

should be run. Simultaneous measurements of instantaneous loads, motions and friction coefficients are needed to explore the limits and suitability of the friction model. Other contact models, mentioned earlier, and other friction mechanisms, (e.g., Challen and Oxley, 1975; Rigney and Hirth, 1979; Kovmopolous et al., 1986) may need to be studied in the context of vibrating contacts. Interactions between the bending vibrations of the block and counter-surface, in combination with contact resonance phenomena could be included.

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DISCUSSION

A. Akay¹ and M. T. Bengisu¹

The authors are to be complimented on their extension of the Greenwood-Williamson model of a rough surface to the case of sliding surfaces. The aim of the paper is to show the coupling between the normal and angular oscillations. The formulation depends on the adhesion theory which is, strictly speaking, applicable under static conditions. Therefore, the resulting equations of motion are not dependent on the sliding velocity. Equations (14b) and (14c) can be solved by using Eq. (15) alone, without Eq. (14a). In other words, normal and angular oscillations can take place under stationary conditions. In such quasi-static context, the results of the paper clearly demonstrate the dynamic effects and the coupling between normal and angular oscillations. When the dependence of friction force on the sliding velocity is taken into account all three modes will be coupled. Stability analyses of such a case clearly demonstrate limit cycles and chaotic behavior.

Authors' Closure

Professor Akay raises some interesting points. Indeed, the

system that is analyzed in the paper only includes coupling between the normal and angular motions that arise due to compliance and friction within the contact. The horizontal suspension is deliberately placed in line with the center of mass of the block to keep the system simple. Movement of the horizontal suspension away from this position, or the addition of other external constraints, can couple all three motions and some out-of-plane motions as well. The present model could therefore be extended to reflect more complicated constraints and to capture multi-axial limit cycles and other complex non-linear responses. Also, if one encounters sticking, loss of contact of momentary reversal of sliding direction, (not considered in this paper), the horizontal motions will automatically become coupled to the others. We believe that a rich variety of dynamic behavior can be found, even within the assumptions of this quasi-static model of the friction process. In cases where the friction coefficient or friction force is speed-dependent, the problem is more complex and involves more parameters. An even larger variety of dynamic responses can be found. We agree with Professor Akay that many applications may demand such a model. However, the present work shows that it may be necessary to explicitly include a model of the contact region to fully describe the high frequency dynamics of sliding systems.

¹College of Engineering, Wayne State University, Detroit, MI.