Author's Closure

The author is grateful to the discussers for their interesting comments about the contribution of dead load to the buckling of arches and in particular for their bringing to his attention the experimental work of Lind and Deutsch and their own analytical work on this question.

Unfortunately, the experimental point representing the maximum load achieved was inadvertently omitted from the graphs in Fig. 10. That point has values of $P = 55.5$, $V = -0.214$, and $G = 42.8$. Furthermore, the load of 55.5 is the applied load at buckling (uncorrected for dead load). For the high-strength-steel model used in the tests, however, the weight of the arch itself was a small fraction (about 5 percent) of the measured buckling load.

Velocity Profiles of Flow at Low Reynolds Numbers[1]

H. C. PERKINS[2] and C. CARTMILL.[3] The authors are to be congratulated for an interesting paper in which they have shed considerable insight into the question of whether "kinks" in the velocity profile are real. Recent experimental work,[4] done subsequent to the authors work, also show these kinks although the entrance geometry and flow passage is different from that analyzed by the authors. The question we should like to raise is what do the authors mean by a steady-state flow? They raise the question of whether an axially symmetric flow with a flat initial velocity profile will ever achieve a steady state. Are they possibly referring here to "fully developed?"

[1] By S. Abarbanel, S. Bennett, A. Brandt, and J. Gillis, published in March, 1970, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 37, TRANS. ASME, Vol. 92, Series E, pp. 2–4.

[2] Professor, Aerospace and Mechanical Engineering Department, College of Engineering, University of Arizona, Tucson, Ariz. Mem. ASME.

[3] Research Assistant, Aerospace and Mechanical Engineering Department, College of Engineering, University of Arizona, Tucson, Ariz.

[4] Burke, J. P., and Berman, N. S., "Entrance Flow Development in Circular Tubes at Small Axial Distances," ASME Paper No. 69-WA/FE-13.

Since the authors have successfully determined a solution, in this paper, to the low Reynolds number steady-state equation of motion there would seem to be an implication that steady state can in fact be achieved.

In short could the authors comment on or amplify their remarks regarding "steady state?"

Authors' Closure

I was interested and gratified to hear that Burke and Berman had found experimental verification of the "kinks." Their paper has not yet reached me but its title suggests that one ought to compare their results with those in an earlier theoretical paper.[5]

The reservations about whether "steady-state" flow could be achieved did indeed refer to fully developed flow. Starting from rest and gradually speeding up the boundary flow to unity, would the flow pattern necessarily tend with time to a limiting form? If so, is only one limiting form possible, viz., that in which the pattern is analytic over the entire quadrant? And finally, would this flow be stable?

None of these questions has been finally answered. There is a widely held belief that physical instability would necessarily have involved numerical instability, but the grounds for this belief are not very clear.

[5] Friedman, M., Gillis, J., and Liron, N., "Flow in the Inlet Region of a Circular Pipe," *Applied Science Research*, Series A, Vol. 19, 1968, p. 426.

Buckling of Vessels Composed of Combinations of Cylindrical and Spherical Shells[1]

MENAHEM BARUCH.[2] The paper is very good and well presented but the discusser would like to discuss some special aspects of the subject.

There is some ambiguity in Table 1 of the discussed paper. It seems that the numbers in the column labeled "Experiment" are based on equation (17) of the paper and not on actual experimental data. Nevertheless, it is stated in the paper that the agreement between theory and experiment is very good. This point needs some discussion.

It is well known that the spherical cap under hydrostatic pressure is sensitive to initial imperfections while the cylindrical shell under the same loading is less sensitive to initial imperfection. The paper deals with a structure which is a combination of both. Is the new structure sensitive to initial imperfections? From the fact that the results obtained by the linear theory used in the paper agree well with experimental results it seems that the combined structure is insensitive to imperfections. Why is it so? To answer this question it is convenient to look at the expressions for buckling pressure under hydrostatic pressure of spherical and cylindrical shells alone

$$\frac{p_{\rm sph}}{E} = k_{\rm sph}\left(\frac{h}{a}\right)^2, \qquad \frac{p_{\rm cyl}}{E} = k_{\rm cyl}\,\frac{a}{L}\left(\frac{h}{a}\right)^{2.5} \tag{1}$$

where $k_{\rm sph}$ is defined from experiments and is only a fraction of the calculated theoretical value obtained from linear theory [1].[3] $k_{\rm cyl}$ is defined by experiments and usually is taken as about 80 percent of the theoretical value calculated by linear theory [2]. Now, if the hemispherical cap and the cylindrical shell are made from the same material and have the same thickness, as in the discussed case than the ratio of the buckling pressures of the two shells will be

$$\frac{p_{\rm sph}}{p_{\rm cyl}} = \frac{k_{\rm sph}}{k_{\rm cyl}}\cdot\frac{L}{a}\cdot\left(\frac{a}{h}\right)^{0.5} \tag{2}$$

For $a/h = 100$ as is given in the paper $k_{\rm sph}$ is about 0.3 [1] and $k_{\rm cyl}$ about 0.73 [2]. For the minimum value $(L/a) = 1$ given in the paper, one obtains

$$\frac{p_{\rm sph}}{p_{\rm cyl}} = \frac{0.3}{0.73}\cdot 1.100^{0.5} = 4.1$$

[1] By A. Harari and M. L. Baron, published in the June, 1970, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 92, Series E, pp. 393–398.

[2] Associate Professor, Department of Aeronautical Engineering, Technion-Israel Institute of Technology, Haifa, Israel.

[3] Numbers in brackets designate References at end of Discussion.

For this case the buckling load of the spherical cap is about 4 times higher than that of the cylindrical shell. It seems that this fact explains the good agreement between theory and experiment.

From equations (1) it is clear that for some different geometrical relations between the radii and the thicknesses of the spherical and cylindrical shells the buckling pressure of the spherical shell will be lower than that of the cylindrical shell, and the combined structure will become more sensitive to initial imperfections. Then one cannot expect an agreement between linear theory and experiment as good as that reported in the paper.

In Fig. 1(a) of the paper the cylindrical shell is shown to be optionally stiffened. If this is done the general buckling load of the cylindrical shell will increase appreciably [3], and the question of the combined structure becoming sensitive to initial imperfections will arise again.

References

1 Vol'mir, A. S., "Stability of Deformable Systems," *Nauka*, Moskow, 1967, p. 989–in Russian, p. 666.

2 Weingarten, V. I., and Seide, P., "Elastic Stability of Thin-Walled Cylindrical and Conical Shells Under Combined External Pressure and Axial Compression, *AIAA Journal*, Vol. 3, No. 5, May 1965, pp. 913–920, equation (2).

3 Baruch, M., and Singer, J., "Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells Under Hydrostatic Pressure," *Journal of Mechanical Engineering Science*, Vol. 6, No. 1, Jan. 1963, pp. 23–27.

Authors' Closure

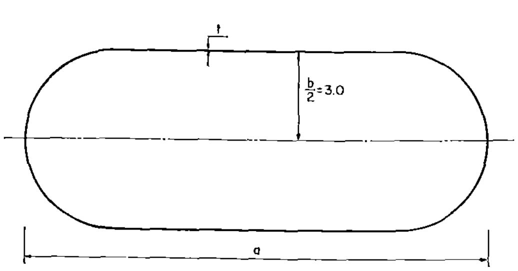
The statement in the paper that agreement between theory and experiment is very good refers to results from an unpublished experimental report by DTMB [4][4] (NSRDC). Since the experimental results agreed very well with the equivalent length formula obtained at NSRDC it was decided that it would be just as instructive to compare the theory with the results obtained by the equivalent length approach. Table 1[5] included with this Note compares our results with actual experimental results. Table 1 also compares our results with results obtained by Bushnell [5] for the experimental model in a report that appeared after our report [6] came out. The results are compared for $L/a > 1$ since as was pointed out in the paper prebuckling rotations have to be included for smaller value of L/a.

Table 1

COMPARISON WITH EXPERIMENTS

a/b	t	THEORY		TEST		EQUIVALENT LENGTH		BUSHNELL	
		p	n	p	n	p	n	p	n
2.0	0.061	8.23	4	8.45	4	8.09	4	7.98	4
2.0	0.080	15.25	4	14.43	4	15.10	4	14.66	4
2.5	0.061	5.40	4	5.76	4	5.37	4	5.25	4
2.5	0.078	10.46	4	9.87	4	10.40	4	10.09	4
3.0	0.061	4.23	3	4.66	4	4.21	3	4.12	3
3.0	0.079	7.56	3	7.99	3	7.50	3	7.29	3
4.0	0.042	1.07	3	1.30	3	1.03	3	1.04	3
4.0	0.062	2.86	3	2.86	3	2.80	3	2.78	3
4.0	0.080	5.60	3	5.68	3	5.67	3	5.52	3

$E = 325000$
$\nu = 0.4$



References

4 DTMB Tests on Cylinders With Hemispherical Ends, unpublished results.

5 Bushnell, D., "Buckling and Vibration of Segmented Shells of Revolution," LMSC N-26-67-5, Jan. 1968.

6 Harari, A., and Baron, M. L., "Buckling of Vessels Composed of Combinations of Cylindrical and Spherical Shells," Nonr-3454(00) FBM, July 1967.

[4] Numbers in brackets designate References at end of Closure.

[5] Table is taken from page 37 of reference [5].

The Response of Narrow-Mouthed Harbors in a Straight Coastline to Periodic Incident Waves[1]

FREDRIC RAICHLEN[2] and JIIN-JEN LEE.[3] The authors have presented interesting results on the response to long periodic waves of a harbor with a narrow opening connected to the open sea. The theory which was developed is considered applicable by the authors so long as the entrance and the characterizing dimension of the harbor are small compared to the wavelength of the periodic plane incident waves. Actually, for a large entrance width this apparent limitation can be overcome by dividing the harbor entrance into several segments (e.g., "n" segments with the length of each segment small compared to the wavelength), and the solution can be considered to be the superposition of the solutions for n harbors with entrances corresponding to the n segments into which the original entrance was divided. Instead of one unknown coefficient, A, in equation (5) there will be n unknowns at the harbor entrance which are determined from the solution of simultaneous equations obtained by matching conditions. The imaginary boundary between the entrance channel and the harbor can also be divided into n segments. Naturally, if the entrance channel width is no longer small the dependence in the y-direction of the velocity potential must be included in equation (6).

A limitation of the general applicability of the theory developed is that the full response curve can be obtained only for harbors for which the eigenfunctions can be predicted a priori. This limits the application of the the authors' theory to cases where the geometry of the harbor is either simple or can be approximated in a reasonably simple fashion. The amplitude distribution within the harbor cannot be predicted without knowledge of these eigenmodes.

The writers, in a recent work, have studied the wave-induced oscillations in harbors of arbitrary shape that contain interconnected (or coupled) basins each with arbitrary shape. (Publication of this work is now under preparation.) This coupled-

[1] By G. F. Carrier, R. P. Shaw, and M. Miyata, published in the June, 1971, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 93, Series E, pp. 335–344.

[2] Associate Professor of Civil Engineering, W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, Calif.

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