better agreement with the actual behavior of most metals than the assumption used by the author that the rate of hardening is a function of the rate of plastic work.

There should be no great difficulty in obtaining exact solutions using the discussers' assumptions, although the computational effort required would be increased substantially. The ambiguity referred to for stress points in the corner of the yield hexagon disappears in the formulation of the problem for the exact solution and can be removed in the approximate solution by making an additional assumption in the following way.

The profile of the shell can be assumed to be a circular arc (the difference in the results obtained using this assumption and those found using a parabolic profile are not significant). Then, when the material at the center of the shell enters the corner, the shell is assumed to deform as in the third stage of deformation (as a portion of a sphere of uniform thickness). Since both strain rates at the center of the shell are known, the maximum shear strain is known and approximate solutions can be obtained in all cases. This method can be used to obtain approximate solutions which agree fairly well with the exact solutions given in the paper.

The author does not agree that, since plastic instability of a *tensile specimen* occurs when the yield stress is reached for $H \leqslant 1$, these values of $H$ are irrelevant in the study of the instability of *finite cylinders*. One could argue with equal logical force that, since plastic instability of an infinite cylinder occurs at yield for $H \leqslant 2$, these values of $H$ are irrelevant in the study of the tension test. Analysis of the tension test shows that the decrease in strength due to reduction in area is sufficient to offset the increase due to strain hardening for $H \leqslant 1$. The author has shown for $H \leqslant 2$ that the reduction in strength of a finite cylinder due to thinning of the wall is offset by the combination of the increase due to strain hardening and the increase due to changes in geometry if the shell is sufficiently short.

# Effect of Small Cross Flow and Surface-Temperature Variation on Laminar Free Convection Along a Vertical Plate[1]

**R. EICHHORN.**[2] The writer would like to pose some questions about the nature of the solution since it appears to him that some of the physical features of real flows are not included.

According to equation (16) of the paper, the cross-flow velocity is only a function of the nondimensional stream variable $\xi = cyx^{-1/4}$. From this, we easily deduce that the boundary-layer thickness for the cross-flow velocity grows in the $x$-direction like $x^{1/4}$ but not at all with $z$, the crosswise distance variable. Tracing back through the authors' equations, this fact is seen to result from the assumption which caused the term $w\, \partial w/\partial z$ to be dropped from equation (2). Physically, of course, this means that the plate has no leading edge along a line of constant $z$ and that the flow in the $z$-direction is of the shear-flow variety. To the writer, such an occurrence seems quite artificial. The most disturbing fact is that in the absence of a leading edge along a line of constant $z$ the boundary-layer thickness of the cross flow is determined solely by the flow in the upward direction.

In line with the foregoing comments, the introduction of $\theta_s$ as a function of $z$ is seen to be necessary in order that the problem solved be nontrivial. It is difficult to see that the solution presented solves the problem of "edge effects" when edges are specifically excluded.

[1] By R. J. Young and K-t Yang, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E, pp. 252–256.

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## Authors' Closure

The authors appreciate the remarks made by Dr. Eichhorn and would like to offer the following answers to his questions:

He is incorrect in stating that the boundary-layer thickness in the cross-flow direction is independent of the crosswise distance variable $z$. The similarity variable $\xi$ does depend on $z$ in view of the definition of $c$ in equation (13) of our paper. Consequently, the flow in the $z$-direction can be considered as a shear flow only when the entire plate is maintained at a uniform temperature. The authors fail to see why this type of flow is artificial.

As indicated in the beginning of our paper, this study was motivated by the desire to find corrections to the two-dimensional solution when both cross surface-temperature variation and cross flow are present. Physically, the cross surface-temperature variation could be due to heat loss through edges when the plate is not sufficiently wide, and the cross-flow could represent probably the simplest type of disturbances in the ambient. We have found these corrections in terms of first-order deviations from the quasi-two-dimension solution. It is indeed unfortunate that Dr. Eichhorn took the view that $\theta_s(z)$ was introduced in our analysis merely for the purpose of obtaining nontrivial results.

It is true in our analysis that the leading edges in the $z$-direction are not considered. We did not attempt to solve the "leading-edge" problem. What we meant by the problem of "edge effects" is only to the extent that the presence of these edges is responsible for the cross-temperature variation in the physical problem, and we have attempted to solve this cross-temperature variation problem in the paper. In order to solve this $z$-direction "leading-edge" problem, either more terms in our perturbation solution must be considered, or an entirely different approach is taken.