$$S = 30 \sqrt[4]{\frac{8g^3}{\pi^2}} \sqrt[4]{\frac{tg^2\beta_1^*}{(atg^2\beta_1^* + b)^3}}$$

putting

$$30 \sqrt[4]{\frac{8g^3}{\pi^2}} = A$$

we get

$$S = A \sqrt[4]{\frac{tg^2\beta_1^*}{(atg^2\beta_1 + b)^3}} \qquad (5a)$$

Our aim is to find for what value of β_1^* the value of S will be a maximum. To simplify the mathematical procedure we have to remember that this will happen when $1/S$ will be a minimum. Equation (5*a*) can be rewritten in the form:

$$\frac{A}{S} = \sqrt[4]{\frac{a^3\theta^3 + 3a^2\theta^2 b + 3a\theta b^2 + b^3}{\theta}} \qquad (5b)$$

Where

$$\theta = tg^2\beta_1^*$$

Written here

$$\frac{a}{b} = K$$

and

$$\frac{A}{S\sqrt[4]{b^3}} = Y$$

equation (5*b*) becomes

$$Y = \sqrt[4]{K^3\theta^2 + 3K^2\theta + 3K + 1/\theta} \qquad (5c)$$

substituting here

$$K^3\theta^2 + 3K^2\theta + 3K + 1/\theta = X$$

and differentiating we get

$$dx = (2K^3\theta + 3K^2 - 1/\theta^2)d\theta$$

From this it follows that

$$\frac{dy}{d\theta} = \frac{dy}{dx}\frac{dx}{d\theta} = \frac{1}{4} X^{-3/4}(2K^3\theta + 3K^2 - 1/\theta^2)$$

From equatinos (5*b*) and (5*c*) it follows that $\frac{1}{S}$ will be a minimum when $\frac{dy}{d\theta} = 0$. This will occur when either

$$X^{-3/4} = 0 \qquad (5d)$$

or

$$\theta^3 + 1.5\theta^2/K - 1/(2K^3) = 0 \qquad (5e)$$

Condition (5*d*) is fulfilled only when $tg^2\beta_1^* = 0$ or $tg^2\beta_1^* = \infty$. Both cases are of no practical interest for the engineer. Equation (5*e*) remains, therefore, the only practical solution.

Finally, substituting $K = \frac{a}{b}$ in equation (5*e*) we get:

S will be a maximum when

$$\theta^3 + 1.5\,\frac{b}{a}\,\theta^2 - \frac{b^3}{2a^3} = 0$$

DISCUSSION

S. Bhaduri[2]

The author is to be congratulated for presenting important information regarding the performance characteristics of inducers and pumps. The author has developed relationship between the net positive suction head NPSH, the volume rate of flow Q, and their optimum values corresponding to the maximum suction specific speed S. Equation (10) may be used to find the required NPSH for any flow rate once the optimum NPSH and the optimum Q are known. This relation seems to be helpful in selecting the mode of operation of a pump. However, it is to be noted that the definition of the net positive suction head given by the equation (2) needs justification. Usually the net positive suction head NPSH is defined as the gage pressure in feet of the liquid taken on the suction nozzle referred to the pump center line, minus the gage vapor pressure in feet corresponding to the temperature of the liquid, plus the flow velocity head at this point. The author did not include the vapor pressure in his equation. It is not indicated whether the pumps for which the inducer characteristics are given are special types operating under conditions where possibility of cavitation is absent. In that case, the results obtained are not applicable to large number of centrifugal and mixed flow pumps.

The inducer is intended to improve the operational characteristics of the impeller by directing the entering flow at an angle such that the flow enters the impeller channel smoothly. For a given impeller speed, there is only one flow rate at which the liquid will approach the impeller meridonally or without prerotation. For the flow rates significantly smaller than the so-called normal flow rate, the liquid needs to acquire prerotation in the direction of the impeller rotation and for the flow rate greater than the normal, the liquid should have a prerotation in the opposite direction. Generally, the inlet flow to the impeller in the actual pumps follows this pattern modified somewhat by the effect of the suction nozzle and suction pipe design. The author did not furnish information regarding the basic features of the inducers used. A sketch describing the various component of the pump unit would have been very helpful.

The comparison between the calculated and tested NPSH values in various pumps has been presented in the paper and valuable information namely the ratios such as S/n_s, $Q_{B.E.P}/Q_{opt}$, and $NPSH_{B.E.P}/NPSH_{opt}$ for the various pumps tested can be easily obtained from the graphs. It is observed that the range of the specific speed ratios for pumps tested is approximately 4.96 to 18.8. The flow rate ratios have values between 1.76 and 3.0 and the NPSH ratios have values between 1.4 and 2.9. The ranges of the values of the ratios are quite small and therefore care should be exercised in applying the results to pumps in general.

The author did not present the performance characteristics of the pumps tested with and without the inducers. This would have been useful in the evaluation of the inducers. It is believed that the inclusion of graphs of total head versus flow rate Q, efficiency versus flow rate Q would have strengthened the quality of this excellent paper.

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Author's Closure

The author wishes to thank Mr. Bhaduri for his kind comments. He hopes that the following will answer Mr. Bhaduri's questions.

1 The expanded form of equation (2) is

$$NPSH = H_A - H_V - C_A^2/2_g = (aCm_1^2 = bV_1^2)/2_g \ldots \quad (2)$$

where:

H_A = absolute static head at the pump inlet;
H_V = vapor pressure divided by specific weight;
C_A = absolute velocity of the liquid at the point where H_A was measured.

2 The test data presented in this paper relate only to pumps tested without inducers and to inducers tested without pump.

3 The range of specific speeds n_s covered by the tested pumps and inducers extend from n_s = 400 (for the $1^1/_2$-HN-122 pump listed in Table 1) to about n_s = 10,000 (for some of the inducers represented in Fig. 1).

4 All pumps mentioned in this paper are standard commercial units, manufactured by Worthington Corporation. Full data relating to the performance characteristics of these pumps are given in the company's catalogs and related literature.