} = Y_{r2} = -2, \quad Y_{s1} = Y_{s2} = -\frac{2}{3},
\]

\[
Y_{p1} = Y_{p2} = \frac{4}{3},
\]

\[
G_{p} = \partial_{\mu} G_{\mu} = -\partial_{\nu} G_{\nu} - i \left[ G_{\mu}^a, G_{\nu}^b \right], \quad G_{\mu}^a = \gamma^a \left( U_{\mu} a_{2} \pm V_{\mu} a_{2} \right),
\]

\[
U_{\mu} = \partial_{\mu} U_{\nu} - \partial_{\nu} U_{\mu} + 2 U_{\nu} \times U_{\mu},
\]

\[
U_{\mu} = \partial_{\mu} U_{\nu} - \partial_{\nu} U_{\mu}
\]

and

\[
\alpha_1 = \frac{i}{(2\pi)^4} \int \frac{d^4p}{p^2 - a_{1}^2} \frac{d^4p}{(p^2 - a_{1}^2) \left| K_{1} \right|^2}, \quad \beta_1 = \frac{-i}{(2\pi)^4} \int \frac{d^4p}{p^2 - a_{1}^2} \frac{d^4p}{(p^2 - a_{1}^2) \left| K_{1} \right|^2},
\]

\[
\alpha_2 = \frac{i}{(2\pi)^4} \int \frac{d^4p}{p^2 - a_{2}^2} \frac{d^4p}{(p^2 - a_{2}^2) \left| K_{1} \right|^2}, \quad \beta_2 = \frac{-i}{(2\pi)^4} \int \frac{d^4p}{p^2 - a_{2}^2} \frac{d^4p}{(p^2 - a_{2}^2) \left| K_{1} \right|^2}.
\]
\[ \alpha_5 = \frac{i}{(2\pi)^4} \int \frac{d^4p}{p^2 - b_1^2\langle |K_1|^2 \rangle - c_1^2\langle |K_1|^2 \rangle} \]

\[ \beta_5 = \frac{-i}{(2\pi)^4} \int \frac{d^4p}{(p^2 - b_2^2\langle |K_2|^2 \rangle - c_2^2\langle |K_2|^2 \rangle)^2} \]

\[ \alpha_4 = \frac{i}{(2\pi)^4} \int \frac{d^4p}{p^2 - b_2^2\langle |K_2|^2 \rangle - c_2^2\langle |K_2|^2 \rangle} \]

\[ \beta_4 = \frac{-i}{(2\pi)^4} \int \frac{d^4p}{(p^2 - b_2^2\langle |K_2|^2 \rangle - c_2^2\langle |K_2|^2 \rangle)^2} \]

where \( \langle |K_j|^2 \rangle \)'s are vacuum expectation values for color-singlet parts of \( K_j \)'s.

The “created” vector fields \( U_\mu, U_\mu', \ U_\mu'' \) and \( V_\mu'' \) enter the “covariant derivatives” in (2.4). This suggests us that the created vector fields can play the role of local gauge fields if they happen to be massless. In the following we require that they are massless, i.e.,

\[ n = \alpha_1 + \alpha_2 + 3\alpha_3 + 3\alpha_4, \quad m = 3\alpha_1 + 3\alpha_2 + \frac{11}{3}\alpha_3 + \frac{11}{3}\alpha_4, \]

\[ p = q = 4(\alpha_3 + \alpha_4). \] (2.6)

Then, after some rescaling such as

\[ U_\mu = \left( \frac{3}{4\beta_1 + 4\beta_2 + 12\beta_3 + 12\beta_4} \right)^{1/2} A_\mu = \frac{g}{2} A_\mu, \]

\[ U_\mu'' = \left( \frac{9}{36\beta_1 + 36\beta_2 + 44\beta_3 + 44\beta_4} \right)^{1/2} B_\mu = \frac{g'}{2} B_\mu, \]

\[ G_\mu'' = \left( \frac{3}{16\beta_2 + 16\beta_4} \right)^{1/2} C_\mu'' = \frac{f}{2} C_\mu'', \] (2.7)

\[ K_1 = (a_1^2\beta_1 + a_2^2\beta_2)^{-1/2} \phi_1, \quad K_2^{\text{color singlet}} = -(3b_2^2\beta_1 + 3b_2^2\beta_2)^{-1/2}\phi_2^{\text{color singlet}}, \]

\[ K_3^{\text{color singlet}} = -(3c_4^2\beta_3 + 3c_4^2\beta_4)^{-1/2}\phi_3^{\text{color singlet}}, \]

we have the Higgs-type Lagrangian

\[ L_{\text{eff}} = L_{\text{Higgs}} = -\frac{1}{16} \text{Tr} (C_{\mu\nu}^2 + C_{\mu\nu}^{\text{singlet}}^2) - \frac{1}{4} A_{\mu\nu}^2 - \frac{1}{4} B_{\mu\nu}^2 \]

\[ + \text{Tr} \left( \frac{\partial - ig'}{2} B - \frac{ig}{2} \tau A - \frac{if}{2} (C^+ - C^-) \right) \phi^{\text{singlet}}_2^2 \]

\[ + \text{Tr} \left( \frac{\partial + ig'}{2} B - \frac{ig}{2} \tau A - \frac{if}{2} (C^+ - C^-) \right) \phi^{\text{singlet}}_3^2 \]

\[ - V_1 (|\phi_1|^2) - V_2 (|\phi_2^{\text{singlet}}|^2, |\phi_3^{\text{singlet}}|^2) \]

\[ + (\text{terms containing } \phi^{\text{singlet}}_{\alpha,\beta}), \] (2.8)

where
Finally we take into account of interactions between \((\ell, \rho R, L, R)\) and \((\phi, A_\nu, B, C^\pm)\), which are given by the starting Lagrangian (2.1), and add them together with the fermion kinetic parts to the \(L_{\text{Higgs}}\) after rescaling (2.7). Then what we get is the following Lagrangian:

\[
L_{\text{total}} = L_{\text{Higgs}} + \sum_{j=1,2} \left\{ \bar{t}_j i\gamma^\mu \left[ \partial_{\mu} - \frac{ig'}{2} Y_{\nu} B - \frac{ig}{2} \gamma^\mu A \right] t_j + \bar{\rho} j i\gamma^\mu \left[ \partial_{\mu} - \frac{ig'}{2} Y_{\nu} B - \frac{ig}{2} \gamma^\mu A \right] \rho_j \right\} -\sum_{j=1,2} G_{j^{(1)}} (\bar{t}_j \phi^i r_j + \text{h.c.}) + \sum_{j=1,2} \left\{ \bar{L}_j i\gamma^\mu \left[ \partial_{\mu} - \frac{ig'}{2} Y_{\nu} B - \frac{ig}{2} \gamma^\mu A \right] L_j + \bar{R}_j i\gamma^\mu \left[ \partial_{\mu} - \frac{ig'}{2} Y_{\nu} B - \frac{ig}{2} \gamma^\mu A \right] R_j \right\} -\sum_{j=1,2} G_{j^{(2)}} \left( \bar{L}_j i\gamma^\mu \partial^\text{singlet}_{\mu} R_j + \text{h.c.} \right) -\sum_{j=1,2} G_{j^{(3)}} \left( \bar{L}_j i\gamma^\mu \partial^\text{singlet}_{\mu} R_j + \text{h.c.} \right) + \text{(terms containing } \phi^\text{d} \text{ ),} \tag{2.10}
\]

where

\[
G_{j^{(1)}} = \frac{a_j}{\sqrt{a_1^2 \beta_1 + a_2^2 \beta_2}}, \quad G_{j^{(2)}} = \frac{b_j}{\sqrt{b_1^2 \beta_3 + b_2^2 \beta_4}}, \quad G_{j^{(3)}} = \frac{c_j}{\sqrt{c_1^2 \beta_3 + c_2^2 \beta_4}}. \tag{2.11}
\]

Here it should be noted that in any calculations based on the above Lagrangian we should not take into account of one-fermion-loop diagrams where bosonic fields are attached as external one, because such diagrams have already been considered. This “rule” will be formulated in the following way.\(^9\) The starting Lagrangian \(L\) can be written as

\[
L = L_{\text{Higgs}} + L - L_{\text{Higgs}} = L_{\text{total}} + L_{\text{counter terms}}, \tag{2.12}
\]

where

\[
L_{\text{counter terms}} = h \left| K_1 \right|^2 + k \text{Tr} \left| K_3 \right|^2 + L \text{Tr} \left| K_3 \right|^3 + m U_\nu^0 + n U_\nu^0 + p U_\nu^{a3} + q U_\nu^{a4} - L_{\text{Higgs}}. \tag{2.13}
\]
The parameters $h, k, \cdots$ are the same as those defined before. The one-fermion loop diagrams can be seen to be always cancelled by $L'_\text{counter terms}$. Therefore, the above “rule” is satisfied if we adopt the Lagrangian of the form (2.12).

The Lagrangian (2.10) apparently contains the Weinberg-Salam Lagrangian for leptons and quarks. It contains furthermore the vector and axial-vector color-gluon theories in the following form:

$$L_{\text{gluons}} = -\frac{1}{16} \text{Tr} (C^+ + C^-) + \sum_{j=1,2} \left[ L_j \bar{i}i^a \left( \partial_\mu - \frac{if}{2} C_\mu^a \right) L_j \right] + \bar{R}_j \bar{i}i^a \left( \partial_\mu - \frac{if}{2} C_\mu^- \right) R_j + \bar{R}_j^\dagger \bar{i}i^a \left( \partial_\mu - \frac{if}{2} C_\mu^+ \right) R_j^\dagger \right]$$

$$+ (\text{Higgs-scalar parts})$$

$$= -\frac{1}{16} \text{Tr} (C^+ + C^-) + \bar{q}i^a \left( \partial_\mu - \frac{if}{2} \kappa (G_\mu^a - \gamma sG_\mu^{a'}) \right) q$$

$$+ (\text{Higgs-scalar parts}),$$

where $q = (p, n, \lambda, \lambda')$, $C_\mu^\pm = \kappa (G_\mu^a \pm G_\mu^{a'})$, $G_\mu^a = \text{vector-color-gluon}$ and $G_\mu^{a'} = \text{axial-vector-color-gluon}$. The vector coupling constants $g, g'$ and $f$ are given by (2.7), i.e.,

$$g^2 = \frac{3}{\beta_1 + \beta_3 + 3\beta_4 + 3\beta_4} \approx \frac{3}{8\beta}, \quad g'^2 = \frac{9}{9\beta_1 + 9\beta_3 + 11\beta_4 + 11\beta_4} \approx \frac{9}{40\beta},$$

$$f^2 = \frac{3}{4\beta_3 + 4\beta_4} \approx \frac{3}{8\beta}.$$  

(2.14)

Here we have set $\beta = \beta_1 = \beta_3 = \beta_4 = \beta_4$, since the $\beta_j$'s do not so strongly depend on “mass-terms” in $\beta_j$'s for large cutoff $\Lambda$ (see also § 3). This shows that the Weinberg angle is fixed to be

$$\sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2} \approx \frac{3}{8},$$

and $g, g'$ and $f$ are related to the fine-structure constant $e^2$ through

$$e^2 = g^2 \sin^2 \theta_W \approx \frac{3}{8} g^2 = \frac{5}{8} g'^2 = \frac{3}{8} f^2 = \left( \frac{3}{8} \right)^2 \frac{1}{\beta}.$$  

(2.17)

### § 3. Spontaneous-symmetry breaking

Our Lagrangian (2.10) is invariant under the local $SU(3)_L^{\text{color}} \times SU(3)_R^{\text{color}} \times SU(2)_L \times U(1)$ gauge group. If the Higgs scalars develop vacuum-expectation values:

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle \text{ color singlet} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle \phi_3 \rangle \text{ color singlet} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_3 \\ 0 \end{pmatrix},$$

(3.1)
both $SU(2)_L$ and $U(1)$ gauge symmetries are spontaneously broken. Then, by choosing the $U$-gauge such that
\[
\phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_3 + \eta_3 \end{pmatrix}, \quad \phi_3^{\text{singlet}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_3 + \eta_3 \end{pmatrix},
\]
relevant fermions acquire masses, i.e.,
\[
m_e = \frac{v_1}{\sqrt{2}} a_1 \sqrt{a_1^2 \beta_1 + a_2^2 \beta_2}, \quad m_\mu = \frac{v_1}{\sqrt{2}} a_2 \sqrt{a_1^2 \beta_1 + a_2^2 \beta_2}.
\]
and
\[
m_e = \frac{v_2}{\sqrt{6}} b_1 \sqrt{b_1^2 \beta_3 + b_2^2 \beta_4}, \quad m_\mu = \frac{v_2}{\sqrt{6}} b_2 \sqrt{b_1^2 \beta_3 + b_2^2 \beta_4},
\]
\[
m_p = \frac{v_3}{\sqrt{6}} c_1 \sqrt{c_1^2 \beta_3 + c_2^2 \beta_4}, \quad m_\rho = \frac{v_3}{\sqrt{6}} c_2 \sqrt{c_1^2 \beta_3 + c_2^2 \beta_4}.
\]
Here we have set a relation $b_1 = b_2$, in order that the $n\lambda$ cross terms are dropped out.

The $W$ and $Z$ boson masses are given by
\[
M_w^2 = \frac{1}{4} g^2 v^2 = \left( \frac{\pi \alpha}{\sqrt{2} G_w} \right) \frac{1}{\sin^2 \theta_w} \approx 38 \left( \frac{3}{8} \right) = (62.05 \text{ GeV})^2,
\]
\[
M_\phi^2 = \frac{M_w^2}{\cos^2 \theta_w} = (78.49 \text{ GeV})^2,
\]
where
\[
v^2 = v_1^2 + v_2^2 + v_3^2.
\]
The axial-vector color-gluon also acquires a mass
\[
M_A^2 = \frac{2}{3} f^2 (v_3^2 + v_3^2) = \frac{v_3^2 + v_3^2}{2 (\beta_3 + \beta_4)},
\]
whereas the vector-color-gluon remains massless. From (3.3) and (3.4), $v_1$, $v_2$, and $v_3$ can be solved as:
\[
v_1^2 = 2 (m_e^2 \beta_1 + m_\mu^2 \beta_2), \quad v_2^2 = 6 (m_e^2 \beta_3 + m_\mu^2 \beta_4), \quad v_3^2 = 6 (m_p^2 \beta_3 + m_\rho^2 \beta_4).
\]
These equations together with (3.5) and (3.7) give us relations
\[
\frac{v^2}{2 \beta} \approx m_e^2 + m_\mu^2 + 3 (m_e^2 + m_\mu^2 + m_\phi^2 + m_\rho^2) = 2 \left( \frac{8}{3} \times 38 \right) \text{ GeV}^2,
\]
\[
M_A^2 = \frac{3}{2} (m_e^2 + m_\mu^2 + m_\phi^2 + m_\rho^2),
\]
hence
\[ M_4 \equiv \left( \frac{8}{3} \times 38 \right) \equiv (101 \text{ GeV})^2. \] (3.11)

Here we have neglected \( m_e \) and \( m_\nu \).

The Higgs scalar \( \gamma_1 \) generates the following mass:
\[ m_{\gamma_1}^2 = 2v_1^2 \frac{a_1^4 \beta_1 + a_2^4 \beta_2}{\beta_1 + \beta_2} \left( \frac{1 + r^4}{r^2} \right) \approx 2v_1^2, \]
because of \( r = a_1/a_2 = m_e/m_\mu \ll 1 \). Hence, from (3.8) we have
\[ m_{\gamma_1}^2 \approx 4(m_e^2 + m_\mu^2) \equiv (2m_\mu)^2. \] (3.12)

The mass terms relevant to \( \gamma_2 \) and \( \gamma_3 \) are
\[ \begin{align*}
\frac{2}{3} \left( b_1^2 \beta_3 + b_2^2 \beta_4 \right) v_2^{\text{singlet}} |\phi_2^{\text{singlet}}|^2 + \frac{2}{3} \left( c_1^2 \beta_3 + c_2^2 \beta_4 \right) v_3^{\text{singlet}} |\phi_3^{\text{singlet}}|^2 \\
+ \frac{4}{3} \left( b_1^2 c_1 \beta_3 + b_2^2 c_2 \beta_4 \right) v_2^{\text{singlet}} v_3^{\text{singlet}} |\phi_2^{\text{singlet}}||\phi_3^{\text{singlet}}|.
\end{align*} \] (3.13)

The Higgs scalars \( \gamma_2 \) and \( \gamma_3 \) will be diagonalized in the approximation of \( m_\mu = m_\mu = m_\pi \ll m_\nu \), because the above equation is rather complicated. As a result we get
\[ m_{\gamma_2}^2 = 2m_\mu^2 \left( s - \frac{3}{2}s' \right) \quad \text{for} \quad \gamma_2' = \left( 1 - \frac{s^2}{2} \right) \gamma_2 + \frac{s}{\sqrt{2}} \gamma_3, \] (3.14)
\[ m_{\gamma_3}^2 = 2m_\mu^2 \left( 2 - s^2 + \frac{15}{2}s' \right) \quad \text{for} \quad \gamma_3' = \frac{s}{\sqrt{2}} \gamma_2 + \left( 1 - \frac{s^2}{4} \right) \gamma_3, \] (3.15)
where \( s = c_1/c_2 = m_\mu/m_\nu (\ll 1) \).

The vacuum-expection values \( \langle \phi_j \rangle^{\text{color singlet}} \) are given by equations:
\[ \frac{\partial V_1(|\phi_1|)}{\partial |\phi_1|} = 0, \quad \frac{\partial V_2(|\phi_2|^{\text{singlet}}, |\phi_3|^{\text{singlet}})}{\partial |\phi_2|^{\text{singlet}}} = 0, \quad \frac{\partial V_2(|\phi_2|^{\text{singlet}}, |\phi_3|^{\text{singlet}})}{\partial |\phi_3|^{\text{singlet}}} = 0, \]
from which, together with (3.3) and (3.4), it follows that
\[ \begin{align*}
h &= -\frac{ia_1^2}{(2\pi)^4} \int_{p^2 - m_e^2}^{A} d^4p - \frac{ia_2^2}{(2\pi)^4} \int_{p^2 - m_\mu^2}^{A} d^4p, \\
k &= -ib_1^2 \frac{1}{(2\pi)^4} \int_{p^2 - m_e^2 - m_\mu^2}^{A} d^4p - ib_2^2 \frac{1}{(2\pi)^4} \int_{p^2 - m_\mu^2 - m_\mu^2}^{A} d^4p, \\
l &= -ic_1^2 \frac{1}{(2\pi)^4} \int_{p^2 - m_e^2 - m_\mu^2}^{A} d^4p - ic_2^2 \frac{1}{(2\pi)^4} \int_{p^2 - m_\mu^2 - m_\mu^2}^{A} d^4p.
\end{align*} \] (3.16)

These are nothing but Nambu and Jona-Lasinio’s self-consistent equations for fermion masses. For a sufficient large \( A \) the right-hand sides of (3.16) take negative signs, so that they are consistent only when “four-Fermi coupling constants” \( h^{-1}, k^{-1} \) and \( l^{-1} \) are all negative. In other words the spontaneous-symmetry breaking
of $SU(2)_L \otimes U(1)$ will be occurred only in this case.

Finally the $\beta_i$'s defined in § 2 can be rewritten, in terms of fermion masses, as:

$$\beta_1 = -\frac{i}{(2\pi)^4} \int \frac{d^4 p}{(p^2 - m^2_\pi)^2}, \quad \beta_2 = -\frac{i}{(2\pi)^4} \int \frac{d^4 p}{(p^2 - m^2_\rho)^2},$$

$$\beta_3 = -\frac{i}{(2\pi)^4} \int \frac{d^4 p}{(p^2 - m^2_\rho - m^2_\pi)^2}, \quad \beta_4 = -\frac{i}{(2\pi)^4} \int \frac{d^4 p}{(p^2 - m^2_\rho - m^2_\pi)^2}. \quad (3·17)$$

They do not so strongly depend on fermion masses for large cutoff $A$. From (2·17) one can see that the typical cutoff $A$ is given by $(A/m)^2 \approx \exp(3042) > 1$. This permits us to set $\beta = \beta_1 = \beta_2 = \beta_3 = \beta_4$.

§ 4. Concluding remarks

Our generated Lagrangian (2·10) is invariant under the local $SU(3)_L$ color $\otimes SU(3)_R$ color $\otimes SU(2)_L \otimes U(1)$ gauge group. We have seen that the spontaneous-symmetry breaking of $SU(2)_L \otimes U(1)$ gauge group makes, among other things, the "created" axial-vector-color-gluon $G^a_{\pi}$ massive, whereas the created vector-color-gluon $G^a_\rho$ remains massless. This shows that the chiral $SU(3)$ color-gauge symmetry is also broken, but the $SU(3)$ color-gauge symmetry is still preserved.\(^5\)

The latter fact means that the vector-color-gluon theory included in (2·14) plays the role of the asymptotically free gauge theory of Gross, Wilczek and Politzer.\(^5\)

In this theory there is also a possibility of quark confinement based on infrared singular nature of non-Abelian gauge theory.\(^10\)

On the other hand, the system of created massive axial-vector-color-gluon $G^a_{\pi}$ together with color-singlet Higgs scalars $\eta_{\pi}'$ and $\eta_{\rho}'$ is parallel to the type II superconductors with magnetic flux lines. This suggests us that our system contains vortex solutions of Nielsen and Olesen\(^6\) which can be regarded as hadrons.\(^6\)

We have predicted the mass of $G^a_{\pi}$ to be $M_A \approx 101$ GeV from (3·11), and those of $\eta_{\pi}'$ and $\eta_{\rho}'$ to be $m_{\eta_{\pi}'} \approx \sqrt{2} m_\pi$ and $m_{\eta_{\rho}'} \approx 2 m_\rho$ from (3·14) and (3·15). If we push the above parallel forward, the Ginzburg-Landau parameters are given by $\kappa_3 = m_{\eta_{\pi}'}/M_A \approx (2/\sqrt{3}) (m_\pi/m_\rho)$ for $\eta_{\pi}'$ and $\kappa_3 = m_{\eta_{\rho}'}/M_A \approx (8/3)^{1/2}$ for $\eta_{\rho}'$. The vortex solutions will be expected to exist when $\kappa_3 > 1/\sqrt{2}$.\(^{10}\)

At the present time, however, we cannot predict anything about correlation between two theories, the asymptotically free gauge theory and vortex model of dual strings. In unified theories there are some problems concerning conservations of parity and strangeness in corrections of order $e^2$. Weinberg\(^{10}\) has shown that parity and strangeness are automatically conserved in order $e^2$ if the strong gauge group is non-chiral. This possibility, however, may be extended to the case where the strong gauge group is chiral. We leave this possibility for further study.

* This possibility is also pointed out by A. Sato, J. Ishida and M. Horibe. (Private communication)
Other main results from our composite theory are:

i) The Weinberg angle is fixed to be $\sin^2 \theta_W = 3/8$, and created gauge-field coupling constants $g, g'$ and $f$ are related to the fine-structure constant $\alpha$ through (2.17), i.e., $g^2 = (5/3) g'^2 = f^2 = (8/3) \alpha^2$. This coincides with the result of Terazawa et al.\cite{1} and hence with that of Georgi and Glashow based on the $SU(5)$ gauge model.\cite{4}

ii) The $W$ and $Z$ bosons acquire masses $M_W = 62.05 \text{ GeV}$ and $M_Z = 78.49 \text{ GeV}$ through Eqs. (3.5) and (3.6).

iii) The mass of Higgs scalar $\eta_1$ is given by (3.12), i.e., $m_{\eta_1} \geq 2m_\pi$.

Some of these results, of course, could be modified if we take into account of further renormalizations of vertex parts, etc.

Finally we make a remark on the choice of auxiliary fields. In the path-integral approach we first face the question of how to choose auxiliary fields. Our unified model has been generated by choosing $K_1, K_2, K_3, U_\nu, U_\mu, U_\tau$ and $V_\mu^a$ as auxiliary fields and for special constraints on coupling constants (2.6) and (3.16). This choice is a minimal one for our purpose. Another choice of auxiliary fields leads to another composite theory which is realized by other constraints on coupling constants.

References

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