

### The Effect of Misalignment on the Fatigue Life of Cylindrical Roller Bearings Having Crowned Rolling Members<sup>1</sup>

**J. H. RUMBARGER,**<sup>2</sup> Mr. Harris is to be congratulated for presenting a long-needed analysis technique for the mathematical treatment of modified line-contact or crowned roller members. The effects of angular misalignment in cylindrical roller bearings cannot be ignored in demanding applications. Shaft bending together with shaft displacements can introduce significant amounts of misalignment in most moderate and heavily loaded applications.

The present treatment is much better than the previous linearized approaches mentioned by the author. The present treatment of laminae with nonlinear stiffnesses is especially useful for determining the load per unit length along the roller when the area of elastic contact under load has been experimentally determined. This can be accomplished by using the expression for semicontact width as a function of roller load (for steel components),

$$b = (2.8692 \times 10^{-4}) \sqrt{L \left( \frac{1}{Ra} + \frac{1}{Rb} \right)} \text{ (in.)} \quad (1)$$

<sup>1</sup> By T. A. Harris, published in the April, 1969, issue of the JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 91, No. 2, pp. 294-300.

<sup>2</sup> The Franklin Institute Research Laboratories, Philadelphia, Pa. Mem. ASME.

Experimental verification of the effects of angular misalignment is lacking in the literature at the present time. Several years ago I was able to obtain the elastic contact impressions or "footprints" of some large diameter rollers loaded into flat plates. Indentation of the flash copper plating on one of the blocks reveals a footprint pattern which can be accurately measured using the micrometer staging on an optical comparator with special backlighting effects. Figs. (1a) and 1(b) show the measured footprint of a cylindrical roller (with straight taper leads) with 0 deg and 0 deg - 5 ft misalignment. The contact shape as computed with the Harris laminae theory is also shown. Reasonable order-of-magnitude type of agreement is obvious. One notable exception is the shape at the ends of the contacts. The actual footprints have wider semiwidths than the calculated values. This can be explained by the simplification of ignoring shear-compatibility between the laminae. As mentioned by the author, a cylindrical roller with no crowning would show stress increases or dog-boning at the roll ends and that the present treatment does not include these effects. The fact that the experimental misaligned footprint is shorter than the calculated one is more difficult to explain but probably results from the same simplification.

The analysis is straightforward with the exception of equation (19) of the text which is used to relate  $\delta_1$  and  $\delta_2$ . From the description of  $\delta_1$  and its pictorial presentation in Fig. 6 of the text, it seems sufficient to use the relationship:

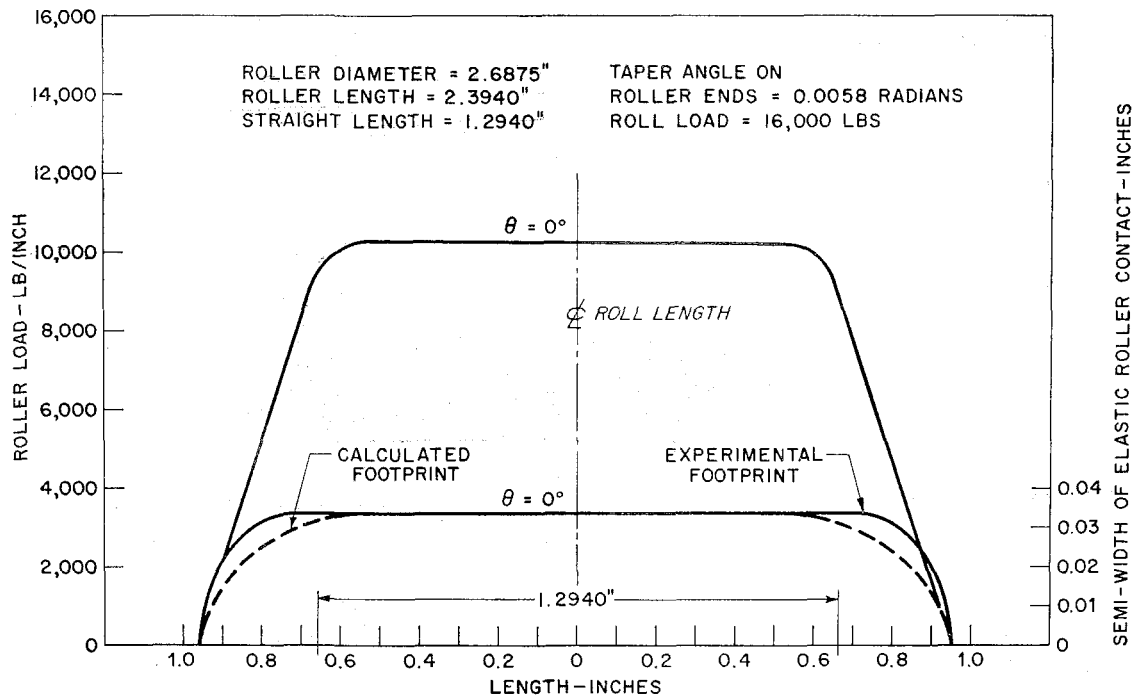
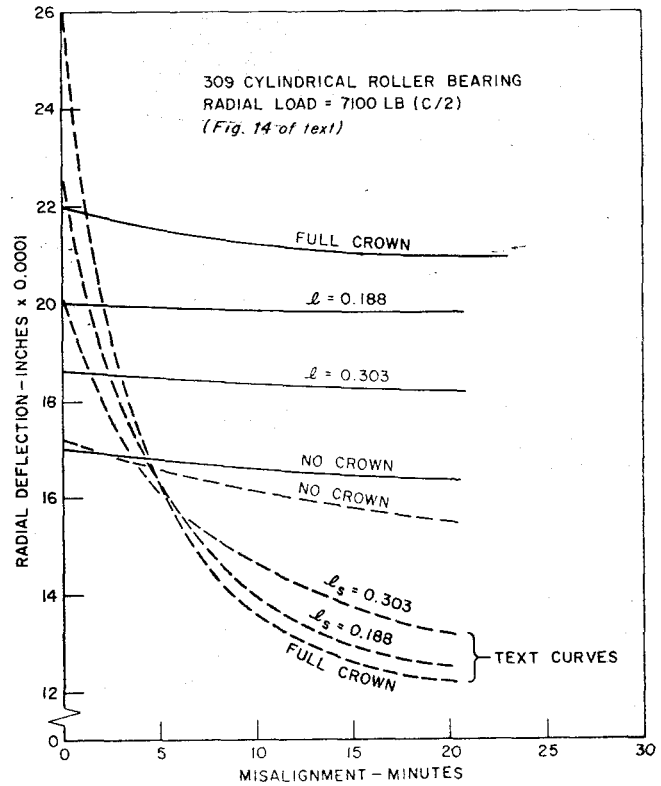
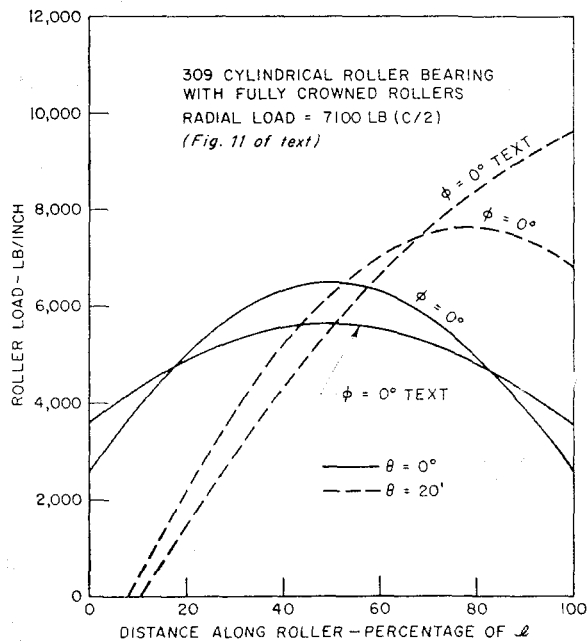
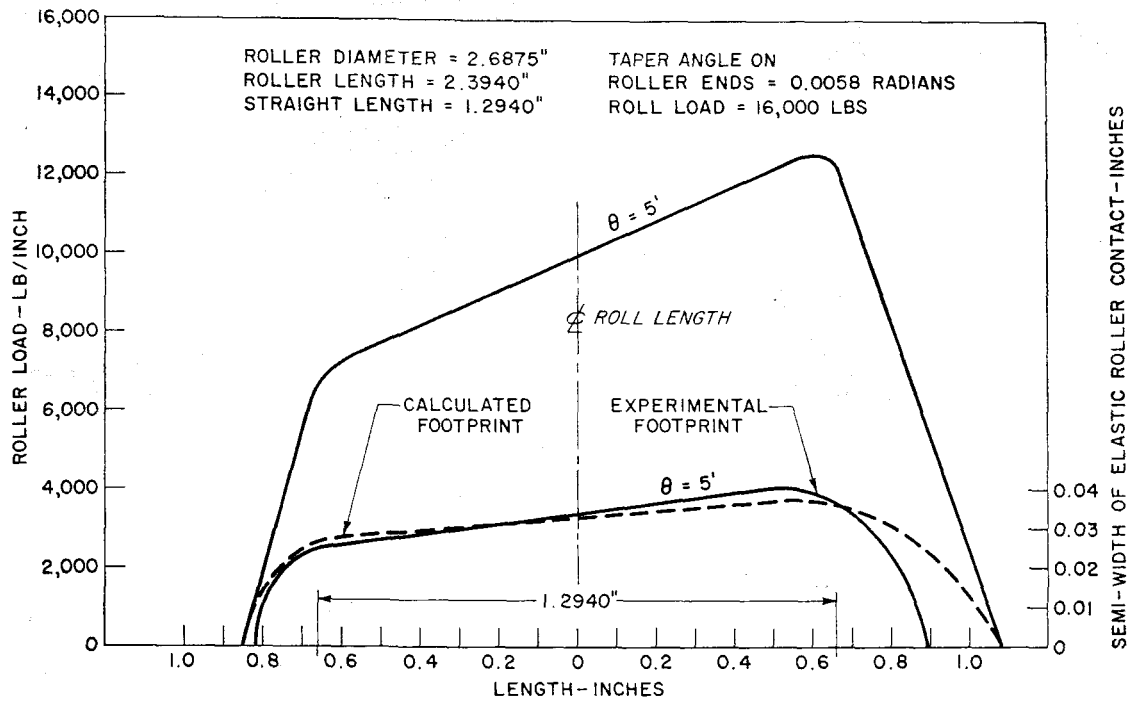


Fig. 1(a)



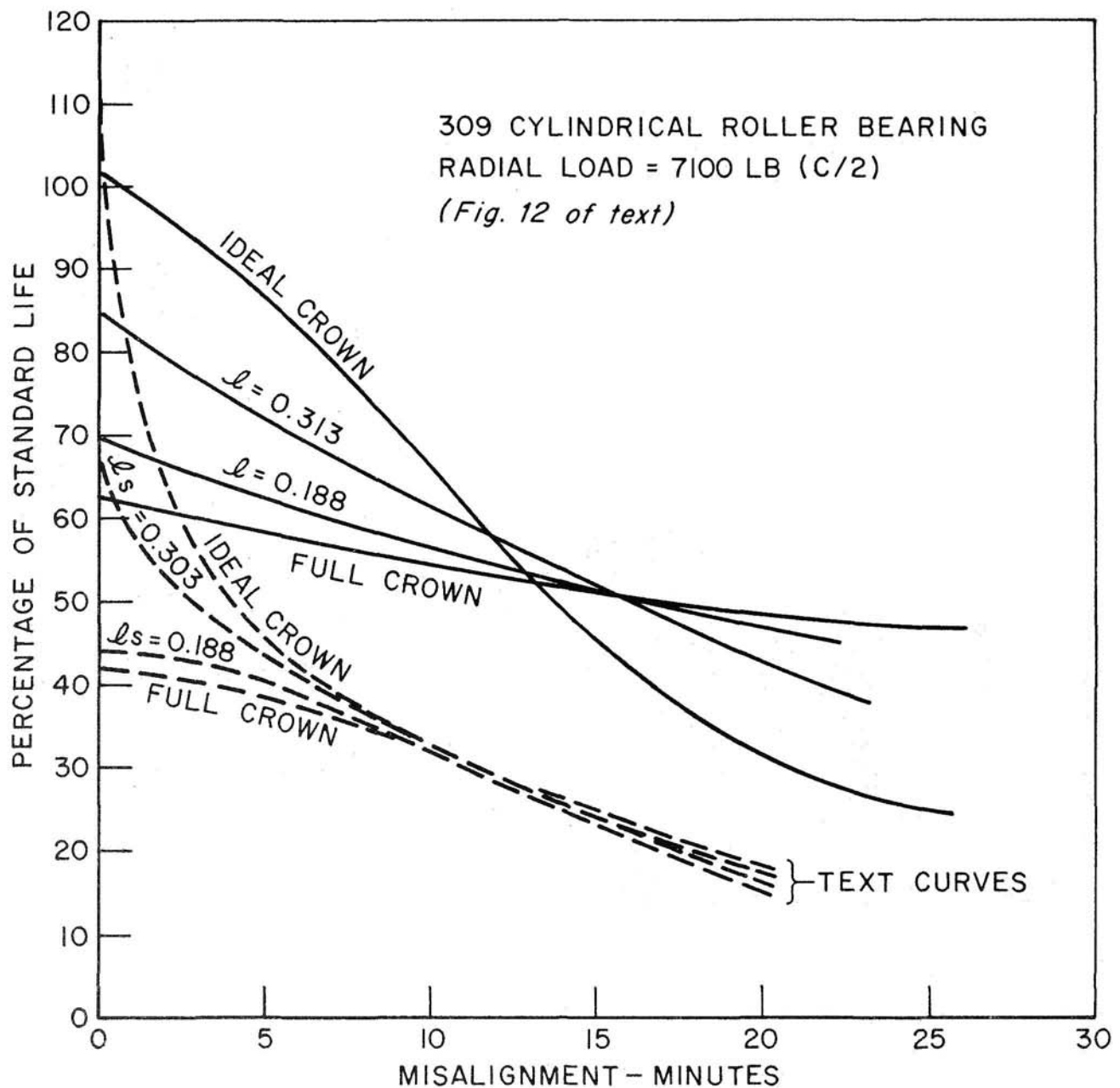


Fig. 4

$$\delta_r = \frac{1}{2} \left( \delta_r \cos \phi - \frac{Pd}{2} \right) \text{ (in.)} \quad (2)$$

The use of equation (19) implies reduction of the crown-drop effects in the final analysis. The equations of the text with the appropriate substitution of equation (2) for equation (19) of the text were programmed on a digital computer. Displacement of the center of the inner race was assumed and the resulting laminae displacements (functions of  $\delta_r$  and  $\theta$ ) were calculated. The resulting roller reaction loads were combined into bearing reactions at the bearing center and compared to the applied load system. A rapidly convergent Newton-Raphson iterative technique provided equilibrium solutions. The curves in Fig. 10 of the text (with no crown effects) were reproduced exactly. The solution of the fully crowned roller had some significant differences from the text curves. Fig. 2 shows the comparison. The text curves do not have as much load relief near the roll ends

although the same crown dimensions are used in both solutions. The  $\theta = 0$  deg,  $\theta = 20$  curve of the fully crowned roller (Fig. 11) is not very different from the uncrowned roller in Fig. 10 of the text. The resulting differences in calculated radial deflection and fatigue life are shown in Figs. 3 and 4. Thus the significance of equation (19) of the text is extremely important to the results of the analysis. The author's comments and clarification of the logic, interpretation, and use of equation (19) would be most helpful in making this analytical technique more understandable and thereby of more general usefulness.

The author's analysis is a powerful addition to the computer systems analysis of rolling element bearing applications. Experimental verification is desirable and will no doubt be available in the near future. The technique of obtaining "footprints" should be applied to a complete bearing as the quickest and most economical means of obtaining experimental verification.

**THOMAS BARISH.**<sup>3</sup> The author has contributed a thorough and useful analysis of a problem that has needed and does still need attention. He is to be complimented.

Question: Does the distortion of the contact pattern produce a variation in roller speed from one end to the other which would result in a positive skewing force of considerable magnitude though small displacement? Does the theory indicate this and the extent of it?

Extensive tests on the author's subject were made 10 years ago, and they confirm the author's conclusions approximately. Test data are presented in Fig. 5 and Table 1.

The first two groups of bearings were put in as a control using the standard curvatures, the first being with inner race-riding-cage, and the second being put in with outer race-riding-cages. The major difference here is the better lubrication within the bearing provided at very high speeds.

If we use the average of these two bearings with the misalignment present, we obtain 47 percent life as against a zero misalignment. The author's Figs. 12 and 13 give 50 and 42 percent.

The last two groups of bearings are the design recommended to correct the trouble: no crown on the rollers at all (except for a short edge relief at each end) and a reverse crown on the outer race only. The first group R211-G9 were made with too small a radius and uncontrolled for the outer race crown, but were tested. The last group with correct crown showed 4.4 × the life of a

standard bearing ABEC rating without misalignment. Most of this increase is due to the more effective use of the roller length at the inner race contact.

In M. Kubinek's "Practice in Czechoslovakia,"<sup>4</sup> they checked both inverse crowned outer rollers as above, and also fully crowned rollers with straight races. In this last case, they used a relatively small radius, approximately 100 × roller length. They are producing fully crowned rollers and claim 40 percent higher dynamic capacity.

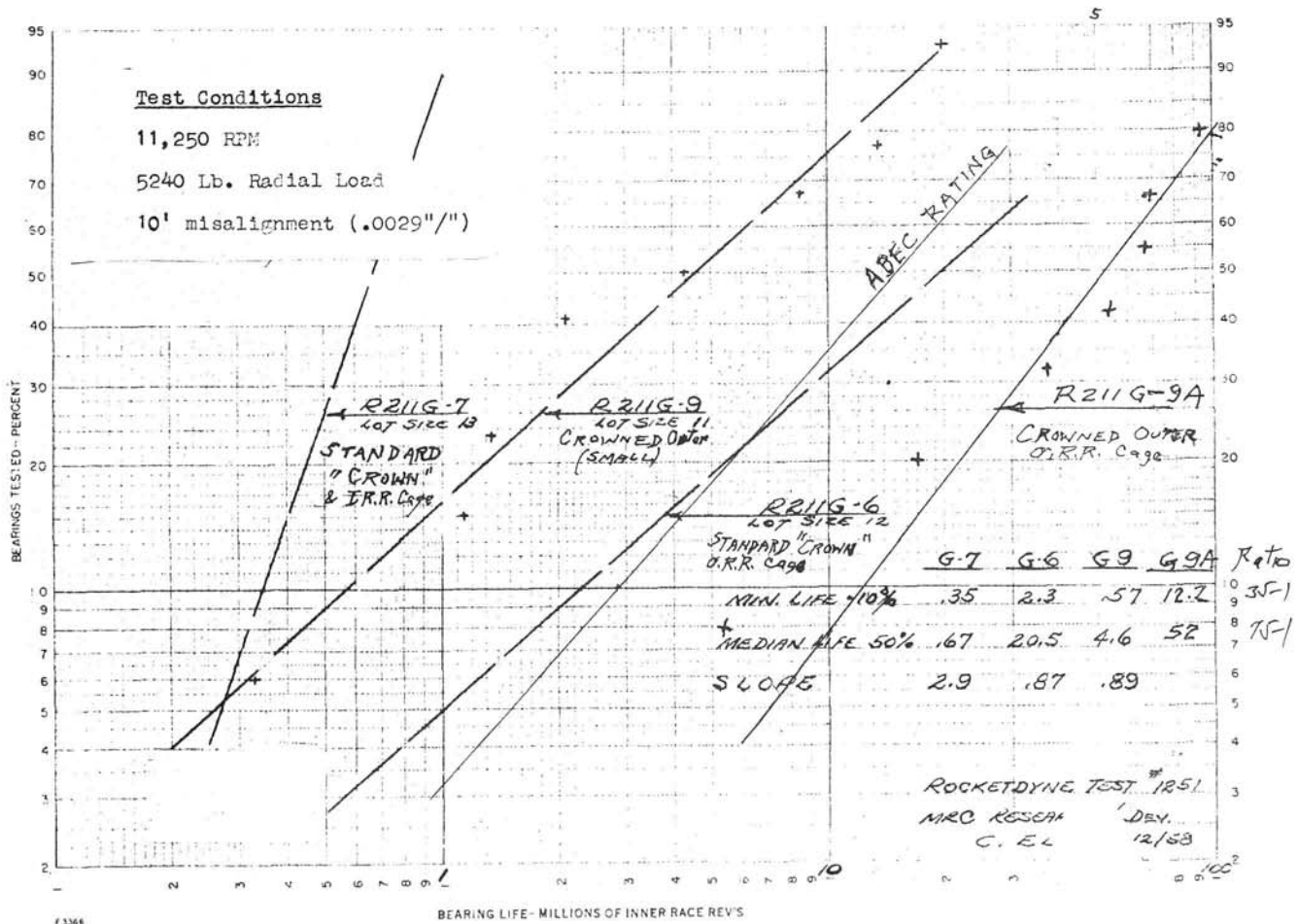
**Table 1 Tests under 10<sup>1</sup> misalignment**

Brg	Crown	Cage Riding	B10 Life Hrs	Life Ratio vs 0 Tilt	Ratio per Harris
ABEC Rating	Standard		2.8 <sup>a</sup>	100%	
R211-G7	Standard	Inner	.35	12%	} 50% & 42%
R211-G6	Standard	Outer	2.3	82%	
R211-G9	Outer only but small radius	Outer	.57	20%	
R211-G9A	Outer only 11" Rad.	Outer	12.2	440%	

\* No Misalignment

<sup>3</sup> Consulting Engineer, Van Nuys, Calif.

<sup>4</sup> *Engineers Digest*, London, Aug. 1963, Vol. 24, No. 8, pp. 63-65.



**Fig. 5**

W. J. DERNER.<sup>5</sup> The author is to be commended for his very fine approach representing the rigorous analysis required in bearing application studies to show the effect of significant operating conditions on bearing life.

At Rollway Bearing Co., such an analysis is routinely performed, and roller specifications are modified to insure that the bearings will perform satisfactorily under the otherwise limiting conditions. Modified crowns are selected after performing a detailed analysis such as given in this paper. It must be recognized that the specification of the essential crown relief is sometimes easier than its accomplishment. The techniques for generating, blending, and verifying roller crown modifications requires the development of a detailed process.

In developing equations (15) and (16), the effect of crown drop was determined to be negligible. We note in Fig. 8 (schematic diagram of misaligned roller, etc.) that the roller half length shown was not modified to account for the effective crown length. Does not the author agree that the effective length of the roller in contact with the race must be considered and that the determination of this modified contact must be related to the load and to the crown modification? More specifically, we contend that the radial interference is not solely related to roller geometry, but also related to load, misalignment, and the crown modification selected. Sample calculations showing the influence of crowning on the maximum radial displacement have been given to the author for his evaluation. We have calculated displacements of 0.00009 versus the authors 0.00014, and 0.00044 versus the authors 0.00071, considering misalignment angles of 0 degree 1 second and 0 degree 5 seconds, respectively. Would the author comment on these differences?

In the attachment giving the results of the Parametric Study, it is stated that an efficient method to determine edge loading has not been developed. It is suggested that a suitable method is available, being found in FORSHUNG—auf dem Gebiete des Ingenieurwesens (Band 10 Berlin, Sept./Okt. 1939—NR5) being an article "Elastic Contact Between Two Semi-Infinite Bodies"—G. Lundberg. It is our contention that this, along with certain experimental methods employed by Rollway, does permit optimizing crowns which add significantly to bearing performance under critical operating conditions. Has the author considered this method in the paper by Lundberg and rejected it for some obvious reason?

Using sample brg. (#309) selected by author in his parametric study, we have calculated values of  $\delta_0$  (interference due to misalignment, in.)—also see Fig. 8 of author's attachment—for misalignment angles of 0°1' and 0°5' both based on (a) a non-crowned roller and (b) a crowned roller whose straight length is 0.303 in. (see Table I data on p. 16 of author's paper).

Noncrowned roller with 0°1'  $\bowtie$  misalign:

$$\left. \begin{aligned} \beta &\cong 12^\circ 10' & \theta &= 0^\circ 1' \\ \delta_0 &\cong 0.00006R \\ R &\cong 2.353 \end{aligned} \right\} \therefore \delta_0 \cong 0.00014$$

Crowned roller with 0°1'  $\bowtie$  misalign [Note:  $l$  taken as 0.303 instead of 0.496]:

$$\left. \begin{aligned} \beta &\cong 7^\circ 30' & \theta &= 0^\circ 1' \\ \delta_0 &\cong 0.00004R \\ R &\cong 2.320 \end{aligned} \right\} \therefore \delta_0 \cong 0.00009$$

Noncrowned roller with 0°5'  $\bowtie$  misalign:

$$\left. \begin{aligned} \beta &\cong 12^\circ 10' & \theta &= 0^\circ 5' \\ \delta_0 &\cong 0.00030R \\ R &\cong 2.353 \end{aligned} \right\} \therefore \delta_0 \cong 0.00071$$

Crowned roller with 0°5'  $\bowtie$  misalign [Note:  $l$  taken as 0.303 instead of 0.496]:

$$\left. \begin{aligned} \beta &\cong 7^\circ 30' & \theta &= 0^\circ 5' \\ \delta_0 &\cong 0.00019R \\ R &\cong 2.320 \end{aligned} \right\} \therefore \delta_0 \cong 0.00044$$

## Author's Closure

Mr. Rumbarger's commentary is most welcome; however, the writer believes that Mr. Rumbarger has not fully contemplated the significance of equation (19).

This formula relates the maximum contact deformation to the maximum interference between a roller and the inner and outer raceways at any azimuth location  $j$ . As the roller-to-raceway load-deflection relationship is nonlinear, the contact becomes relatively stiffer at greater loads. It can be seen from Fig. 5 that under misalignment, it is usually the roller end which undergoes the heaviest loading. It appears to the author that Mr. Rumbarger incurs error in his interference equation by omitting the effect of misalignment on interference. The decreased radial stiffness and increased life which is obtained is a result of this omission.

In reply to Mr. Barish's query, as a result of misalignment each roller assumes an angle of tilt (pitch) relative to the shaft axis according to its azimuthal location on the pitch circle. Consequently, sliding motions tend to occur between the rollers and raceway as a function of the axial position along the roller contact and azimuthal location. A skewing moment will occur at each roller location owing to the variation of sliding velocity with axial position. Using EHD friction force analysis, the magnitude of each skewing moment can be estimated.

It is further interesting to note that the reduction in fatigue life caused by misalignment as estimated by the author's analysis seems to be in the ball park with reported experimental results referenced by Mr. Barish.

Mr. Derner's commentary concerning the routine performance of an analysis similar to that presented by the writer is certainly disheartening to the writer. Not having seen such an analysis in print has led the author to conclude his was an original effort. Mr. Derner's rhetorical questions relating to the use of effective lengths of the roller-to-raceway contacts and the connection between radial interference, load, misalignment, crowning, and roller geometry must necessarily be answered in the affirmative.

While it is true that G. Lundberg developed a method to estimate the stress-deflection pattern for contact of two semi-infinite bodies, the analysis is not so general as to permit its utility with other than idealized bodies of revolution. This means that actual crowning geometry used in bearings tends to negate the utility of the Lundberg analysis. Moreover, the author's experience with any analysis concerning contact stresses at discontinuities such as those occurring at roller ends and crown blends in crowned cylindrical roller bearings is that the peak contact stresses predicted are several orders of magnitude larger than the bearing steel can tolerate and yet survive. It is therefore the author's viewpoint that either (a) plastic deformations tend to relieve the peak stresses by providing more stressed area, or (b) stresses of such magnitude do not exist in the first place. Probably, the truth lies somewhere in between these extremities. In any event, an experimental program is required to more accurately determine the effect of contemporary roller crowning methods on relief of stress concentrations at the roller ends and crown blends.

It is interesting to observe that the bearing radial deflections developed from Mr. Derner's analysis are substantially less than the author's analysis predicts. Mr. Rumbarger's analysis shows radial deflections substantially greater than those of the author (at least for crowned roller bearings). The writer can explain away the variance between Mr. Rumbarger's analysis and his

<sup>5</sup> Engineering Manager, Rollway Bearing Co., Syracuse, N. Y.

own; however, since Mr. Derner's commentary is not accompanied by appropriate mathematical backup, the same cannot be done in the latter case. The writer suspects, however, that Mr. Derner's analysis does not consider the total effective roller length, notwithstanding his commentary to the contrary, and therefore a stiffer bearing results from his analysis.

## Influence of Water on Fatigue-Failure Location and Surface Alteration During Rolling-Contact Lubrication<sup>1</sup>

**J. M. HOPKINS.<sup>2</sup>** The authors are to be complimented for conducting an investigation of value to bearing users, bearing investigators, and lubrication investigators. However, we feel that some comment needs to be made relative to the significance of the probability of failure of pole and equator area.

Conclusion "A," based on the failure analysis presented in Table 3, states "approximately 90% of failures per unit surface area of region occur on the poles and equator. . . . These regions have reduced fatigue strength."

Admittedly, a predominance of failure occurring in end fiber areas warrants suspicion; however, the "reduced fatigue strength" conclusion is not justifiable without a comparable life analysis of balls failed in end fiber and side fiber areas.

In previous work conducted by our laboratory,<sup>3</sup> we found the rolling contact fatigue life differences between balls failed in the end fiber and side fiber areas to be relatively insignificant in balls of standard manufacture. We also found variations in the failure probability of end fiber to side fiber failures in nonstandard manufactured balls.

We would also like to direct the authors' attention to the paper by Gulbransen and Copan<sup>4</sup> which we feel suggests an alternate failure mode to that proposed by the authors.

**R. J. PARKER<sup>5</sup> AND E. V. ZARETSKY.<sup>5</sup>** The authors have written an excellent paper on a well-conducted experimental program. The effect of very small water concentrations on roller-element fatigue life is clearly shown.

The work verifies and extends the work of Grunberg and Scott<sup>6</sup> which shows reduced fatigue life with several lubricants for larger concentrations of water than those reported by the author. For these tests,<sup>6</sup> no reduction in life was observed when stainless steel balls were used as specimens. These results<sup>6</sup> support the authors' corrosion hypothesis.

<sup>1</sup> By P. Schatzberg and I. M. Felsen, published in the April, 1969, issue of the *JOURNAL OF LUBRICATION TECHNOLOGY*, TRANS. ASME, Series F, Vol. 91, No. 2, pp. 301-307.

<sup>2</sup> Marlin-Rockwell Corp., Division of TRW Inc., Jamestown, N. Y.

<sup>3</sup> Hopkins, J. M., "Method of Producing Improved Bearing Components by Elimination or Control of Fiber Orientation," Marlin-Rockwell Corporation, Jamestown, N. Y., Oct. 15, 1963.

<sup>4</sup> Gulbransen, Earl A., and Copan, Thomas P., "Effect of Stress on Unusual Crystal Habits of Corrosion Products on Iron, Nickel, Chromium and Stainless Steel," Westinghouse Research Laboratories, Pittsburgh, Pa., 1958.

<sup>5</sup> NASA-Lewis Research Center, Cleveland, Ohio. Mr. Zaretsky is a Mem. ASME.

<sup>6</sup> Grunberg, L., and Scott, D., "The Acceleration of Pitting Failure by Water in the Lubricant," *Journal of the Institute of Petroleum*, Vol. 44, No. 419, Nov. 1958, pp. 406-410.

The stress-life relationship found by the authors is of particular interest. The exponents  $n = 11.7$  and  $11.5$  are not surprising. Unpublished fatigue data from the discussers' laboratory indicate an exponent as high as 14 in the five-ball fatigue tester in the stress range of 650,000 to 800,000 psi maximum Hertz stress. The discussers' analysis of other data from a four-ball fatigue tester [3] indicated an exponent of 13 at maximum Hertz stresses between 600,000 and 800,000 psi. The accumulation of such data showing larger exponents than 9 or 10 indicate that the bearing materials may be more sensitive to stress changes at higher stress levels than has been generally accepted.

The authors' data tend to refute the contention put forth in the authors' closure of [3], "that no power law with a single exponent fits the data well throughout the overload region (high Hertz stress region, e.g., maximum Hertz stress greater than 600,000 psi), and that this is indicative of a change in mechanism of the fatigue failure phenomenon compared with that prevailing at lower loads where a power law fits very well." A single exponent does fit the authors' data in a very high stress region. In addition, the authors' results suggest two different failure mechanisms exhibiting the same stress-life relation.

The discussers would like to ask the authors if a recalculation of Hertz stress was made based on the altered surface geometry due to plastic deformation and wear? If so, what effect did these alterations have on the reported stress-life exponent?

**FRED G. ROUNDS.<sup>7</sup>** In order to find an explanation for the detrimental effect of water on fatigue life presented in an earlier paper [1], the authors examined the failed balls by a number of techniques. We have used many of the same techniques to examine failed balls but our purpose was somewhat different. In our case, we wanted to explain the effects of oil additives. The results of our surface analyses tend to parallel those presented by the authors. To study the effects of straight mineral oils, synthetic fluids and additive blends on fatigue failure location, 375 upper balls from 4-ball fatigue tests were etched. The results of our examinations indicated that a region about 15 deg from the equator was as prone to fatigue failure as the polar area on a failure rate per unit area basis. Thus we confirm the poor fatigue characteristics of the equatorial region of balls. Although our equatorial region was somewhat larger than that found by the authors, the difference in equatorial region size may merely reflect differences in the amount of upset that occurred during the initial cold heading operation used to make the balls.

As did the authors, we also examined the ball tracks by light microscopy at about 100 $\times$  but the white bands noted by the authors were not seen even though no attempt was made to dry the oils prior to test. Do the authors have any ideas about the composition of these bands and how they are formed? When the ball tracks were examined at much higher magnification with an electron microscope, the general surface appearance of balls run at 1,200,000 psi Hertz max. closely resembled the surface appearance of thrust ball bearing races run at loads from 300,000 to 500,000 psi Hertz Max. in friction tests.<sup>8</sup> However, the high load test electron micrographs had one additional feature, fine cracks were often detected in the electron micrographs taken at points on the running track well away from the fatigue spall. These cracks are probably similar to those found by Reichenbach and Syniuta.<sup>9</sup>

<sup>7</sup> General Motors Research Laboratory, Warren, Mich.

<sup>8</sup> Rounds, F. G., "Effects of Additives on the Friction of Steel on Steel Part 1. Surface Topography and Film Composition Studies," *ASLE Trans.*, Vol. 7, 1964, pp. 11-23.

<sup>9</sup> Reichenbach, G. S., and Syniuta, W. D., "An Electron Microscopy Study of Rolling Contact Fatigue," *ASLE Trans.*, Vol. 8, 1965, pp. 217-223.