Transverse Momentum in the Multiple Production Processes and Urbaryon Rearrangement Model

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Several characteristic features of $p_T$ distribution are studied on the basis of the multibody amplitude of urbaryon rearrangement model which is a generalization of the all-angle formula for two-body hadronic reactions at high energies. Several characteristic structures of amplitudes of the model are compared with other models. The seagull behaviour observed in the longitudinal momentum dependence of the average $p_T$ of pion is explained as a result of long range correlation among the hadrons linked with each other by rearranged urbaryons.

It is also shown that the increase of average $p_T$ with particle mass and the dependence on unitary spins such as strangeness and charm are well reproduced by the model amplitudes.

§ 1. Introduction

One of the characteristic features of the high energy multiple production processes which has been observed in the cosmic ray experiments is that the transverse momentum $p_T$ of produced particles is restricted around several hundreds MeV/c.\(^1\) At present, accelerator experiments including ISR have yielded a great deal of precise data on the $p_T$ distribution. Main characteristics of the $p_T$ distribution may be summarized as follows:\(^2\)

1) The $p_T$ distribution $d\sigma/dp_T$ has a maximum at around several hundreds MeV/c.

2) The invariant cross section $E d\sigma/dp_T dp_T^2$ decreases exponentially in the small $p_T$ region ($p_T \leq 2$ GeV) and turns to power behaviour in the large $p_T$ region.

3) At large $p_T$ the cross section increases slowly with energy.

4) The average $p_T$, $\langle p_T \rangle$, increases with incident energy especially in the cosmic ray energy regions ($\sqrt{s} >$ TeV).

5) The $p_T$ distribution and $\langle p_T \rangle$ depend on the longitudinal momentum $p_L$ of the particle. Especially, the $\langle p_T \rangle$ of pion takes a minimum at $p_L = 0$, which is referred to as 'seagull behaviour'.

6) The $\langle p_T \rangle$ depends on the kind of particles.

In this paper, we shall discuss these characteristic features of the $p_T$ distribution on the basis of the urbaryon rearrangement model which reproduces satisfactorily the all-angle behaviour of two-body hadronic reactions at high energies.\(^3\) In the urbaryon rearrangement model, the hadrons participating in the process

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are strongly correlated through rearranged urbaryons which are represented by lines connecting the hadrons. This correlation effect would reveal one of the important differences of the amplitudes of the urbaryon rearrangement model from others such as the parton model and the multi-Regge model. The correlations among different channels are also taken into account by the multibody Veneziano amplitudes, but this model has correlations among rather a small number of channels compared to the urbaryon rearrangement model.

As is seen later, though our calculations are given only for exclusive process with three or four final particles, most of the above-mentioned characteristic features of $p_T$ distribution of inclusive reactions can be explained by regarding the final hadrons as clusters. It is also shown by the present authors that the $p_T$ dependence of power behaviour in large $p_T$ regions is successfully reproduced by the urbaryon rearrangement model with clustering effects.

In the next section the $n$-body urbaryon rearrangement amplitudes (URA) are constructed. Several characteristic structures of the URA are compared with other models in § 3. In § 4, the calculated $p_T$ distributions for pion and nucleon are shown and we give a comprehension of the occurrence of seagull behaviour in the $p_T$ distribution of pion. In § 5 the $p_T$ distributions are shown for the processes where the strange or charmed urbaryons are rearranged. Section 6 is devoted to discussion.

§ 2. The $n$-body urbaryon rearrangement amplitudes

From a viewpoint of the composite model of hadrons, the amplitude of a hadronic multiple production process $12\rightarrow 34\cdots n$ is given by a sum of URA as

$$T(12\rightarrow 34\cdots n)/s=\sum_{K} C_{K} T_{K}(s_1, \cdots, s_n) \quad \text{(Additivity)},$$

where $s=s_i$ is the square of total energy of the system and $s_2, \cdots, s_r$ are squared transfer momenta from one of the initial particles to one or more of the final particles or squared subenergies of two or more final particle’s systems in the process. Coefficients $C_{K}$ are obtained by counting the contributions from urbaryon rearrangement (UR) of type $K$, each of which is expressed by a UR diagram. Each UR diagram is specified by a set of numbers $(\bar{n}_{ij}) (i<j; i,j=1,2,\cdots,n)$. The $\bar{n}_{ij}$ represents number of urbaryons rearranged from a hadron $i$ to a hadron $j$. Here we have assumed the Okubo-Zweig-Iizuka rule which was introduced firstly by Okubo in the Sakata scheme of composite model. This rule forbids the occurrence of $\bar{n}_{ii}=0$.

On a way of rearrangement, an urbaryon which is represented by a line with

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*) The factor $1/s$ on the left-hand side is required to reproduce the $s$-dependence of the quasi-elastic processes which are caused by no urbaryon rearrangement. In order to guarantee the crossing symmetry, the factor should be replaced by a crossing symmetric function which reduces approximately to $1/s$ when the $s$-channel is the incident channel.
an arrow in the UR diagram passes through various channels and interacts with the others in the channels. From this picture, the UR amplitude is constructed by a product of factors each of which corresponds to an urbaryon line as

\[ T_K(s_i, \ldots, s_n) = g_K \prod_{i<j} \left[ \frac{s_{ij} - s_{ij}/\lambda_{ij}}{s_{ij} - s_{ij}/\lambda_{ij}} \right]^{n_{ij} - 1} \quad \text{(Factorizability)} \] (2.2)

In (2.2), \( g_K \) is a constant and \( s_{ij} = (p_i + p_j)^2 \), \( p_i \) and \( p_j \) being the four momenta of hadrons \( i \) and \( j \). It has been shown that experimental data of all-angle behaviour of the various two-body hadronic reactions are well reproduced if the constant parameters in (2.2) are taken as follows:

\[ \begin{aligned}
\xi_{ij} &= 1.2 & \text{if both of } i \text{ and } j \text{ are mesons or baryons,} \\
\lambda_{ij} &= 1.5 & \text{if one of } i \text{ and } j \text{ is a baryon and the other is a meson,} \\
2^2 \times 2m_Q^2 & \quad \text{for both of } i \text{ and } j \text{ are mesons,} \\
2 \times 3 \times 2m_Q^2 & \quad \text{for one of the } i \text{ and } j \text{ is a baryon and the other is a meson,} \\
3^2 \times 2m_Q^2 & \quad \text{for both of } i \text{ and } j \text{ are baryons,}
\end{aligned} \] (2.3)

(2.4)

where \( m_Q \) is considered the mass of rearranged urbaryon. The exponent \( \Gamma_{ij} \) in (2.2) represents the correlation effects on the urbaryon rearranged from hadron \( i \) to hadron \( j \) and given by a multiplication of correlation factor \( \gamma_{n_\beta}(s_\beta)/n_\beta \) over all the channels \( \beta \) through which the urbaryon is rearranged:

\[ \Gamma_{ij} = \prod_\beta \left[ \frac{\gamma_{n_\beta}(s_\beta)}{n_\beta} \right]. \] (2.5)

Each correlation factor depends on the squared transfer momentum or subenergy of the channel as well as on the number of urbaryons \( n_\beta \) passing through the channel \( \beta \). If, in the small \( |s_\beta| \) region, the \( \gamma_{n_\beta}(s_\beta) \) is related to the linear Regge trajectory \( \alpha_{n_\beta}(s_\beta) \) corresponding to the composite system of \( n_\beta \) urbaryons and/or antiurbaryons as

\[ \gamma_{n_\beta}(s_\beta) = 1 - \alpha_{n_\beta}(s_\beta) = 1 - \alpha_{n_\beta}(0) - \alpha' s_\beta, \] (2.6)

then the UR amplitude shows behaviour similar to that of the Regge pole amplitude with intercept \( \alpha_{n_\beta}(0) \) and slope \( \alpha' \). In the large \( |s_\beta| \) region the effect of correlation from the channel \( \beta \) will disappear on the urbaryon. This leads to

\[ \gamma_{n_\beta}(s_\beta)/n_\beta \rightarrow 1 \quad (|s_\beta| \gg s_{\beta_0}), \] (2.7)

where \( s_{\beta_0} \) denotes some critical value of \( s_\beta \) around which the \( \gamma_{n_\beta}(s_\beta) \) transfer its behaviour from the one shown (2.6) to the one given by (2.7) and vice versa (see Fig. 1). One of the possible expressions of \( \gamma_{n_\beta}(s_\beta) \) which satisfies such picture is given by

\[ \gamma_{n_\beta}(s_\beta) = \gamma_{n_\beta}(0) - \alpha' s_\beta - \alpha' d \ln \{1 + \exp[(s_{\beta_0} - s_\beta)/d] - \exp[s_{\beta_0}/d]\}, \] (2.8)
\[ \gamma_{s_0}(0) = 1 - \alpha_{s_0}(0) \quad \text{and} \quad s_{b_k} = \left(1 - \alpha_{b_k}(0) - n_{b_k}\right)/\alpha'. \]

It happens that in one or more channels no urbayons pass through the channels. This is a case of disconnected type of UR. In this case, the UR diagram is separated into several connected parts. For this disconnected type the URA are assumed to be given by a product of connected type sub-amplitudes and diffraction amplitudes. For example, if \( n_{b_k} = 0 \) in a UR diagram such as shown in Fig. 2, the URA are given by

\[ g_0 T_C(s_1, \ldots, s_s) i\theta(-s_{b_k}) \exp(B_{b_k}s_{b_k}) T_{C'}(s_1, \ldots, s_s), \]

(2.9)

where \( T_C(s_1, \ldots, s_s) \) and \( T_{C'}(s_1, \ldots, s_s) \) are the UR amplitudes of the connected type given by (2.2) corresponding to UR diagram \( C \) and \( C' \) respectively. Two urbayons which belong respectively to different subsystems \( C \) and \( C' \) interact with each other through diffractive scattering described in terms of the Pomeron exchange. Total effect due to scattering of two subsystems \( C \) and \( C' \) is represented by a factor \( i\theta(-s_{b_k}) \exp(B_{b_k}s_{b_k}) \). The step function \( \theta(-s_{b_k}) \) is introduced in order to forbid the contribution from a diagram such as shown in Fig. 2(b) where no urbayons transfer through the time-like channel \( b_k \), i.e., \( n_{b_k} = 0 \) for \( s_{b_k} > 0 \). The slope parameter \( B_{b_k} \) are assumed to be given by

\[ B_{b_k} = \begin{cases} 4B_0 & \text{for meson-meson collision} \\ 6B_0 & \text{for meson-baryon collision} \\ 9B_0 & \text{for baryon (antibaryon)-baryon collision,} \end{cases} \]

(2.10)

where the parameter \( B_0 \) is adjusted by two-body diffraction scattering and is chosen as \( 0.5 (\text{GeV}/c)^{-2} \). The factor \( (\xi - t/\lambda_0)^{-\theta} \) multiplied to \( \exp(B_{b_k}s_{b_k}) \) in the nucleon-nucleon diffractive scattering amplitude makes the slope of \( \ln(d\sigma/dt) \) steeper for smaller \( |t| \) which is in favour of experiments.

§ 3. Characteristic structures of urbayon rearrangement amplitudes

In this section we note several aspects of characteristic structures of URA introduced in § 2 comparing with those of the one-particle-exchange (OPE) type
models and the Veneziano models.

3.1 Dual structure

In the OPE type models, scattering in the resonance region is described by the sum of resonance amplitudes and OPE or Regge pole exchange amplitudes which have poles in the s-channel and t or u-channels, respectively. The two amplitudes having poles in the s-channel and t or u-channels are independent of the other. In contrast to this, the existence or absence of poles in a certain channel gives constraints for the existence of poles in the other channels in the URA and Veneziano amplitudes (VA). Table I shows in what channels the poles of the amplitudes do appear for the two-body reactions.

<table>
<thead>
<tr>
<th>channel</th>
<th>( s )</th>
<th>( t )</th>
<th>( u )</th>
</tr>
</thead>
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<tr>
<td>OPE model</td>
<td>( \frac{1}{s-s_g} )</td>
<td>( \bigcirc )</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>( \bigcirc )</td>
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<tr>
<td></td>
<td>( \frac{1}{t-t_x} )</td>
<td></td>
<td>( \bigcirc )</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{u-u_g} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Veneziano model</td>
<td>( V(s, t) )</td>
<td>( \bigcirc )</td>
<td>( \bigcirc )</td>
</tr>
<tr>
<td></td>
<td>( V(t, u) )</td>
<td></td>
<td>( \bigcirc )</td>
</tr>
<tr>
<td></td>
<td>( V(s, u) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table I. Circles denote the existence of poles or cuts. Figures for URA represent numbers of urbaryons \( n \) in the relevant channel \( \beta \).

** The OPE type model used hereafter represents the models of one-particle-exchange type including the Regge pole exchange, the interference models and the multiperipheral Regge exchange models, etc.

** Strictly speaking, the URA have cuts in general because the exponent of \( \Gamma_H \) defined by (2.5) is not integer.
The essential difference between the OPE type amplitudes and the URA or VA lies in the choice of primary amplitudes. For example, the $K^+n \rightarrow K^0p$ reaction is described in terms of superposition of meson exchange and baryon exchange amplitudes in the OPE type models. This type of approaches, however, leads to the ‘double counting’ which has been excluded by analyses of $\pi N$ scattering based on the finite energy sum rule. On the other hand, the $X$-type amplitude and the $Y(t,u)$ amplitude describe the reaction in the UR model and the Veneziano model, respectively. These amplitudes give significant contributions to both $|t| \approx 0$ and $|u| \approx 0$ region in a unified way.

Similar differences are seen in the multiple production process. Except for the $s$-channel resonance decay types, there are nine types of OPE graphs for the $K^-p \rightarrow \pi^+\pi^-A$ reaction. Each of the nine amplitudes gives a dominant contribution to a respective region shown around the Van Hove plots in Fig. 3.

There are five types of URA which contribute to the $K^-p \rightarrow \pi^+\pi^-A$. Each of them gives contributions over several regions of the Van Hove plots changing its correspondence to the OPE amplitudes as illustrated for the $H_3$-type URA. The VA contributes also over several regions but in somewhat different way which is discussed in the next subsection.

### 3-2 Differences between URA and VA

As to the dual character, many similarities and correspondences are seen between the URA and the VA. As is well known, the $n$-body Veneziano amplitudes have one-to-one correspondence to the connected UR diagrams in the cases where only mesons are concerned. Therefore, in these cases the VA also correspond to the URA. The correspondence between the VA and URA also exists if an incoming baryon is connected with the outgoing baryon by two urbaryon lines and no other baryons participate in the UR diagram. This is the case of the $H$-type or $X$-type URA in the processes $MB \rightarrow BMM \cdots M$. If four or more baryons participate as in the case of $\overline{BB} \rightarrow \overline{BB}$ process, the URA have pole contributions representing the exotic states, although the correspondence between URA and VA has validity to a certain extent. In the case where two baryons are connected by one urbaryon line as the $Z$-type, there also appear exotic poles. For the $W$-type URA, where at least two baryons are indirectly connected by urbaryon lines through intermediation of three or more mesons, there is no correspondence to the multibody VA. For example the $W$-type URA for $MB \rightarrow BMM$
Table II.

<table>
<thead>
<tr>
<th>channel</th>
<th>$K^-p$</th>
<th>$K^-\to\pi^-$</th>
<th>$K^-\to\rho^-$</th>
<th>$p\to\pi^+$</th>
<th>$p\to\pi^-$</th>
<th>$\pi^+\pi^-$</th>
<th>$\Lambda\pi^+$</th>
<th>$\Lambda\pi^-$</th>
<th>correspondence</th>
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<td>$K^{*+}$</td>
<td>exotic</td>
<td>$p$</td>
<td>$\Delta^{++}$</td>
<td>$N$</td>
<td>$K^+$</td>
<td>$\rho^+$</td>
<td>$\Sigma^+$</td>
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<td>4</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$Z_2$</td>
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<td>4</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
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</tr>
<tr>
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<td>5</td>
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<td>4</td>
<td>3</td>
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<td>3</td>
<td>5</td>
<td>D</td>
</tr>
<tr>
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<td>3</td>
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<tr>
<td>$B$</td>
<td>$\pi^+\Sigma^-\Lambda$</td>
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<td>$\pi^+\Lambda$</td>
<td>$\pi^+\Lambda$</td>
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<tr>
<td>$C$</td>
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<td>$\pi^+\Lambda$</td>
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<td>$E$</td>
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<td>$F$</td>
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<td>$Z_4'$</td>
</tr>
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process has six baryon-exchange channels\(^6\) \((n_p=3)\) while the five-body VA can accept trajectories only in five channels.

In Table II, the URA are compared to the VA in the \(K^-p\to\pi^-\pi^+A\) process.

For six-body or more multibody amplitudes, there appear non-exotic channels which contain three or more hadrons. In these cases, the VA have many poles in such channels while the URA have only cuts and are considered to describe effectively the contributions of these poles.

### 3.3 Correlations between hadrons

In UR model, two kinds of correlations arise in the incoming and/or outgoing two hadrons. The first is represented by urbaryon lines connecting the two hadrons. An urbaryon line connecting two hadron \(i\) and \(j\) which represents pair annihilation or creation of urbaryon in the channel \(s_{ij}=(p_i+p_j)^2\) corresponds to a factor \(F(s_{ij})^{-r_{ij}}\) in the URA. This factor continues to play the role through the whole region of the process irrespective of the value of \(s_{ij}\). This brings out a striking contrast to the OPE model where the dominant amplitude in a certain region does not contain any channel variable with large magnitudes in that region. In this sense in the UR model the incoming and/or outgoing hadrons have long range correlations if they are connected by urbaryon lines. As shown in the next section, this long range correlation is important for explanation of the seagull behaviour observed in the \(\langle p_T\rangle\) distribution.

Another kind of correlation is introduced through the exponent \(r_{ij}\) defined by (2.5). This correlation represents total effect on the urbaryon from the other urbaryons propagating in the same channels. This effect is important only in the region with small channel variables.

### 3.4 Singularity structure

The URA have singularity at \(s_{ij}=\bar{\xi}_{ij}\lambda_{ij}\) which correspond to the pair annihilation and/or creation of urbaryon lines. The degree of the singularity of pole is \(\bar{\rho}_{ij}r_{ij}\) which depends on the variables in the coupled channels. This is contrast to the VA. The VA have infinite numbers of simple poles in the non-exotic channels at equal intervals of \(s_{ij}\). The sum of pole contributions is regarded as the sum of pole contributions in a crossed channel. In Fig. 4 the imaginary part of \(H\)-type URA for the two-body reaction is compared with those of the VA, \(V(s,t)\). It is also compared with the imaginary part of the resonance correlation model\(^{10}\) (the peripheral model) which is written as \(J_0(b\sqrt{-t})\) and corresponds to the sequence of poles with

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**Fig. 4.** Imaginary part of \(H\)-type URA.
equal spacing of $\sqrt{s}$. As shown in Fig. 4, the imaginary part of the $H$-type URA has only one zero, though the gross behaviour of URA is very similar to the other amplitudes in the small $|t|$ region.

The numerical values of singularity position of the URA are

$$\xi_{ij} = \begin{cases} 
1.2 \sim 1.0 \text{ GeV}^2 & \text{for } MM \text{ channel} \\
1.8 \sim 2.2 \text{ GeV}^2 & \text{for } MB \text{ channel} \\
2.7 \sim 3.4 \text{ GeV}^2 & \text{for } BB \text{ channel.} 
\end{cases} \quad (3.1)$$

To avoid divergences at the poles, imaginary parts of the parameters $\xi_{ij}$ have to be introduced for the channel $s_{ij} > 0$. The values of $\xi_{ij}$ in (3.1) suggest that the poles or cuts in the URA should be regarded as the total effect from those of many resonances in the channel but not to a single resonance. This structure of the URA enhances the mass distribution of two or more hadron’s system around $\xi_{ij}$. This effect can be interpreted as the cluster structure observed in the multiple production process by adjusting the imaginary parts of $\xi_{ij}$.

The outgoing hadron in the UR diagram can be also regarded as the cluster if the mass is taken as that of the cluster. Taking account of these structures of URA, it is conjectured that the URA for exclusive processes with rather a small number of produced hadrons have behaviour very close to those of inclusive processes as illustrated in Fig. 5.

3-5 Symmetry breaking

The breakings of the unitary symmetry in the URA are introduced through three different ways:
(a) The parameters $\lambda_{ij}$ are related to the effective masses of urbaryons inside hadrons. The mass of urbaryons $p, n, \bar{n}$ and $c$ are taken as

$$m_p = m_n = m_q, \quad m_i = m_q + \Delta m_i, \quad m_c = m_q + \Delta m_c,$$

where

$$m_q = 0.35 \sim 0.4 \text{ GeV}, \quad \Delta m_i \approx 0.15, \quad \Delta m_c \approx 1.0 \text{ GeV}$$

which are consistent with mass relations of hadrons. The breaking of $\lambda_{ij}$ gives a scale change of $s_{ij}$ and enhances relatively the production of large mass particle in the region where $s_{ij}$ is large.

(b) The correlation functions $\gamma_{ij}(s_j)$ are considered to correspond to the Regge
trajectories in the small $|s_j|$ region. Therefore, similar symmetry breaking effects are introduced as the Regge trajectories in the small $|s_j|$ region. Therefore, similar symmetry breaking effects are introduced as the Regge pole model. In the actual calculation, we assume the following relations among the intercepts $\alpha(0)$ and slopes $\alpha'$:

\[ \alpha'_{\phi} \alpha'_{\rho} = \alpha'^{2}_{\kappa}, \quad \alpha'_{\phi} \alpha'_{\rho} = \alpha'^{2}_{\rho}, \]

\[ \alpha_{\phi}(0) + \alpha_{\rho}(0) = 2\alpha_{\kappa}(0), \]

\[ \alpha_{\phi}(0) + \alpha_{\rho}(0) = 2\alpha_{\rho}(0). \]

These symmetry breaking effects are important in the small momentum transfer regions.

(c) The symmetry breaking due to the mass differences of the incoming and outgoing hadrons. This breaking affects the cross section by kinematics and also the amplitudes through the channel variables $s_j$.

§ 4. The $p_T$ distributions of pion and nucleon and the seagull behaviour

Awaya and one of the authors\(^6\) studied the longitudinal momentum distributions of $NN\rightarrow\pi NN$ and $\pi N\rightarrow\pi\pi N$ processes by calculating the longitudinal phase space distribution on the basis of various types of the URA. In this section we examine the $p_T$ distributions of these processes in terms of invariant cross sections $Ed\sigma/dp_T^2d\pi_L$ given by URA.

Typical calculated results for the $p_T$ distributions are shown in Fig. 6. They exhibit the following characteristic behaviour.

(a) The invariant cross section of nucleon has a roundish peak at $p_T=0$ while that of pion has a sharp peak especially near $x = 2p_L/\sqrt{s} = 0$.

(b) In the medium $p_T$ region (0.2 GeV/c $\leq p_T \leq$ 2.0 GeV/c) the invariant cross section decreases exponentially as $e^{-b p_T}$. The values of exponent $b$ are about 6 both for pion and nucleon which fit strikingly to experiments.

(c) In the large $p_T$ region, the exponential behaviour of cross section turns to the power behaviour $p_T^{-N}$ where $N=10\sim14$.\(^5\)
These features of $p_T$ dependence are just like the experimentally observed inclusive cross sections. This fact supports the conjecture mentioned in subsection 3-3 that the URA of exclusive process are also responsible to the inclusive processes if some of the outgoing hadrons are replaced by clusters of hadrons with a suitable parameter change. A realization of this conjecture is made recently by the present authors in the large $p_T$ region.\(^1^0\)

The calculated longitudinal momentum dependence of average $p_T$, $\langle p_T \rangle$ is shown in Fig. 7. The seagull behaviour\(^8\) is reproduced clearly in the $\langle p_T \rangle$ of pion similarly to the experimental data. The behaviour does not appear in the calculated $\langle p_T \rangle$ distribution for nucleon as well as experimental observations. In Fig. 8, the growth of seagull behaviour with incident momentum $p_{\text{inc}}$ is shown for $\pi N \rightarrow \pi \pi N$ reaction. At incident momenta below 3 GeV/c, the seagull behaviour is not seen because the $p_T$ distribution is much restrained by kinematical boundaries. Above 3 GeV/c the dip of $\langle p_T \rangle$ at $x = 0$ becomes clear. In Fig. 9, the $s$-dependence of average $p_T$ at $x = 0$ and that of all $x$ regions are compared. From these curves one can see that the seagull behaviour becomes more and more remarkable with increasing energy.

Here let us explain the reason why the seagull behaviour does appear so clearly in the $\langle p_T \rangle$ distribution of pion and not in those of nucleon. To serve as an illustration let us take the $X$-type URA for $NN \rightarrow N\pi N$ reaction. The URA
is expressed as

\[ T \propto (\xi_{X^N} - s_{23}/9\lambda_0)^{-2r_{23}} (\xi_{X^N} - s_{13}/9\lambda_0)^{-2r_{13}} (\xi_{X^N} - s_{15}/9\lambda_0)^{-2r_{15}} \times (\xi_{N} - s_{34}/6\lambda_0)^{-r_{34}} (\xi_{N} - s_{14}/6\lambda_0)^{-r_{14}}, \]  

(4.1)

where \(s_{ij}\) are defined in Fig. 10. Here we note that all factors in (4.1) have effects for all regions of the process though the exponents \(T\) varies significantly. The \(p_T\) and \(p_L\) dependence of the squared momentum transfers from the initial nucleon to the final nucleon and pion \(s_{23}\) and \(s_{24}\) are shown in Fig. 10. The contour lines of \(s_{24}\) crowd densely in the small \(x\) region. This means that rapid change of \(s_{24}\) with increasing of \(p_T\) of pion. This is not the case in the nucleon \(p_T\) dependence of \(s_{23}\). Therefore the magnitudes of URA decrease rapidly with increase of around \(x=0\) region and the \(\langle p_T \rangle\) of pion is made small at \(x=0\).

The argument given here is only applicable when the amplitudes describe in a unified way throughout all over the region of a process such as the UR model and the Veneziano model. On the other hand, in the OPE type model the variable \(s_{24}\) is not involved in the amplitude which dominates at around \(x=0\). These situations are illustrated in Fig. 10.

Consequently the occurrence of seagull behaviour is considered a manifestation of duality structure and long range correlation among the hadrons participating in the process connected by urubyon lines in the UR diagrams. The seagull behaviour also implies that at least in the small \(p_T\) region the amplitudes which describe the multiple production processes depend directly on the invariant variables, i.e., the squared momentum transfers and squared subenergies, not directly on \(p_T\) nor \(p_L\).

§ 5. The \(p_T\) distributions of hadrons composed of the strange or charmed urubyon

The effects of the breaking of unitary symmetry of the URA are introduced through several ways as discussed in subsection 3-5.

In Fig. 11, the \(p_T\) distributions are shown for the processes \(\pi N \rightarrow K\bar{K}N\), \(\pi N \rightarrow \psi KA\), \(\pi N \rightarrow D\bar{D}N\) and \(\pi N \rightarrow \psi DB_c\). The masses of charmed meson \(D = (c\bar{n})\) and charmed baryon \(B_c = (c\bar{p}n)\) are taken as 1.9 GeV and 2.4 GeV, respectively. As shown in Fig. 11, in the cases where one or two \(\lambda\) urubyons are rearranged the \(p_T\) distributions decrease very slowly in the small \(p_T\) region (\(p_T < 0.5\) GeV/c) com-
Compared to those of ordinary hadrons and have comparable or larger magnitudes in the larger $p_T$ region. This tendency is strengthened further for the processes where the charmed baryons are rearranged. This is consistent with recent experimental information of $\psi$ production.\(^{19}\) The invariant cross sections of these processes decrease rather rapidly with increasing $p_T$.

The calculated average $p_T$ for various hadrons are shown in Fig. 12. They
are compared with experimental data.

As shown in Fig. 13, the integrated cross sections of three-body production processes which are caused by rearrangements of the λ or c urbaryons have small magnitudes especially for that of two charmed urbaryon rearrangement and rapidly decrease with increasing energy. However, since they give significantly large contributions to the large \( p_T \) regions, it is expected that they play the role to contribute to the observed rising of invariant cross section of inclusive reactions. Similar contributions are also expected if we take the effects of cluster or jet production in the multiple production processes.\(^5\) In order to clarify which of two origins, i.e., new heavy particle production or cluster (jet) production is more important it is needed to study the s-dependence of the rising of invariant cross sections in large \( p_T \) region around the thresholds of charmed hadron productions.

§ 6. Discussion

In preceding sections we have studied the invariant cross sections especially \( p_T \) distribution of multiple productions on the basis of URA which are the generalization of all-angle formula for high energy two-body reactions. The URA describe well coherently the hadronic reaction in the small momentum transfer region as well as in the large momentum transfer region. Therefore URA offer a useful method to study the multiple production process where both regions are simultaneously included. It is shown that the characteristic features of \( p_T \) distributions of multiple production with large multiplicities summarized in § 1 are already exhibited by the calculated invariant cross sections for three-body production processes.

The occurrence of seagull behaviour, the dip structure in the \( \langle p_T \rangle \) distribution of pion at \( x = 0 \), is explained by duality structure of amplitudes or the long range correlations among initial and final hadrons which remain up to the large momentum transfer region.

Furthermore, in the multiple production where strange and/or charmed urbaryons are rearranged, the URA are much suppressed in small \( p_T \) region. In these processes the cross sections decrease slowly with increasing \( p_T \) and give large \( \langle p_T \rangle \). Thus, at certain large \( p_T \) they have comparable or larger magnitudes than ordinary hadron production processes. Therefore we can assign the significant contribution of charmed hadron productions to one of the origins of the increase of invariant cross sections of inclusive reactions at large \( p_T \). If the outgoing hadrons are replaced by the clusters of hadrons in the URA the increase of the cross sections at large \( p_T \) can also be explained.\(^6\) Both of the charmed hadron production and cluster production can contribute to increase \( \langle p_T \rangle \) with energy which is observed by experiment (the feature 4) in § 1). Though to conclude which production is the main origin of increase of \( \langle p_T \rangle \) the more detail experimental analyses are required, the rapid increase of \( \langle p_T \rangle \) in the cosmic ray region (\( p_{\text{inc}} \)
Transverse Momentum in the Multiple Production Processes

$\geq$TeV) may be considered to suggest the existence of further much freedom of heavier urbaryons than charmed one\(^{06}\) or rapid increase of masses of clusters.

In the large $p_T$ region, the decrease of experimental invariant cross section turns to the power behaviour from exponential one. This change of behaviour is also well reproduced by URA.

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