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Return Periods of Hydrological Events

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A relation between return periods of hydrological events in annual maxima and partial duration series respectively is shown to be exactly fulfilled when the events take place according to a Poisson process. Furthermore it is shown that the relation can be used as a good approximation when dealing with more regular processes than the Poissonian. Finally a record of rainfall depths is analysed.

Introduction

When evaluating extreme hydrological events a number (say *n)* of annual maxima, denoted an *annual maxima series,* is a common basis for the analysis. If we, however, instead of the *n* annual maxima use the *n* largest events that have occurred during the *n* years, regardless of some of those events not being annual maxima, we are dealing with a series denoted the *annual exceedance series.* As a third possibility for the basis of an extreme value analysis a *partial duration series,* consisting of all events exceeding a certain base level, can be used. Evidently the annual exceedance series is a special form for a partial duration series, where the base level is chosen so that the number of events equals the number of years. In the following we shall therefore only distinguish between annual maxima and partial duration series. When analysing a record of a hydrological variable we will, corresponding to a given level, normally find different return periods depending on which of the series we use as basis for the analysis. For large values of the variable the difference in return periods is insignificant, whereas for more frequently reached levels this is not the case. Assuming the events in the partial duration series to occur according to a Poisson process, we shall show that a previously proposed relation between the return periods is exactly fulfilled. Furthermore, on the condition that the base level is not chosen too large, we shall show that the relation is insensitive to deviations from the assumption of a Poisson process towards processes more regular than the Poissonian. Finally, as an illustration of the theory, a long record of rainfall events will be analysed.

Return Periods in Annual Maxima and Partial Duration Series

Let us consider a hydrological variable D and assume that a partial duration series consisting of totally N events larger than a base level d_0 and obtained during n years is available. We shall further assume that all the events $D > d_0$ are independent and that the probability distribution of D is stationary, i.e.

$$P\{D \leq d \mid D > d_0\} = F(d) \tag{1}$$

is fulfilled regardless of the time of the year the event is occuring.

Two types of stochastic processes shal now be considered as models for the partial duration series: (i) a very regular process with a fixed number of events larger than d_0 ocurring every year, and (ii) a random process where only the expected number of event $D > d_0$ per year is fixed.

(i) With exactly κ events larger than d_0 in a year, where κ can take on all integer values greater than or equal to 1, the distribution function for the yearly maximum D_{max} becomes

$$P\{D_{max} \leq d\} = G(d) = (F(d))^{\kappa} \tag{2}$$

The return period T for the event $D_{max} > d$ in the annual series, defined as the expected time interval in years between such events, can now be expressed by the function $F(d)$,

$$T = \frac{1}{P\{D_{max} > d\}} = \frac{1}{1-G(d)} = \frac{1}{1-(F(d))^{\kappa}} \tag{3}$$

In most practical situations the occurrence of the κ events is irregulary distributed over the year due to seasonal effects. Because of this seasonal variation pattern an introduction of a return period in the partial duration series is not straightforward. However, the difficulties can be overcome by defining the return period in years for the event $D > d$, T_* as the reciprocal of the expected number of such events per year. Thus we have

$$T_* = \frac{1}{\kappa\, P\{D>d\}} \qquad \frac{1}{\kappa(1-F(d))} \tag{4}$$

Elimination of $F(d)$ between Eqs. (3) and (4) gives

$$\frac{1}{T} = 1 - (1 - \frac{1}{\kappa T_*})^{\kappa} \; ; \qquad T_* \geq \frac{1}{\kappa} \tag{5}$$

an expression depending on κ.

(ii) The Poisson process is a frequently applied model for the occurrence of hydrological events. Introducing the Poisson model means that the number of events larger than the base level d_0 per year now is a random variable following a Poisson distribution. The parameter in the distribution equals the expected number of events $D > d_0$ per year, denoted by κ. As κ now means an expected number instead of a fixed number, κ can take on all real values greater than zero. Considering a new threshold $D > d_0$, the number of events $D > d$ per year, J_d follows a Poisson distribution too. We find

$$P\{J_d = j\} = \frac{\lambda_d^{\,j}}{j!} \; e^{-\lambda_d} \tag{6}$$

where the parameter λ_d equals the expected number of events $D > d$ per year,

$$\lambda_d = E\{J_d\} = \kappa\; P\{D > d\} = \kappa(1 - F(d)) \tag{7}$$

Defining return periods in the partial duration series as before we obtain for the event $D > d$

$$T_* = \frac{1}{\lambda_d} = \frac{1}{\kappa(1-F(d))} \tag{8}$$

In the case of a Poisson process the annual maxima series no longer is a simple subset of the considered partial duration series. The distribution function for the yearly maximum. D_{max} can nevertheless easily be found. By utilizing Eq. (6) we get

$$P\{D_{max} \leq d\} = G(d) = P\{J_d = 0\} = e^{-\lambda_d} = e^{-\kappa(1-F(d))} \tag{9}$$

The return period for the event $D_{max} > d$, defined as the expected time interval between such events, then becomes

$$T = \frac{1}{1-G(d)} = \frac{1}{1 - e^{-\kappa(1-F(d))}} \tag{10}$$

By eliminating $F(d)$ between Eqs. (8) and (10) we obtain

$$\frac{1}{T} = 1 - e^{-1/T_*} \tag{11}$$

a relation originally proposed by Langbein (1949). It should be noted that Eq. (11) is independent of κ and exactly fullfilled in the case of a Poisson process.

Comparing Eq. (11) with Eq. (5) for different values of κ, as illustrated in the figure, shows that the difference between the relations is insignificant for large values of κ. This means that we also can apply Eq. (11) for processes more regular than the Poissonian if we choose the base level appropriately low.

Analysis of Rainfall Record

On basis of rainfall observations at Gentofte, Copenhagen through a period of $n = 39$ years (1934 - 72) the maximum depth observed within 10 minutes during each rainshower has been listed. By extracting all events larger than a base level of 2.5 mm a partial duration series consisting of $N = 227$ events has been obtained. Thus the average number of 10 minutes rainfall depths larger than 2.5 mm per year is $\kappa = 5.82$.

First we shall test the hypothesis that the number of events per year follows a Poisson distribution with the observed value of κ as parameter. The usual X^2-test of godness of fit, when using 10 classes, gives a value $X^2 = 14.1$. The 95% quantile in the X^2-distribution with 8 degrees of freedom equals 15.5. Thus the hypothesis is accepted on a 5% level of significance.

We may not expect that eq. (11) is valid. This can be tested by calculating both T_* and T on basis of respectively the partial duration and the annual maxima series and plotting corresponding values. With a κ-value as large as 5.82 we may expect all the annual maxima to be larger than the base level, which actually is the case. We will accordingly be able to plot 39 points in the figure. In order to calculate the return periods both series are ranked. Denoting by m and M the rank from above of an observation in respectively the annual maxima and the partial duration series and by applying the Weibull plotting position, the estimators of the return periods turn out as

$$T = \frac{n+1}{m} \; ; \quad T_* = \frac{N+1}{\kappa M} \tag{12}$$

From the figure it is seen that the agreement with Eq. (11) is fine. The only deviation woth noting is found for return periods about 10 years and is due to the fact that the third largest event in the partial duration series not being an annual maximum, an incident with a low probability of occurrence.

It can finally be mentioned that the agreement still is very fine if we choose a somewhat smaller κ-value. Gradually, corresponding to an increasing base level, fewer points can, however, be plotted as a consequence of annual maxima falling below the base level. For instance if we choose the annual exceedance series corresponding to $\kappa = 1$ only 24 of the 39 annual maxima have magnitudes above the base level.

Errata Vol. 8. No. 1, page 61, D. Rosbjerg -
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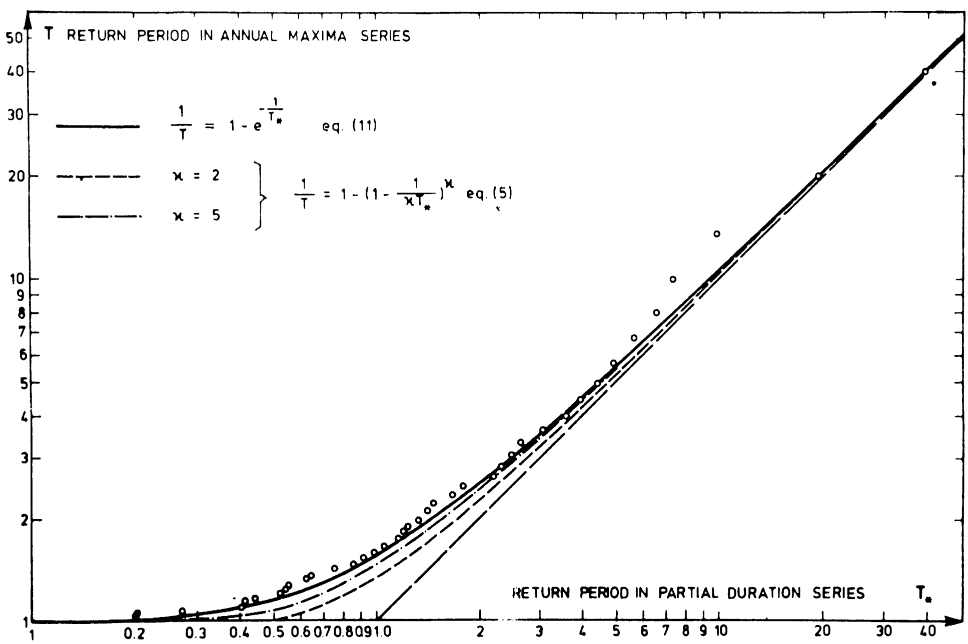
Please insert below drawing on page 61.



References

Langbein, W.B. (1949) Annual floods and the partial duration flood series, *Am. Geophys. Union Trans., Vol. 30,* pp. 879 - 881.

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