3 The shear stress τ is computed by equation (A6a) if its value is less than the critical shear stress τ_c. If the shear stress is larger than τ_c but less than FS, it is computed by equation (A6b). However, if the shear stress is larger than FS, equation (A6c) is used.

References

1 Poritsky, H., Hewlett, C. W., Jr., and Coleman, R. E., Jr., "Sliding Friction of Ball Bearings of the Pivot Type," *Journal of Applied Mechanics*, Vol. 14, No. 4, TRANS. ASME, Vol. 69, Dec. 1947, pp. 261–268.

2 Jones, A. B., "Ball Motion and Sliding Friction in Ball Bearings," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 81, No. 1, Mar. 1959, pp. 1–12.

3 Reichenbach, G. S., "The Importance of Spinning Friction in Thrust-Carrying Ball Bearings," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 82, No. 2, June 1960, pp. 295–301.

4 Harris, T. A., "Ball Motion in Thrust-Loaded Angular Contact Bearings With Coulomb Friction," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series D, Vol. 93, No. 1, Jan. 1971, pp. 32–38.

5 Harris, T. A., "An Analytical Method to Prevent Skidding in Thrust-Loaded Angular Contact Ball Bearings," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series D, Vol. 93, No. 1, Jan. 1971, pp. 17–24.

6 Miller, T., Parker, J., and Zaretsky, E. V., "Apparatus for Studying Ball Spinning Friction," NASA TN D-2796, 1965.

7 Dietrich, M. W., Parker, R. J., and Zaretsky, E. V., "Effect of Ball-Race Conformity on Spinning Friction," NASA TN D-4669, 1968.

8 Dietrich, M. W., Parker, R. J., Zaretsky, E. V., and Anderson, W. J., "Contact Conformity Effects on Spinning Torque and Friction," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 91, No. 2, Apr. 1969, pp. 308–313.

9 Allen, C. W., Townsend, D. P., and Zaretsky, E. V., "Elastohydrodynamic Lubrication of a Spinning Ball in a Nonconforming Groove," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 92, No. 1, Jan. 1970, pp. 89–96.

10 Allen, C. W., Townsend, D. P., and Zaretsky, E. V., "Comparison of Conventional and Microasperity Elastohydrodynamic Lubrication of a Ball Spinning in a Nonconforming Groove," NASA TN D-6761, Apr. 1972.

11 Johnson, K. L., and Cameron, R., "Shear Behaviour of Elastohydrodynamic Oil Films at High Rolling Contact Pressures," *Proc. Inst. Mech. Eng.*, Vol. 182, No. 14, 1967–1968, pp. 307–330.

12 Allen, C. W., Townsend, D. P., and Zaretsky, E. V., "New Generalized Rheological Model for Lubrication of a Ball Spinning in a Nonconforming Groove," NASA TN D-7280, 1973.

13 Townsend, D. P., Allen, C. W., and Zaretsky, E. V., "Friction Losses in a Lubricated Thrust Loaded Cageless Angular Contact Bearing," Proposed NASA TND, 7356, 1973.

14 Wolveridge, P. E., Baglin, K. P., and Archard, J. F., "The Starved Lubrication of Cylinders in Line Contact," *Proc. Inst. Mech. Engrs.* 1970–1971, Vol. 185, 81/71, p. D428.

15 Eskinazi, Salamon, *Principles of Fluid Mechanics*, Allyn and Bacon, Boston, 1968, p. 438.

16 Drutowski, R. C., "Energy Losses of Balls Rolling on Plates," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 81, No. 1, June 1959, pp. 233–239.

17 Dawson, D., and Higginson, G. R., *Elastohydrodynamic Lubrication*, Pergamon Press, 1966.

DISCUSSION

R. M. Barnsby[2]

The authors have introduced a useful rheological model that limits the shear stress in order to give realistic power losses at high pressure, while permitting large ratios of shear to normal stress at low pressure. This is a considerable improvement over the exponential model used by T. A. Harris [5]. The authors' proposal of decreasing the pressure-viscosity coefficient (α) at high pressures has been implemented by limiting the shear stress to a constant fraction of the normal stress, thereby making the cutoff pressure a direct function of the shear rate. Has the corresponding reduction in the film thickness been taken into account, or do the authors feel this effect to be insignificant? The reduced value of α at some cutoff pressure may also be applied directly to the exponential model for isothermal flow. This leads to the pressure-temperature-viscosity model described in [18]. Did the authors consider this earlier model in their present analysis, and if so, do they have reasons for rejecting it? The significance attached to the spinning torque as a major contributor to the total bearing torque emphasizes the importance of a sound rheological model.

The concise separation of friction sources described in the introduction to the Analysis section is commendable. The spin torque derivation excludes translational sliding in the elastohydrodynamic contact, although sliding is listed as a possible friction source. It appears that the calculated friction due to sliding has been taken from the authors' references [4, 13]. The kinematics is assumed unchanged by the "microslip" necessary to balance the shear force due to rolling. On the other hand, the rolling resistance is quoted as being the most significant contributor to bearing torque without a cage for the synthetic paraffinic oil. What is the authors' feeling regarding the validity of the sliding velocities used in this situation? Reconsideration of the sliding velocities might account for those calculated values of torque that are lower than experimental values, particularly if there is opportunity during test for the three balls to move together.

In the calculation of cage drag, the Petroff formula assumes constant viscosity as well as constant radial clearance and full lubrication. The torque induced by the cage on the outer race is derived from analyzing the cage/inner-race drag and initially assuming ball equilibrium under cage drag forces. The authors compensate by including an effective coefficient of friction between ball and pocket. Are the authors aware of experimental evidence confirming the value of this coefficient?

A logical attempt has been made to interpret the experimental results by direct comparison of results at different conditions. A tabulation of the friction sources used for calculating torque under each run condition, for comparison with experimental data, might have clarified the comparison, particularly with regard to conclusions 2 and 3. It is recognized that calculated values of cage torque for the two lubricants do not agree with experimental values, and indeed conclusion 2 is based entirely on experimental results showing that cage drag is more significant with the less viscous lubricant. In attempting to explain the high calculated values, it is suggested that the more viscous synthetic paraffinic oil does not fill the bearing cavity at high speed and does not enter the cage-land area in large quantities. On the other hand, for a bearing without a cage, the negligible effect of oil-jet lubrication over thin-film lubrication, for the less viscous di-2-ethylhexyl sebacate, is attributed also to lack of oil retention. It is concluded that the resistance due to rolling is negligible for this fluid. Is it the authors' opinion that increased oil flow, without a cage, affects only the rolling resistance and then only for the more viscous fluid?

It is hoped that the suggestions made will be of use to the authors in further development of their rheological model. This discusser welcomes the opportunity to comment and would be particularly interested in any application the authors intend to make of their model to predict cage drag forces at high speed and high temperature.

Additional Reference

18 McGrew, J. M., Gu, A., Cheng, H. S., and Murray, S. F., "Elastohydrodynamic Lubrication—Preliminary Design Manual," AFAPL-TR-70-27, Nov. 1970.

[2] Pratt & Whitney Aircraft Div., United Aircraft Corp.

J. W. Kannel[3]

Bearing torque is a very significant factor in many instrument bearing applications, such as in space-oriented missions. In this regard, the authors' discussion is quite timely. The authors discuss various possible sources of drag in a bearing, including ball-spin effects and cage drag. One source of torque not discussed is that associated with ball speed variation (BSV). Under some conditions of bearing alignment or radial-axial loading, BSV can require that gross slip occur at the ball-race interface and hence can cause significant bearing torque.

With respect to spin torque, I am surprised that the authors have found that torque varies as much between the two lubricants as they have. Essentially, the torques (even without cage drag) vary by as much as an order of magnitude. Examination of the lubricant properties reveals that the α values are very similar for the two fluids, and the F (upper-limit traction coefficient) factors differ only by about 35–40 percent. There is, of course, considerable difference in the values of μ_0 (base viscosity), but this will affect film thickness more than traction. Essentially, lubricant shear stress should only be related to viscosity to the $3/11$ power [see equation (1*a*)], since h increases with viscosity to the $8/11$ power. I would appreciate some comment by the authors on this subject.

The method used at Battelle to calculate bearing-spin torque is to use pressure–viscosity data inferred from disk-traction measurements. (The exact method will be published in the near future.) A curve of this type is given in Fig. 13. The lubricant behavior is modeled from this curve for the pressure range of interest (in this case, $\approx$ 100,000 psi or less) by the model $\mu = \mu_i e^{\gamma p}$, where μ_i is an intercept viscosity and γ is the slope of the intercept line as shown. If we approximate the slip velocity (Δu) due to spin by the equation

$$\Delta u = c_1 + c_2 y + c_3 y^2 \qquad \text{(D-1)}$$

where $c_{1,2,3}$ are constants related to bearing geometry, then

$$T = \int_A \int \tau y dA = \frac{\mu_i}{h} c_2 \int_A \int y^2 e^{\gamma p} dA \qquad \text{(D-2)}$$

or

$$T \approx 0.8 \frac{\mu_i c_2}{h} ab^3 \exp\,[0.514\,\gamma p_h + 0.033\,(\gamma p_h)^2 + 0.0014\,(\gamma p_h)^3] \qquad \text{(D-3)}$$

where p_h is the maximum Hertz pressure.

With the use of the model represented by equation (D-3), reasonable torque prediction for a bearing can be obtained. However, invariably these torques are notably lower than those seen in actual bearing test despite the fact that empirical traction data are used for the lubricant modeling.

[3] Battelle Memorial Institute, Columbus, Ohio.

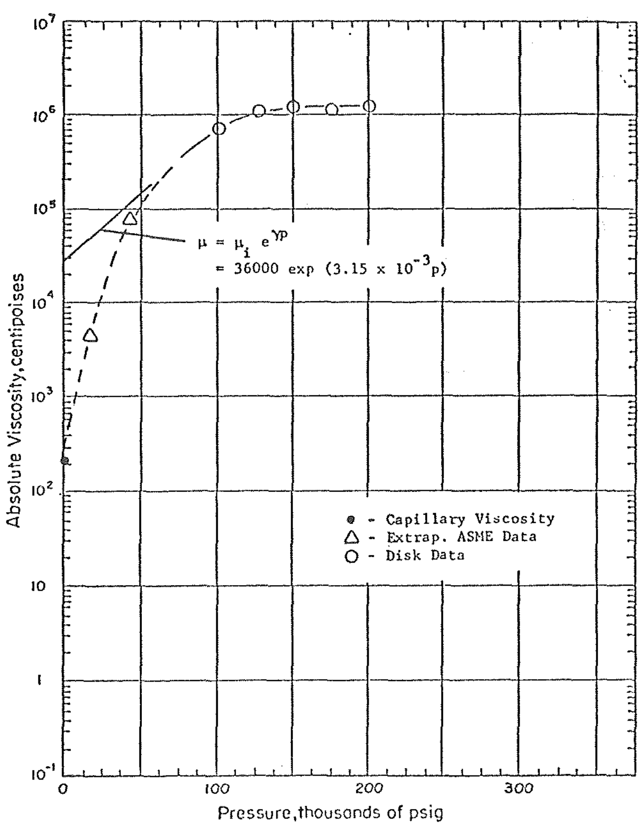

Fig. 13

Authors' Closure

The authors wish to thank the discussers for their comments. In reply to Dr. Barnsby's question on film thickness, this is primarily determined by the pressure viscosity coefficient in the inlet region at which point α has its normal value. There should be no appreciable reduction in the film thickness because of the change in α at the high pressures. Verification is found in the case of the original composite model in the paper by H. S. Cheng [19].

The rheological models studied have been limited to those having some type of limiting shear stress/normal stress relationship. The model referred to in reference [18] was compared with our model in [20].

Dr. Barnsby questions the possible change in bearing kinematics due to the "microslip" necessary to balance the forces due to rolling drag. This condition can occur at very light bearing loads; i.e., below 44 N (10 lb), where there is insufficient traction force to balance the rolling drag. However, at higher loads; i.e., 44 N (10 lb) or higher, it can be shown that at relatively small values of slip velocity the shear stress/pressure relationship (τ/S) approaches the limiting value equivalent to the lubricant factor, F (.07 for the synthetic paraffinic oil), which produces an ample traction force to overcome the rolling drag. Considering the case of the 44 N (10 lb) load for the synthetic paraffinic oil, the limiting traction force approaches 9 N (2 lb), giving a torque of 0.12 N-m (1 lb-in). This is much greater than the torque due to rolling drag at that load level. Hence, there would be no change in the bearing kinematics.

The effect of the cage/ball friction was neglected in the analysis. The quoted value was intended merely to give an upper bound to the additional torque from this source.

The authors believe that the major effect of increase oil flow or a more viscous oil is an increase in the rolling resistance. At very high speeds the fluid-dynamic drag of the balls as they orbit the bearing can become very significant [21].

In reply to Mr. Kannel, the spin torque does not vary appreciably between the two lubricants; the major difference is due to the rolling resistance. As mentioned earlier, the rolling resistance is small with the low viscosity lubricant but is significantly larger than the spinning torque with the synthetic paraffinic oil. Inclusion of the rolling resistance in the Battelle procedure might bring the calculated values into close agreement with the experimental results.

Additional References

19 Cheng, H. S. "Isothermal Elastohydrodynamic Theory for the Full Range Pressure-Viscosity Coefficient," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 94, No. 1, Jan. 1972.

20 Gu, A. L. "An Improved Method for Calculating the Spin Torque on a Fully Lubricated Ball-Race Contact," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 95, No. 1, Jan 1973, p. 106.

21. Allen, C. W. "Film Thickness and Traction in the Elastohydrodynamic Lubrication of Ball Bearings and Spur Gears," NASA CR-134648, June 1974.