such that the residual force at each bearing vanishes (or is sufficiently small) for \( \Omega = \Omega_m \). In this way, by using at most \( m+2 \) balancing planes and operations, it is possible to insure smooth operation of the rotor for all speeds up to its maximum operating speed.

Consider now the alternative procedure—for example the "Method of Minimum Planes"—whereby the rotor is first balanced as a rigid rotor and then balanced in its first \( m \) modes as a flexible rotor. The number of balancing planes required is \( m+2 \). From equation (44), the residual bearing force \( R_n' \) at the maximum operating speed is

\[
R_n' = \Omega_m^2 \sum_{r=m+1}^{m+2} \frac{\varepsilon_r(\Omega_m^2 - 2j\mu_r\omega_m)}{\Omega_m^2 + 2j\mu_r\omega_m} \int_0^L A \phi_r(x) dx
\]

(51)

Once more \( R_n' \) may be within the appropriate tolerances. If not then, as explained above, the residual forces can be suitably reduced by using at most two more balancing planes. The rotor is then, again, in a suitable state for smooth operation throughout its design speed range.

Thus, the two methods which have been the subject of much heart-searching have achieved equivalent results for our flexible rotor as practical experience confirms, both methods will balance a flexible rotor. There would, however, be some advantages in the direct modal approach without a preliminary rigid rotor balancing stage. For the simple modal technique requires only \( m, m+1, \) or \( m+2 \) correction planes to achieve a tolerable state of balance. By contrast, the additional rigid rotor balancing necessitates in total \( m+2, m+3, \) or \( m+4 \) such planes. The direct modal approach is likely, therefore, to require up to two less balancing operations.

In addition, the rigid rotor balancing can only really be performed satisfactorily in a soft bearing machine, whereas the needs of subsequent high-speed testing dictate the use of a hard bearing machine. Moreover, it is practically inconvenient and expensive to place a rotor successively in soft and hard bearing facilities. If this approach is not followed and rigid rotor balancing is attempted at low speed in a hard bearing machine, when \( \omega \) is typically fairly low, then the bearing forces that are apparently balanced by the rotor's behavior as a rigid body include some contribution from the first few terms in the modal series in equation (37) and do not solely represent \( \Omega \int_0^L A \phi(x) dx \). In these circumstances, rigid rotor balancing is producing an additional average balance at low speed. In so far as the bearing forces are proportional to \( \Omega^2 \) and, therefore, more significant at higher speeds, it would seem sensible to perform no more than one average balancing operation, and that at the high speed \( \Omega_m \).

**DISCUSSION**

**W. Kellenberger**

The discusser would like to thank Messrs Bishop and Parkinson for their contemporary contribution to the problem of balancing flexible rotors with principal modes. Since the discusser is mentioned by name, he should like to add his comments, especially as regards the chapter headed "Method of Minimum Planes."

The discusser's article written in 1967, which was quoted, was at that time written in the belief that it would fill a gap in knowledge of the theory of flexible balancing and he is still convinced that it succeeded in doing this, at least to some extent.

The fact that there are obviously two schools of thought regarding balancing with principal modes and the inability to distinguish between the two, as existed a year ago, has induced both sides to reconsider their ideas. The results are now to be found in the present paper and in the discusser's*15 paper.

Without going into details, the difference between the two methods (\( N \) method and \( N + 2 \) method) may be summarized as follows.

The rotating bearing forces \( L_L \) and \( L_R \) of a rotor supported by two bearings, allowing for unbalance, can be expressed as follows:

\[
\begin{align*}
L_L &= \omega_m^2 \sum_{n=1}^{m} \{ \} + \omega_m^2 \int_0^L u(x) dx + \sum_{k=1}^{K} \frac{Z_k}{l} U_k \\
L_R &= \omega_m^2 \sum_{n=1}^{m} \{ \} + \omega_m^2 \int_0^L u(x) dx + \sum_{k=1}^{K} \frac{Z_k}{l} U_k \\
\end{align*}
\]

(52)

There is a finite number of the infinite principal modes. The object of balancing is to reduce the rotating bearing forces as far as possible. By successively eliminating the major principal modes \( \phi_n(x) \), it is possible to reduce individual terms in the sum \( \sum_{n=1}^{m} \{ \} \). In practice, this can be done in two ways.

**N Method.** A finite number of principal modes \( n = 1, 2, \ldots N \) are balanced. The residual unbalance that then remains, i.e., the error, is composed of two components, i.e., a residual sum

\[
\sum_{n=N+1}^{m} \{ \} \]

and the two expressions inside the square brackets in equation (52). For this method \( N \) balancing planes are needed.

**N + 2 Method.** Likewise a finite number of principal modes \( n = 1, 2, \ldots N \) are balanced. In addition, however, the "rigid rotor" is also balanced, i.e., the flexible rotor running at low speed (on flexible bearings if need be). This dispenses with the terms in square brackets in equation (52), at all speeds in fact. Now the error is only composed of the residual sum \( \sum_{n=N+1}^{m} \{ \} \).

For this additional process two more balancing planes are required.

It is fairly evident that, under these conditions, the error in the \((N + 2)\) method is smaller than that of the \( N \) method. Of course the process is more complex. Whether this additional effort is worth while depends solely on the distribution of the unbalance in the rotor and the calculated accuracy of balancing. Examples are quoted in the discusser's paper. Under these circumstances the \((N + 2)\) method cannot be treated as an "unnecessary" refinement. Furthermore, it should be fairly obvious that the preceding balancing of the "rigid" rotor, i.e., elimination of the square brackets in equation (52), is not only performed so that smaller forces are obtained at low speeds, but far more as to eliminate the influence of the terms in square brackets in equation (52) at high speeds.

The question as to which of the two methods is "better," can probably not be decided generally in the light of the circumstances described. Here only practice can prove.

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*15 Brown Boveri & Company, Ltd., Baden, Switzerland.

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15 Brown Boveri & Company, Ltd., Baden, Switzerland.
In the theoretical borderline case where all principal modes are balanced, the two methods are identical. When the infinite sum \( \sum_{n=1}^{\infty} | \cdot | \) vanishes, the terms in square brackets in equation (52) also vanish automatically, thereby dispensing with the need for "rigid" balancing.\(^{19}\)

Thus the only real reason for the difference between the two methods is that in practice it is not possible to balance all principal modes.

K. Federn\(^{10}\)

Some facts and views concerning the Bishop-Parkinson paper may be expressed as follows: From this discusser's view, they are always in congruence with the mathematical statement by the authors, even if sometimes not with their conclusions.

The term "State of Balance" has been defined in standard form as follows: A rotor is perfectly balanced when its mass distribution is such that it transmits no vibratory force or motion to its bearings as a result of centrifugal forces (ISO/TC108/Recommendation R 1925, June 1969). Thus, merely avoiding a limited number of peaks in bearing-force curves at angular frequencies \( \omega_1, \omega_2, \ldots \omega_r (\omega_r > \Omega_m) \), as shown in the author's paper, should not be called "Balancing." It is only dynamic straightening, or elimination of deflections in the modes.

It is a precise and accurate definition and recognized by all that a flexible rotor is in a perfect state of balance, when all its modal unbalance components \( \delta_r \) in the modes \( \phi_r(x) \) are eliminated. But this statement is valid only if "all" includes infinity \( \left( r = 1, 2, 3, \ldots \infty \right) \), and does not cover adequately the case of an actual rotor that can only be operated at a limited speed, e.g., \( 1.25 \Omega_m \), whereby \( \Omega_m \) is the service speed.

Many practical factors are ignored (not the least of which is the cost) by merely "achieving an adequate balance whereby the general level of the bearing forces is reduced to a tolerable level." It is not only a "question of how refined an adequate technique must be," but a question of how to achieve an acceptable balance-state level with a minimum of procedures, correction steps, and time, in the most direct and safe way.

"A pair of bearings appropriately instrumented and located in a suitable test environment," suited for modal balancing of flexible rotors, and well designed and supported to ensure utmost security in strength and performance, are already "exceedingly costly devices." In a decade when small moon quakes are reliably sensed, teletransferred, and registered, completion of the modal balancing device by adding the necessary gauges and electronic instrumentation to sense unbalance force and moment before reading the resonance peaks in Fig. 9 should introduce only minor costs and additional complexity.

The era when sensing vibration signals as information for unbalance corrections tied to resonance is gone; it has already been surpassed in the field of balancing by balancing machines with electro-mechanical transducers for vibrations and it has definitively been surpassed by hard-bearing, force-measuring balancing machines.

Balancing is a procedure which primarily concerns itself with mass errors of the type \( m_r \cdot \delta_r \), not with effects, i.e., forces, of the type \( m_r \cdot \omega_r \cdot \Omega^2 \). (See the authors' equations (4), (5), (6), (12), (13) to (17(b))). Therefore, an understanding of balancing principles is facilitated not by considering deflections at critical speeds and peaks in force curves as in Fig. 9, but by considering local unbalances \( \delta_r \), their modal components \( \delta_r \), and their frequency-ratio including representations \( W_L \) and \( W_R \) for the bearings \( L \) and \( R \):

\[
W_L = \frac{P_L}{\Omega^2} \sum_{r=1}^{\infty} \omega_r^2 \cdot \Omega^2 - \Omega^2 + 2i\mu_r \omega_r \Omega \times \left[ \frac{x^2}{b} \int_{0}^{1} A_r(x)\phi_r(x)dx - \frac{1}{b} \int_{0}^{1} A_r(x)\phi_r(x)dx \right]
\]

\[
W_R = \frac{P_R}{\Omega^2} \sum_{r=1}^{\infty} \omega_r^2 \cdot \Omega^2 - \Omega^2 - 2i\mu_r \omega_r \Omega \times \left[ -\frac{x^2}{b} \int_{0}^{1} A_r(x)\phi_r(x)dx + \frac{1}{b} \int_{0}^{1} A_r(x)\phi_r(x)dx \right]
\]

(\( b \) being bearing distance; \( x, x_1 \) coordinates of the bearings \( R \) and \( L \)). These terms for \( W_L \) and \( W_R \) lead to curves similar to those shown in Figs. 8 and 9. \( W_L \) and \( W_R \) do not become zero for \( \Omega = 0 \). They can be measured with hard bearing balancing machines as well as with soft bearing balancing machines.

In agreement with the author's equation (30) and Matthieu's proposal (14) the unbalance representations \( W_L \) and \( W_R \) for the two bearings \( L \) and \( R \) of the flexible rotor can also be expressed by the terms and expansions

\[
W_L = C_L + \sum_{r=1}^{\infty} \omega_r^2 \cdot \Omega^2 - \Omega^2 + 2i\mu_r \omega_r \Omega \times \int_{0}^{1} A_r(x)\phi_r(x)dx \frac{x^2}{b} - \frac{1}{b} \int_{0}^{1} A_r(x)\phi_r(x)dx
\]

\[
W_R = C_R + \sum_{r=1}^{\infty} \omega_r^2 \cdot \Omega^2 - \Omega^2 - 2i\mu_r \omega_r \Omega \times \int_{0}^{1} A_r(x)\phi_r(x)dx \frac{x^2}{b} - \frac{1}{b} \int_{0}^{1} A_r(x)\phi_r(x)dx
\]

with

![Fig. 8](image-url)

![Fig. 9](image-url)

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$U_L = \sum_{r=1}^{m} \omega_r \frac{1}{b} \int_{0}^{l} A_{r}(x)\phi_{r}(x)(x - z)dx$

$U_R = \sum_{r=1}^{m} \omega_r \frac{1}{b} \int_{0}^{l} A_{r}(x)\phi_{r}(x - x_L)dx.$

$U_L$ and $U_R$ can be eliminated by rigid-body balancing at low speed. Matthieu has noted that the series in equation (306) converges far better than the series in equation (29b) of the foregoing. His interesting observation is not refuted by the fact that the series converge in equation (29b) also depends on the magnitudes of $\omega_r$ with respect to $\omega_r$, after low-speed pre-balancing. The terms $\omega_r$ and $\omega_r$ are of the same order, whereas the terms $\omega_r^2\Omega_r$ and $\Omega_r\Omega_r\Omega_r\omega_r^2$ differ significantly, especially for large values of $r$ with $(\Omega_r^2 - 2\mu\omega_r\Omega_r) < \omega_r^2(\Omega_r$ being angular velocity for low-speed balancing) and for all critical speeds, where $b < r$.\textsuperscript{21}

W. T. Thomson, an acknowledged expert in vibration theory of continuous systems and author of a widely used university textbook, has written a chapter 9.6 “Mode-Acceleration Method” in his book on “Vibration Theory and Applications” (1965). Starting with the words “One of the difficulties encountered in any mode summation method has to do with convergence of the procedure,” he pointed out that if the force $F$ and moment $M$ are isolated of a series

$$y(x, t) = \sum a(t)\phi(x)$$

to give

$$y(x, t) = F(x)\int_{0}^{l} a(x) + M(a, t)\beta(a, x) - \sum q_i(t)\phi_r(x)$$

“the convergence is improved over the mode-summation method,” and “a fewer number of normal modes are needed” (Thomson). Just that is accomplished by rigid-body balancing. Just that concept was the idea behind “the earliest papers in this field” [T, 13]. After pre-balancing and eliminating $F$ and $M$ (or $U_L$ and $U_R$ in equation (306) in the foregoing) only internal unbalance moments $D_r(x) = M_r(x)/\Omega$ remain which can be expanded in a series of functions $\phi_r(x)$ which converge well.

$$D_r(x) = \sum_{r=1}^{m} \frac{1}{\omega_r^2} E \int_{0}^{l} \phi_r(x)dx = \sum_{r=1}^{m} \omega_r^2 \phi_r(x)$$

High-speed balancing then only has to take care of these functions. Thus “the need for rigid-body balancing ahead of any form of high-speed balancing” did not have to be assumed, without discussion, but was obvious.

Only a limited number of modal components of unbalance can be considered: the better convergence of these series is of paramount significance for performance and efficiency of any flexible rotor balancing procedure. Thus, rigid-rotor balancing at low speed has not “the merit of eliminating forces which are small compared with those that can arise at a critical speed” (p. 6, 2nd paragr.) but rather the merits of decisively improving the convergence of the speed-ratio weighted modal components of unbalance and their sum from $r = m + 1$ to infinity.

Even if balancing is performed in service bearings, these actually have different dynamic stiffness in different directions (primarily the vertical and horizontal directions). Thus the mode shapes and functions $\phi_{r}(x)$ are different for diametrical axial planes. Mass corrections which improve the state of balance in the principal direction may be detrimental to the state of balance in the other principal direction, as long as the convergence of the mode-term series is poor. As Dodd and Moore have shown in a VDI-sponsored discussion in Berlin in March 1970, compromises can be reached by optimizing the position of correction planes but this procedure involves substantial trial-and-error efforts.

When modal balancing is applied after elimination of resultant unbalance and resultant unbalance moment, the convergence of the mode-term series is sufficiently good that adequate balancing is possible where bearings and their supports do not have service stiffness, or are insufficiently isotropic.

Finally, after rigid-body balancing, measuring runs for high-speed balancing are not limited “as close as possible to a critical speed” as with the pure modal technique, due to better convergence. This is easily shown in Figs. 10 and 11.

If a point of $\Omega_L$ in the left side Fig. 1 is reduced to zero by appropriate mass correction at an angular velocity $\Omega_L$ near but not at first critical speed $\omega_1$, the rotor may show a pronounced induced unbalance peak at critical speed $\omega_1$. However, if in a rotor, $U_L$ and $U_R$ are eliminated by low-speed balancing at $\omega_1$, and one zero-point on the $(\Omega_L^2 - \Omega_L^2)$ curve is already at low speed (as in the right-hand side Figure) then the rotor is modal-balanced at $\Omega_L$ with corrections in three planes (not upsetting the zero-point at $\Omega_L$). This, a second zero-point is reached on the $(\Omega_L^2 - \Omega_L^2)$ curve; only negligible unbalance effect may be expected at critical speed $\omega_1$.

Under service conditions, large turbo-rotors seldom run without being rigidly coupled to other rotors; thus, their mode shapes under service conditions differ from their mode shapes during balancing as a single rotor, even if performed in service bearings. Both convergence of the speed-ratio factors of the expansion terms for the residual local unbalance will cause less deviation from the original state of balance, and will result in less re-balancing effort after the rotors are coupled.

Including low-speed measuring runs and low-speed mass corrections in the balancing procedure leads to better convergence and thereby relieves the restrictive requirements of modal unbalance measurements at critical speeds; even modern computer-aided
balancing procedures can successfully be applied, as shown by Rieger in the paper presented at the Toronto conference.

A rotor which runs above the nth critical speed usually needs corrections in at least \((m + 2)\) planes. This has been widely observed by balancing experts. Furthermore, the rotor furnishes independent information about these mass correc tions if appropriate procedures are used. This has also been confirmed by the authors in their additional comments, when describing their averaging balancing procedure. This is also confirmed by rigid rotors, which do not run through a critical speed where \(m = 0\).

In general, they need correction in two planes.

It is common knowledge that balancing rigid rotors is far less expensive and time-consuming than balancing flexible rotors for consecut ive modes. Unbalance can always be detected and removed with less effort and better reproducibility at low speed than at high speed.

The function values \(\phi_i(x_k)\) \((i = 1, \ldots, j, k = 1, \ldots, (J + 2))\) in equation (17(a)) can easily be found in a "direct way" by "dummy balancing masses," as used by the authors. The writer cannot see the "further drawback" in Kellenberger's method. The authors believe they have found (on p. 6, last paragraph) "the need to know the characteristic functions \(\phi_i(x)\) for these modes."

Making use of a computer when balancing large expensive rotors should nowadays be considered progress and not a drawback.

For rigid rotors, balancing criteria based upon residual unbalance in two planes have been established (ISO/TC 108 DR1940, June 1969) and have found widespread application and acceptance. Balancing criteria, including criteria for unbalance resultant, resultante unbalance moments and modal unbalance components or modal internal unbalance moment components could help standardize balancing procedures and tolerances for flexible rotors.

But it will never be possible to establish such unbalance criteria without explicitly considering rigid-body unbalance components or classifying the rotors under consideration in many groups of rotors, flexible, quasi-flexible and so on, to be handled separately, as the minutes of so many sessions of ISO/TC108/ SCI/WGI have already demonstrated.

"Modal Approach to Balancing" was successfully in use before recent papers were published. Nobody expressed the opinion that "the literature on shaft vibration has become somewhat complicated." Nobody was concerned about "more theory than one" or mourned that "the situation has become confused." All experienced in the art know that successfully balancing flexible rotors is not "to some extent a matter of luck" but the result of modern equipment, care and thorough knowledge.

The following remarks refer directly to authors' conclusions:

Moore and Dodd certainly have extensive experience in turbo-generator balancing but unfortunately only by using their own technique. If they were experienced in other approaches to balancing of large flexible rotors on modern balancing equipment, they would probably recognize their advantages, especially in saving time.

Bearings should not be evaluated by their rigidity. Higher rigidity only means that the same magnitude of vibrations is caused by greater vibratory forces. It is mainly these forces, not the vibrations, that are detrimental to the environment and cause damage. Advocates of rigid-body pre-balancing are not primarily concerned with the forces at low speed but with the decisive help this simple additional procedure provides by improving the reproducibility of high speed conditions, and by improving the results under different bearing conditions and under different assembly conditions in and "above the first critical speed.

It is true that rigid-body pre-balancing is not absolutely necessary before modal balancing, but in nearly all cases it is far less time-consuming than the averaging technique performed at service speed after modal balancing.

The vibrations at service speed (nonresonant) when preceded by modal balancing should not be considered as being of minor importance. Their reduction to tolerance levels is not an unnecessary refinement but a welcome side effect if accurate rigid-body balancing is performed due to better convergence of the subsequent modal balancing steps.

There are already many balancing machines used for high-speed modal balancing of large turbo-rotors. Their procurement is primarily considered by those who have already experienced their advantages.

Familiarity with flexible rotor balancing would seem to recommend rigid-body pre-balancing. If pre-balancing is not performed the final correction after modal balancing must be done very carefully in a manner not to influence the unbalance modes already eliminated. The paper of the author demonstrates how difficult this can be since a rotor is far more sensitive to unbalance errors at critical speed than at low speed.

Now that Bishop and his co-authors, especially Parkinson [1], have done much thorough research in flexible rotor balancing and have published many papers "in the endeavor to give the experimentor a reliable theoretical guide to his work of assessment," it should again be possible to reach agreement. There may exist different ways of balancing flexible rotors but there can only be one theory, a correct one.

Authors' Closure

We wish to thank Dr. Kellenberger for his stimulating comments on our paper. Indeed we have had the pleasure of many lively discussions with him on the matters in question.

As the discusser remarks, the two methods of balancing produce equal results, if a rotor is balanced in all its modes. In practice, of course, it is only possible and indeed necessary to balance a rotor in its lower modes, say for \(n = 1, 2, \ldots, N\), where \(N\) is at most usually only four or five. In these circumstances one must ask which method is likely to produce the best result in terms of acceptable residual bearing forces, speed of balancing, smallest number of balancing corrections and operations and so on.

We feel that the situation is largely covered in the Appendix to our paper and in our comments to the paper by Dr. Kellenberger. In brief, however, if the unbalance in the first \(N\) modes is corrected by a modal technique then, as Dr. Kellenberger indicates, the residual forces on the rotor arise from the sum

\[
\sum_{n=N+1}^{\infty} \left\{ \right. \]

and from the two expressions in the square brackets in equation (52). It must be stressed at this point that, as a result of attaching correction masses to the rotor to balance the lowest \(N\) modes, the contents of the square brackets in equations (52) and

\[
\sum_{n=N+1}^{\infty} \left\{ \right. \]

are not the same as they were before balancing commenced. If the residual forces at service speed are too large for the balancing tolerances, then these forces can be sufficiently reduced at service speed by two more corrections without upsetting the balance in the first \(N\) modes. In this way the \(N\) method may require effectively between \(N\) and \(N + 2\) correction planes and operations.

Similarly with the \((N + 2)\) method, if the rotor is balanced as a rigid body and also in its lowest \(N\) modes, then, as Dr. Kellenberger points out, the residual bearing forces only come from the series

\[
\sum_{n=N+1}^{\infty} \left\{ \right. \]

. Once more, however, the contents of the square brackets in this series are not the same as their values before balancing. Moreover these contents are also likely to differ from those resulting from the application of the \(N\)
forces to be within specified tolerances. Consequently the aim in a "hard bearing" machine.

even at low speeds can cause its own problems, if it is attempted that the residual forces at the maximum operating speed are rotor balancing. As noted in the paper, true rigid rotor balancing the general observation here. It is precisely on the grounds of balancing is defined as a process which is concerned with the forces 2" for the appropriate number of critical speeds and to ensure this sort achieved at low speeds and it really requires a special facility.

We thank Professor Federn for his lengthy contribution to the discussion. Some of the comments are similar to those expressed by Dr. Kellenberger and others refer to matters which are fully discussed in the paper and its Appendix. We consider that these aspects are adequately treated in our reply to Dr. Kellenberger and in the Appendix and we do not wish to bore the reader with a repetition of the points at issue. In particular the merits of balancing a flexible shaft as a rigid rotor at low speed, before commencing any form of modal balancing, is considered in the Appendix to the paper. Indeed the Appendix was intended to demonstrate how closely related the two methods in contention are and to establish a limited agreement between the protagonists. We are, therefore, surprised by the tenor of some of Professor Federn's remarks.

Professor Federn's position in this field is very well known and very entrenched. It appears to be based squarely on the assumption that a flexible rotor, flexibly supported, has "rigid body" modes and should therefore be statically and dynamically balanced. Such a rotor does not, in fact, possess such modes; moreover the concepts of static and dynamic balancing are irrelevant where they are concerned.

We will now consider briefly some of the detailed points enumerated by Professor Federn.

1 and 2. As Professor Federn reports, the term "State of Balance" has indeed been defined in standard form, although it must be observed that not all standard definitions are ideal or physically satisfactory. In this case, however, the definition describes a rotor which is "perfectly balanced" and Professor Federn correctly states that a rotor is perfectly balanced when all its modal components of unbalance, , , , ... are eliminated. The practicing engineer, however, is not concerned with producing a perfectly balanced rotor, which in any case would be impossible. He is interested in ensuring that the rotor is sufficiently, although imperfectly, balanced for the residual bearing forces to be within specified tolerances. Consequently the aim of balancing is "to remove the peaks from curves like that of Fig. 2" for the appropriate number of critical speeds and to ensure that the residual forces at the maximum operating speed are corrected by the method outlined in the Appendix.

3 and 4. Excluding the first sentence, we agree entirely with the general observation here. It is precisely on the grounds of the extra cost and additional complexity which are incurred, whether they be minor or not, that we object to preliminary rigid rotor balancing. As noted in the paper, true rigid rotor balancing even at low speeds can cause its own problems, if it is attempted in a "hard bearing" machine.

5. This statement is at variance with paragraph 1, where balance is defined as a process which is concerned with the forces on, and the motion of, the bearings. Thus the figures employed by Professor Federn to illustrate this paragraph (and also the figures in paragraph 10) are misleading. The curves of the forces plotted against rotor speed \( \Omega \) pass through the origin \( \Omega = 0 \), so that the results of low speed balancing on those forces are not so dramatic as its effects on the magnitudes of \( \omega_L \) and \( \omega_R \) at low speeds. An understanding of balancing principles is facilitated by considerations of the modal character of the bearing forces and vibration.

6, 7, 8. The interesting comment by Matthieu concerning the convergence of the series in equations (30), (37), (39), (44)-(47) is fully discussed in the paper. We merely stress here that at a given rotor speed \( \Omega \) the magnitudes of the forces \( R \) and \( R' \) (in equations (44) and (46)) also depend on the magnitudes of \( a_i \), \( \alpha_i \) and on the proximity of \( \Omega \) to \( \Omega_m (\omega) \) ... . That is only a few terms in the series (44)-(47) are likely to be of importance, when \( \omega_m < \Omega_m(\omega) < \omega_m \). 9, 10, and 12. Professor Federn correctly explains some of the problems which can arise, if the bearings in which a rotor is balanced have different dynamic stiffnesses in different directions. The application of modal balancing has been extended to these systems elsewhere.11 We cannot accept that the additional complexity in balancing these systems only arises if "the convergence of the mode-term series is poor." The reason for balancing a rotor in its service bearings, with either method, is to reduce the need for subsequent field balancing which may arise if the mode shapes of the rotor in service differ markedly from those of the rotor in the balancing facility.

10. "Measuring runs for high speed balancing" are performed at speeds "as close as possible to the critical speeds" to simplify the detection of the corresponding modal components of unbalance. This has nothing to do with the convergence of the modal series. The first figure used by Professor Federn to illustrate his claim in paragraph 10 is a gross misrepresentation of modal balancing. When a rotor is balanced at a speed \( \Omega_m \), near its first critical speed, the result is not to reduce the bearing forces to zero at the speed \( \Omega_m \), but to remove the first mode unbalance. As a consequence the residual bearing forces at \( \Omega_m \) will be small, but nonzero, and the peak at \( \Omega_m \) will be similar to that shown in the second figure of this paragraph.

11. This paragraph refers to a problem which is potentially of considerable practical importance, no matter which method of balancing is adopted.

13. This was indeed noted in our Appendix, but we fear that the method advocated by Professor Federn is likely to require \( m + 4 \) not \( m + 2 \) correction planes.

14. Yes, but balancing flexible rotors initially as rigid rotors is likely to be more expensive and time consuming. This paragraph is a truism whose relevance escapes us.

15. At present practical data on the dynamic characteristics of the bearing oil film, bearing pedestals and foundations are not known accurately enough for computed mode shapes to be of much practical value.

16. The last sentence of this paragraph is a complete misrepresentation of our comment that "we are concerned with a form of behavior whose severity, with an uncorrected rotor, is to some extent a matter of luck."

In the interests of brevity we refrain from discussing fully Professor Federn's comments on our Conclusions. We feel that the combination of the paper, its Appendix, and our comments on Professor Federn's paragraphs 1–17 deals adequately with these final points. We must draw attention, however, to the misleading comments on Conclusion 7. One of the problems of pre-balancing is the additional complication introduced at the modal balancing stage by the need to avoid upsetting the prior rigid rotor balance. In straightforward modal balancing this need does not arise.

Finally, we merely repeat something that understandably he is unhappy about, viz. balancing machines are not needed for flexible rotors. This may be unpalatable, but it appears to us to be true.