Phenomenological Analysis of Nucleon Diffraction Dissociation in the Drell-Hiida-Deck Model

Tachishige HIROSE, Kimiyo KANAI and Tetsuro KOBAYASHI

Department of Physics, Tokyo Metropolitan University, Tokyo 158

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We quantitatively investigate diffraction processes \( \pi^+ p \rightarrow \pi^+ (\pi^0 n) \) and \( \pi^- p \rightarrow \pi^- (\pi^0 p) \) at 16 GeV/c and some other reactions at different energies by the Drell-Hiida-Deck (DHD) model with a single adjustable parameter. It is found that without any absorption effects our DHD model can fairly well explain global features of diffraction dissociation: low \( \pi N \)-mass enhancement, sharp forward peak in momentum transfer squared, slope-mass correlation and decay angular distributions in the Gottfried-Jackson frame. Some comments are made on the limit of applicability of our model.

§ 1. Introduction

Salient features of diffraction dissociation (abbreviated as \( dd \) hereafter) in inelastic hadron reactions have been revealed from experimental data recently accumulated in the high energy region from several GeV to the ISR.\(^1,2\) Nature of Pomeron would be clarified through detailed experimental knowledge of \( dd \) together with that of elastic scattering.

The purpose of the present paper is to investigate quantitatively how successfully we can explain experimental data on \( dd \) by the Drell-Hiida-Deck (DHD) model,\(^3\) here our DHD model is based upon the elementary pion exchange.

There exist various DHD model analyses\(^4,5,6,7,8\) for nucleon \( dd \) and meson \( dd \). As compared with meson systems, however, for nucleon systems there does not seem to exist a thorough analysis including a partial wave analysis. Increased knowledge about nucleon resonances obtained from \( \pi N \) phase shift analysis has also motivated us to investigate a diffractively dissociated \( \pi N \) system.

We compare quantitatively the DHD model with full experimental data like mass, momentum transfer squared and decay angular distributions of the diffractively dissociated \( \pi N \) system in \( \pi^+ p \rightarrow \pi^+ \pi^- n \),\(^9\) \( \pi^- p \rightarrow \pi^- \pi^0 p \)\(^10\) at 16 GeV/c, \( K^- n \rightarrow K^- \pi^- p \) at 12 GeV/c\(^11\) and \( p p \rightarrow p \pi^+ n \) at 1500 GeV/c.\(^11,12\)

We have found that the DHD model can fairly well reproduce the global features of the \( \pi N - dd \) system, particularly, in the small momentum transfer region. Our model would be regarded as a first step toward further detailed studies of \( dd \) mechanism.

In § 2 we formulate the DHD model. Section 3 is devoted to quantitative comparisons of the DHD model with experimental data. Section 4 contains arguments on possible variation of parametrizations. Discussion and concluding remarks
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are given in § 5.

§ 2. Structure of the DHD amplitude

Consider the reaction \( \pi(p_1) + p(p_2) \rightarrow \pi(q_1) + \pi(q_2) + N(q_3) \), where letters in parentheses stand for respective momentum. The DHD amplitude is represented by the pion exchange accompanied by elastic scattering (Fig. 1). Apart from irrelevant numerical factors we can write the scattering amplitude as follows:

\[
T = M_{\pi\pi}(s_1, t_1, t_2) J_s(t_3, M^2)^{\pi^+N}(t_2). \tag{2.1}
\]

Kinematical variables are defined as \( s = (p_1 + p_2)^2 \), \( s_1 = (q_1 + q_2)^2 \), \( M^2 = (q_2 + q_3)^2 \), \( t_1 = (p_1 - q_1)^2 \) and \( t_2 = (p_2 - q_2)^2 \), and the pion mass and nucleon mass are designated by \( \mu \) and \( M \), respectively.

In Eq. (2.1) \( M_{\pi\pi} \) represents a \( \pi\pi \) elastic scattering amplitude dominated by Pomeron exchange and is reasonably assumed to have the form

\[
M_{\pi\pi}(s_1, t_1, t_2) = \Lambda c^{\alpha_p(t_1)} F(t_2) \tag{2.2}
\]

with the Pomeron trajectory \( \alpha_p(t_1) = 1 + 0.3t_1 \). \( F(t_2) \) stands for virtual pion effects in \( \pi\pi \) scattering. Factorization and experimental \( \pi N \) and \( NN \) total cross sections give us \( \Lambda = 14 \text{ mb} \approx 36 \text{ (GeV/c)}^{-2} \), and \( B = 3 \text{ (GeV/c)}^{-2} \) is adopted from the one pion exchange model analysis.\(^{12}\) The parameter \( s_0 \) is chosen to be \( 1 \text{ (GeV)}^{-2} \) throughout the present analysis.

As for the pion propagator \( J_s \) we assume the form

\[
J_s(t_3, M^2) = \frac{(M^2)^{\alpha_\pi(t_3)}}{t_2 - \mu^2}, \tag{2.3}
\]

where \( \alpha_\pi(t_2) = \alpha_\pi'(t_2 - \mu^2) \) with \( \alpha_\pi' = 1 \text{ (GeV/c)}^{-2} \). Since the production process of \( \pi N \) system should be described by nucleon resonances in addition to the DHD

mechanism, duality constrains the pion exchange to be real. If one introduces a phase factor into Eq. (2.3), the troublesome double counting will occur in the whole amplitude. The phase factor has an influence on the interference of the pion exchange with other terms, but no influence on the differential cross section based on the DHD amplitude.

Finally, the pion-nucleon vertex function is

\[ V_{\pi NN}(t_z) = GF''(t_z) \bar{u}(q_3) \gamma_5 \sigma \mu (p_3) \psi^\nu, \quad (2.4) \]

where \( G^2/4\pi = 14 \) and \( F''(t_z) \) is a vertex form factor which satisfies \( F''(n^2) = 1 \). Hereafter we define the overall form factor by \( F(t_z) = F'(t_z) F''(t_z) \) and parameterize it as

\[ F(t_z) = e^{f(t_z - n^2)}, \quad (2.5) \]

where \( f \) is the sole adjustable parameter in our formalism.

In the \( \pi^- n \) rest frame, i.e., \( q_z + q_t = 0 \), the differential cross section for Fig. 1 is expressed as

\[ d\sigma = \frac{(\sqrt{2} G)^2}{4\pi} \frac{1}{(2\pi)^3} \frac{|q_3|}{32B^2} |M_{\pi n}|^2 \times \left[ \frac{-t_z}{t_z - n^2} (M^2)_{\pi n} e^{f(t_z - n^2)} \right]^2 dM d(-t_1) d(\cos \theta) d\phi, \quad (2.6) \]

where \( B = [(p_1 \cdot p_\pi)^2 - m^2 n^2]^{1/2} \) and, \( \theta \) and \( \phi \) are the decay polar and azimuthal angles, respectively.

§ 3. Comparison with experimental data

3.1 \( \pi^- p \rightarrow \pi^- (\pi^- n) \)

Unambiguous predictions of the DHD model will be expected for this reaction, because the Pomeron dominates \( \pi^- \pi^- \) elastic scattering. We have calculated Eq. (2.6) and compared it with experimental data at 16 GeV/c, i.e., 6460 events corresponding to the \( I = 1/2, \pi^- n \) system purified from the \( J(1236) \) by means of the multidimensional analysis. 9

3.1.1 Mass distributions

Figure 2 shows mass distribution with \( t_1 \)-cut \( (t_1 = t_1 - t_{min}) \) \( 0 < |t_1'| < 0.2 \) (GeV/c)^2. The most reasonable \( dd \pi^- n \)-mass distribution has been obtained by choosing \( f = 1.8 \) (GeV/c)^2 which is adopted in common with all cases dealt with in this paper. It is clearly seen that the DHD model can well reproduce the low mass enhancement characteristic of diffractively dissociated systems.

For larger mass region \( (M \geq 1.5 \text{ GeV}) \), however, contributions besides the DHD mechanism, e.g., nucleon resonances will become significant.

From Fig. 3 it is seen that the better agreement is obtained for smaller \( |t_1'| \) region; the peak position in \( d\sigma/dM \) moves into smaller mass as \( |t_1'| \) decreases.
Fig. 2. Invariant mass $M(\pi^+ n)$ distributions for $0<|t'_1|<0.2(\text{GeV/c})^2$. Each curve represents our DHD-model calculation for different values of the parameter $f$ which appears in the form factor $e^{ft'_1-n^2}$.

Fig. 3. Invariant mass $M(\pi^+ n)$ distributions in various $|t'_1|$ regions. Solid lines represent the DHD-model calculation for $f=1.8$.

Fig. 4. Momentum transfer squared distributions in various $\pi^+ n$ mass regions. Solid lines represent the DHD-model calculations. The value represents an average value of slope parameter of the DHD-model calculation in $0.05<|t'_1|<0.2(\text{GeV/c})^2$. 
3.1.2 Momentum transfer squared distribution and slope-mass correlation

The momentum transfer squared distribution can be expressed approximately by an exponential form

$$\frac{d\sigma}{dMdt_1'} \sim \exp(-b(M)t_1')$$

(3.1)

for small $|t_1'|$. We can see from Fig. 4 that the DHD model can well explain the observed characteristic features of various mass regions: (i) the sharp forward peak in the momentum transfer squared distribution with $b(M) > b_{el}$ for $M < 1.35$ GeV, $b_{el}$ being the slope parameter of elastic scattering and (ii) the slope parameter $b(M)$ decreases systematically with increasing $M$. This can be attributed to the simple kinematical relation inherent to the double-peripheral diagram adopted by us.\(^7\)

Generally speaking, for small $|t_1'|$ and low $M$ regions our DHD model is quantitatively in good agreement with experimental data. Characteristic features (i) and (ii) mentioned above appear to be similar to both nucleon $dd$ and meson $dd$. The striking feature around $|t_1'| \approx 0.2 (\text{GeV}/c)^2$ observed in momentum transfer squared distribution for nucleon $dd$ cannot be seen in that for meson $dd$. This may suggest that some other contributions, for instance, the nucleon resonance or absorption are needed for $|t_1'| > 0.2 (\text{GeV}/c)^2$ as the second order effect.

3.1.3 Decay angular distributions

Let us examine decay angular distributions from diffractively dissociated $\pi N$ system in the Gottfried-Jackson frame, where the direction $\mathbf{p}_1 \times \mathbf{q}_1$ has been chosen to be the positive $y$ axis,\(^8\) $\mathbf{p}_1$ and $\mathbf{q}_1$ being unit vectors along $\mathbf{p}_1$ and $\mathbf{q}_1$, respectively.

Figures 5 and 6 show the comparison of the DHD model calculations with experimental data on various mass regions. On the basis of the assumption of the $t$-channel helicity conservation for production of the nucleon resonances it

![Fig. 5. Decay azimuthal angle $\phi_{az}$ distributions in the Gottfried-Jackson frame in various ($\pi^+ n$) mass regions. Solid lines indicate the DHD model calculations.](https://academic.oup.com/ptp/article-abstract/57/4/1334/1933451)

\(^8\) Our $\phi_{az}$ is different from that of Ref. 7) by $\pi$.\(\)
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1.339 at 16 GeV/c

1.5 < M < 1.6

16 GeV/c

\[ \text{Fig. 6. Decay polar angle } \cos \theta_{GJ} \text{ distributions in the Gottfried-Jackson frame in various mass regions. Calculated curves have been renormalized to the cross section in each mass region.} \]

might be reasonable to add a certain amount of flat angular distributions to the \( \phi_{GJ} \) distribution predicted by the DHD model so as to obtain the same magnitude of cross section as the experimental one. In the \( \cos \theta_{GJ} \) distribution, however, we simply normalized the cross section obtained from the DHD model to the experimental one, because we do not know the spin structure in each mass region of \( \pi N \) system.

Here we have the kinematical relation

\[
s_j = c(M, t'_1) \cdot d(M, t'_2) \cdot \sin \theta_{GJ} \cdot \cos \phi_{GJ},
\]

where \( c(M, t'_1), d(M, t'_2) \) and \( e(M, t'_1) \) are known positive functions of \( M \) and \( t'_1 \). Since the cross section is proportional to \( s_j \cos \theta_{GJ} \), it is maximized at \( \cos \phi_{GJ} = -1 \) for fixed \( \phi_{GJ} \). Thus the \( \phi_{GJ} \) distribution has a peak at \( \phi_{GJ} = \pi \).

Strong violation of the t-channel helicity conservation recently observed in \( \pi V \) system\(^{10,13} \) can be understood on the bases of the pion exchange DHD model.

3.2 \( \pi^- p \rightarrow \pi^- (\pi^0 p) \)

The experimental data with which we compare the DHD model are 3945 events of the \( dd \) in \( \pi^- p \rightarrow \pi^- (\pi^0 p) \) at 16 GeV/c\(^{39,10} \) they are purified by subtracting the \( J(1236), \rho \) and \( g \).

The DHD model for this process needs Reggeons besides the Pomeron. The \( \rho \) cannot contribute to \( \pi^- \pi^0 \) elastic scattering part due to the decoupling of a \( \pi^- \pi^0 \) state to the isospin one state. The Reggeon \( f \) will be a candidate, because resonances higher than \( g \) would contribute to \( M_{zz} \).

Assuming the production of system with isospin 1/2, one obtains

\[
R = \frac{\sigma(\pi^- p \rightarrow \pi^- (\pi^0 n))}{\sigma(\pi^- p \rightarrow \pi^- (\pi^0 p))} = 2
\]

from isospin considerations. The DHD model based upon the Pomeron and the \( f \) in \( M_{zz} \) gives \( R \approx 1.5 \), which is consistent
The contribution of the Pomeron is separately indicated by the broken-dotted curve.

Adding the Reggeon $f$ to the Pomeron exchange, we attempt to parametrize $M_{\pi\pi}$ as follows:

$$M_{\pi\pi} = A \left\{ e^{a_1 - (t/l_2)\pi\pi P(t_1)} \left( \frac{s}{s_0} \right)^{a_f(t_1)} 
+ r \frac{\Gamma(1 + \alpha_f(0))}{\Gamma(1 + \alpha_f(t_1))} \alpha_f(t_1) \left( 1 + e^{-i\pi a_f(t_1)} \frac{s}{s_0} \right)^{a_f(t_1)} \right\},$$

(3.2)

where $\alpha_f(t_1) = 0.5 + t_1$ is the $f$-trajectory and $A = 36$(GeV/c)$^{-2}$ and $B = 3$(GeV/c)$^{-2}$ as before. We choose here $r=1$ in the context of the model of the $f$-dominated Pomeron.$^{15}$

The $\pi^p$-mass distributions thus obtained are illustrated in Fig. 7 together with the prediction from $M_{\pi\pi}$ with Pomeron only. The inclusion of the $f$ seems to make agreement better.

Figure 8 shows the mass distribution in each $|t_1'|$ region; it is seen that the DHD model is satisfactory in small $|t_1'|$ and low mass regions. As for the momentum transfer squared distributions, Fig. 9 demonstrates the qualitatively good description of experimental data as Fig. 4 for $\pi^+p \rightarrow \pi^+(\pi^0n)$.
3.3 Other reactions

Is the good agreement of our DHD model for the reaction $\pi N \rightarrow \pi (\pi N)$ accidental or not? Applying our DHD model to other reactions than pion induced ones at other energies we have found that the model can successfully explain the global feature of experimental data as in the case $\pi N \rightarrow \pi (\pi N)$.

3.3.1 $K^+ n \rightarrow K^+ (\pi^- p)$

We compared the DHD model with experimental data based upon 6454 events for $K^+ n \rightarrow K^+ (\pi^- p)$ at 12 GeV/c.\(^{(1)}\) For $M_{K}$, elastic $\pi K$ scattering amplitude, we took into account the Pomeron, $f$ and $\rho$; the same parametrizations for the Pomeron and $f$ as Eq. (3.2) and the $\rho$ which is exchange degenerate with the $f$ have been adopted. From the factorization, parameters $A$ and $B$ are determined to be 29.45 $(\text{GeV/c})^{-2}$ and 2.75 $(\text{GeV/c})^{-2}$, respectively.

Figures 10 and 11 show the $\pi^- p$-mass distribution and momentum transfer squared distribution, respectively.\(^{(9)}\)

3.3.2 $pp \rightarrow p(\pi^+ n)$

To see how good our DHD model is in very high energy region we compare it with the ISR data on $pp \rightarrow p(\pi^- n)$.\(^{(1,2)}\) We have assumed only the Pomeron contribution, since other Reggeon contributions will safely be neglected at the ISR

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\(^{(9)}\) Experimental data contain the $K^*$ resonance.
Fig. 10. Invariant mass $M(\pi^+p)$ distributions in different $|t'_1|$ regions. Curves represent the DHD-model calculation which contains the Pomeron, Reggeons $f$ and $p$.

Fig. 11. Momentum transfer squared distributions in various mass regions. Solid curves represent the prediction of the DHD model.

Fig. 12. Invariant mass $M(\pi^+n)$ distributions for $-t'_1>0.05$ (GeV/c)$^2$. Solid curve shows the DHD-model calculation using the Pomeron only.

Fig. 13. Invariant mass $M(\pi^+n)$ distributions observed in the forward (a) and backward (b) hemisphere for $-t'_1>0.05$ (GeV/c)$^2$. Solid curves show our DHD-model calculation using the Pomeron only.
§ 4. Effects of possible variation of parametrizations

We have through assumed the effective pion exchange part to be

\[ \sqrt{\frac{t_2}{t_2 - f^2}} \eta(M^2, t_2) e^{(t_2 - M^2)/f^2}, \]

where \( \eta(M^2, t_2) = (M^2)^{- \gamma(t_2)} \) and \( f = 1.8 \, (\text{GeV}/c)^2 \). It is now the position to examine what kind of effects will be produced by a possible variation of these parametrizations.

Since the \( f \) is only parameter in our formalism, first we should see a change when it is varied around \( f = 1.8 \, (\text{GeV}/c)^{-2} \) which is the most reasonable value in our analysis. As seen from Fig. 2 \( f = 1 \, (\text{GeV}/c)^{-2} \) corresponding to Berger and Pirilä seems to yield too large cross section, while \( f = 3 \, (\text{GeV}/c)^{-2} \) gives rise to rather small cross section. Figure 15 shows that in the low mass region the larger value of \( f \) results in large \( b(M) \). In the decay polar angle distribution the forward peak becomes sharper for larger \( f \), on the other hand the azimuthal angle distribution is little in-
fluenced by the variation of $f$.

Next, we examine the following variation in Eq. (4-1): case (a) $\sqrt{-t_2/(t_2-\mu^2)}$ and case (b) $\mu/(t_2-\mu^2)$. In both cases (a) and (b) we attempted to fix the parameter $f$ using the two parametrizations listed in Table I: case (i) $\eta(M^2, t_2) = (M^2)^{\alpha_0(t_2)}$ and case (ii) $\eta(M^2, t_2) = 1$.

First of all, case (b) (IV and IV' in Table I) is ruled out in comparison of the prediction with data: (1) The cross section corresponding to $f=0$ is still much smaller than experimental data on $\pi^+p \rightarrow \pi^+(\pi^+n)$ as shown in Fig. 16 and (2) $\cos\theta_{GJ}$ distribution indicates a sharp peak as seen in Fig. 17. For case (a), the fairly good agreement in mass- and $\cos\theta_{GJ}$-distributions is obtained by choosing $f=2.3$

Table I. Combinations of possible forms of the pion propagator and numerical values of the parameter in the form factor:

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>(a) $\sqrt{-t_2/(t_2-\mu^2)}$</th>
<th>(b) $\mu/(t_2-\mu^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) $M^2 \alpha_0(t_2)$</td>
<td>1</td>
<td>II</td>
</tr>
<tr>
<td>(ii) 1</td>
<td>I'</td>
<td>II</td>
</tr>
</tbody>
</table>

(cf., Eq. (2·6) and see Figs. 16 and 17).

Fig. 16. Invariant mass $M(\pi^+n)$ distributions corresponding to various parametrizations; curve (1): $\sqrt{-t_2/(t_2-\mu^2)}e^{\xi_0(t_1-\mu^2)}$, curve (2): $\mu/(t_2-\mu^2)$, curve (3): $\sqrt{-t_2/(t_2-\mu^2)}(M^2)^{\alpha_0(t_2)}$ and curve (4): $\sqrt{-t_2/(t_2-\mu^2)}(M^2)^{\alpha_0(t_2)}(1+4.5Q^2/1+4.5Q^2)^{\alpha_0}$. The curves (1), (2) and (3) correspond to the combinations (III'), (IV') and (IV) in Table I, respectively.

Fig. 17. The decay polar angle $\cos\theta_{GJ}$ distributions. Each curve represents the same parametrization as in Fig. 16.
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\( (\text{GeV/c})^{-2} \) in contrast with \( f=1.8 \ (\text{GeV/c})^{-2} \) adopted so far (see Figs. 16 and 17). Thus, \( \gamma(t_2, M^2) = (M^2)^{\alpha(t_\beta)} \) with \( f=1.8 \ (\text{GeV/c})^{-2} \) (II in Table I) and \( \gamma(t_2, M^2) = 1 \) with \( f=2.3 \ (\text{GeV/c})^{-2} \) (III' in Table I) cannot be distinguished on a purely phenomenological ground. Some theoretical arguments are needed to clarify this point.

Other functional forms, of course, will be possible for the form factor; we have, for instance,

\[
F(t_2) = \left(1 + RQ^2\right)^{1/2} \left(1 + RQ^2\right)^{-1/2},
\]
where

\[
Q^2 = -t^2(4m^2 - t^2)/4m^2, \quad Q_i = -t_i(4m^2 - t_i)/4m^2
\]
and \( R \) is a free parameter. This form is frequently adopted in the one-pion-exchange model analysis of \( \pi\pi \) scattering.\(^{10}\) \( R \approx 4.5^{(8)} \) gives almost the same mass- and decay angular distributions for \( \pi^+p \to \pi^+(\pi^+n) \) at 16 GeV/c as those of exponential form factor with \( f=1.8 \ (\text{GeV/c})^{-2} \).

§5. Discussion and concluding remarks

We have carried out phenomenological analysis of nucleon diffraction dissociation \((dd)\) by means of the DHD model, which is characterized by a single adjustable parameter \( f \) contained in the pion form factor. Reaction mechanism governing \( dd \) over a wide range of energy manifests itself in a particular manner, i.e., low mass enhancement in the dissociated system, strong slope-mass correlation, forward enhancement in the decay polar angle \( \cos\theta_G \) distribution and enhancement at \( \phi_G = \pi \) in the azimuthal angle distribution.

In the first place, we compared in detail our model with the experimental data on \( \pi^- p \to \pi^- (\pi^- n) \) and \( \pi^- p \to \pi^- (\pi^- p) \) at 16 GeV/c through mass distribution, differential cross section and decay angular distributions. From these investigations we found that \( f=1.8 \ (\text{GeV/c})^{-2} \) is the best choice for our parameter. This value of \( f \) could also enable us to explain reactions \( K^- n \to K^- (\pi^- p) \) at 12 GeV/c\(^{10}\) and \( pp \to p (\pi^- n) \) at 12 GeV/c\(^{13}\) and 1500 GeV/c.\(^{12}\)

It might therefore be concluded that our model can describe the characteristic features of the nucleon \( dd \) observed in \( \pi^- , K^- \) and \( p \)-induced reactions as far as we are concerned with the small mass and small momentum transfer regions.

Uehara\(^6\) and Berger and Pirilä\(^8\) emphasized absorption effect even in the small \( |t_1'| \)-region to explain the dip at \( |t_1'| \approx 0.2(\text{GeV/c})^2 \) observed in the reaction \( pp \to p (\pi^- n) \) at 100 GeV/c. In contrast with our DHD model, they used a flat Pomeron and a fixed pion form factor \( e^{i(t_2 - \lambda^2)} \), so that their DHD calculation without absorption gives rise to the cross section roughly twice as large as ours. This indicates that the comparable amount of absorption correction to the DHD contribu-
tion itself must be introduced to reduce the excess of the cross section. Systematic investigation of the slope parameter $b(M)$ for $pp\rightarrow p(\pi^- n)$ from 12 GeV/$c^{10,20}$ reveals that $b$ at 100 GeV/$c^{10}$ adopted by Berger and Pirila appears to be too high. Reliable experimental information would be desirable on this point.

If one wishes to make a further refinement of the DHD model, one might introduce a certain amount of absorption effect. Nevertheless, we have to evaluate quantitatively the success and failure of the naive DHD model by referring to reliable experimental data. Furthermore, one must be careful enough not to deteriorate the simple structure of a DHD formulation being able to explain characteristic behavior of $dd$.

In order to understand $dd$ mechanism beyond the restricted phase space regions, i.e., low mass and small momentum transfer, we will encounter hard but unavoidable task of introducing nucleon resonances which are already seen in a mass distribution; in this case the duality point of view should not be ignored.

The contribution of the nucleon exchange to $\pi^- p\rightarrow \pi^- (\pi^- n)$ seems to be small because the enhancement at $\phi_{GJ}=\pi$ as seen in Fig. 5 can be regarded as typical to the pion exchange. This can also be confirmed in $\cos\theta_{GJ}$ distributions at 16 GeV/$c$ (Fig. 6) and at the ISR (Fig. 13), where the forward contribution (mainly due to the pion exchange) dominates over the backward contribution (mainly due to the nucleon exchange). Problems on the nucleon exchange were discussed in detail by Minaka et al.\cite{18}

In addition to problems mentioned above we have other difficult problems such as spin structure and peripherality in the impact parameter space.\cite{19} We should exploit our DHD model to reach a unified understanding of $dd$ mechanism by successfully solving these problems.

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References

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