analytical model is good; however some inaccuracy is noted due to the oversimplified model of the shaft flexibility and difficulties in determining the stiffness parameters of the system.

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APPENDIX

Structural Properties of the MIT AE Rotor

Dimensional Parameters

$$M = \text{total mass of rotor assembly} = \int_v dm = 25.4 \text{ kg}$$

$$S = \text{axial mass imbalance} = \int_v z\, dm = -0.741 \text{ kg}\cdot\text{m}$$

$$I_d = \text{rotor pitch moment of inertia} = \int_v (x^2 + z^2) dm$$

$$= 0.297 \text{ kg}\cdot\text{m}^2$$

$$m_0 = \text{blade modal mass} = r_T^2 \int_{r_H}^{r_T} \gamma^2 dr = 1.43 \times 10^{-3} \text{kg/m}^2$$

m_1 = blade consistent mass coupling to disk pitch

$$= r_T \int_{r_H}^{r_T} r\gamma dr = 2.41 \times 10^{-3} \text{kg}\cdot\text{m}^2$$

m_2 = blade consistent mass coupling to disk translation

$$= r_T \int_{r_H}^{r_T} \gamma dr = 1.01 \times 10^{-3} \text{kg}\cdot\text{m}^2$$

$$m_\Omega = \text{blade mass foreshortening} = \int_{r_H}^{r_T} r \int_{r_H}^{r} \left[\frac{\partial \gamma}{\partial r}\right]^2 dr\, dm$$

$$= 3.70 \times 10^{-3} \text{kg}\cdot\text{m}^2$$

$K_x = K_y$ = shaft translational stiffness = 5.68×10^7 N/m
$K_\eta = K_\xi$ = shaft pitch stiffness = 1.40×10^6 N·m/rad
$K_{x\eta} = -K_{y\xi}$ = shaft coupling stiffness = 6.43×10^6 N/rad
k_B = blade modal stiffness = $\omega_B^2 m_0$ = 6.43×10^3 N·m
α = effective stagger angle = 36 deg

Nondimensional Parameters

μ_{TP}^2 = shaft inertial coupling = 0.073
μ_{TB}^2 = blade bending/disk translation inertial coupling = 0.021
μ_{PB}^2 = blade bending/disk pitch inertial coupling = 0.054
ρ^2 = blade mass/rotor inertia ratio = 0.055
l = Southwell coefficient = 1.93
$\bar{\omega}_T^2$ = disk translation/blade bending frequency ratio = 0.497
$\hat{\omega}_P^2$ = disk pitch/translation frequency ratio = 2.11
κ^2 = shaft elastic coupling = 0.520

DISCUSSION

N. Klompas[1]

By their excellent investigation into a dynamic interaction that has been ignored too long and their clear and thorough presentation, the authors have made a significant contribution to potential advancement of engine design. While they have demonstrated that two important mechanisms traditionally treated separately may interact and should be combined, an actual engine presents a much more complicated structure and more interacting mechanisms which also are currently ignored. Progress toward integration of valuable research, such as represented by this paper, into the analysis of a whole engine requires physcial interpretation of results into the context of engine needs. My published work [15–20] toward evolving a physical basis for analyzing a whole engine with accounting for exceptional mechanisms should be helpful in providing necessary interpretation.

While the present study incorporates the bladed disk and shaft into a single analysis of the system, the previous papers [15, 16] had accounted for bladed disk flexing by assuming independent characterization of effective gyroscopic moment which would be expressed as the product of the gyroscopic moment assuming rigidity and a magnification factor which depends on the speed of precession with respect to the disk (ω_R) and the speed of spin (Ω). For a bladed disk where the deflected forms due to the body forces producing the gyroscopic moment and due to the inertial forces of flexing are similar, the magnification factor would be the same as for the classic spring/mass. When the relative speed of precession passes through the natural circular frequency of the bladed disk spinning while rigidly supported at the juncture to the shaft, the magnification factor goes through magnitude peak and sign reversal.

[1] 29246 Lake Park Drive, Farmington Hills, MI 48018; Mem. ASME.

The possibility of this peak coinciding with a critical speed of the whole engine has been intriguing and the present paper suggests proximity to such a coincidence (point A) as a criterion for the existence of dynamic interaction which should signal caution in accepting critical speeds calculated by assuming bladed disk rigidity. Although the experimental data do not show significant error in the calculated natural frequency of the backward whirl mode responsible for the coincidence, strong interaction may be inferred from other observations.

The experimentally determined natural frequencies show coalescence of forward and backward frequencies at approximately 400 Hz with spin speed of approximately 120 Hz, a point close to point A. The blade natural frequency at the same speed is shown also as approximately 400 Hz. The previous paper [15] has shown analytically that forced whirl passing through the resonance of a bladed disk with system natural frequency close to coinciding does not produce sharp change in whirl amplitude and the present experimental results show that the system becomes indifferent to changes in direction of the strong effective gyroscopic reaction. Therefore, it is now concluded that the dynamic stiffness at the shaft becomes overwhelming and the natural frequency represents the bladed disk alone. The thesis that the coalescence of forward and backward whirl natural frequencies occurs at the natural frequency of the bladed disk alone is now asserted.

Significant errors in calculated resonant whirl speeds are seen at points more remote from point A. The forward speed relative to the stationary frame of reference at the point of coalescence is approximately 445 Hz, or 16 percent higher than that calculated assuming rigidity. For the lower, or "bounce" mode the critical speed for synchronous backward whirl—the whirl speed relative to the disk would be twice spin speed—is approximately 22 percent higher than that calculated for the rigid disk. Such errors with respect to the usual assumption of disk rigidity are to be expected where the higher natural frequency of the bladed disk raises the coupled natural frequency. However, the opposite effect is also possible and is represented by the splitting of the higher mode.

The current practice in analysis of the complete engine, as typified by Hibner [21], is to assume rigid bladed disks and circular orbits. Therefore, backward whirl is not accounted for unless counterrotating spools are incorporated. However, it occurs often because asymmetry of engine support structure causes interaction between forward and backward whirl so as to produce coupled whirl of elliptical modeform; experimental evidence of ellipticity has been shown previously [17, 18].

The current practice has led to previous studies on the effect of bladed disk flexibility on synchronous forward critical speeds, such as represented by Palladino and Rossettos [7], with the consistent finding of inconsequence. The present data confirm this finding.

Previous papers [18–20] derive physical interpretation, and possible integration into the analysis of the whole engine, of exceptional mechanisms that might arise. The method of modifying a basic linear response analysis of a complete engine to account for a mechanism involving secondary interaction or nonlinearity would be useful in extending the assumed mechanism of disk flexing to shroudless bladed disks. While the effective gyroscopic moment accounts for blade flexing which produces a moment only in the plane of precession, shroudless fans would involve also effects due to translational displacement of the centroid in the plane and would produce cross coupling between the plane of precession and the orthogonal whirl plane. Whereas this cross coupling would exist only at locations of flexing shroudless bladed disks, the previous accounting for steady torque [19] has demonstrated analysis with similar cross coupling distributed along the length of a shaft.

A significant exceptional mechanism might be the excitation of harmonics by asymmetry of the effective gyroscopic moment due to blade mistuning. This possibility has been discussed previously [18] and is implied by the present paper. The method of accounting for exceptional mechanisms [19, 20] should provide effective means of determining whether an engine would be sensitive to such asymmetry.

The present paper performs a great service by revealing potential shortcomings in current design practice. Future work should include developing and validating analytical models of complete engines.

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