7 "Spanlose Formung der Metalle, Eigenspannungen in Metallen," by G. Sachs, Handbuch der Metallphysik, Band 3, Erste Lieferung, Leipzig, Germany, 1937, pp. 63–70; Akademische Verlags Gesellschaft, M.B.H.

8 "Einiges über die Fliessbewengung beim Pressen von Stangen und Rohren Sowie beim Ziehen," by H. Unckel, *Zeitschrift für Metallkunde*, vol. 20, 1928, pp. 323–330.

9 "Plastic Flow in Metals," by J. J. Jelinek, A. J. Latter, E. G. Thomsen, and J. E. Dorn, OPRD Research Project NRC-548, May, 1945, p. 69.

10 "Photoelasticity," by M. M. Frocht, John Wiley & Sons, Inc., New York, N. Y., vol. 1, 1946, p. 28.

11 "A Treatise of Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge University Press, 1931, p. 95.

Discussion

B. W. Shaffer.[4] This is an interesting study of the behavior of lead during the extrusion process. The figures, obtained by studying grid deformation, vividly show the flow pattern of particles during this process.

In Fig. 8 of the paper the authors analyzed the extrusion of a billet which is symmetric about the center line shown to the right of the figure. Therefore but one half of the process is shown. Nevertheless, because this center line is an axis of symmetry, all shear lines should intersect it at an angle of 45 deg. This condition, however, was not satisfied by the semicircular arc shown at the boundary between the rigid and plastic regions. Fig. 8, therefore, is incorrect and it cannot possibly be a solution to the problem being studied.

Three correct solutions to the extrusion process with $^1/_3$ reduction have been published. R. Hill[5] published two. Each of his solutions has a stagnant, or dead-metal region in the square corner between the die and cylinder wall, and each satisfies all boundary conditions. E. H. Lee[6] published a third solution to the same problem. His analysis, shown in Fig. 9 of this discussion, also satisfies all boundary conditions, but does not show a dead-metal region.

[4] Assistant Professor of Mechanical Engineering, New York University, New York, N. Y. Jun. ASME.

[5] "A Theoretical Analysis of the Stresses and Strains in Extrusion and Piercing," by R. Hill, *Journal of the Iron and Steel Institute*, vol. 158, 1948, pp. 177–185.

[6] "Theory of Perfectly Plastic Solids," by W. Prager and P. G. Hodge, John Wiley & Sons, Inc., New York, N. Y., 1951, pp. 181–182.

The extrusion pressures required by these solutions were not the same. In fact, the one without a dead-metal region required the least pressure. Since flow will follow that pattern which requires the minimum applied force, extrusion with a $^1/_3$ reduction,

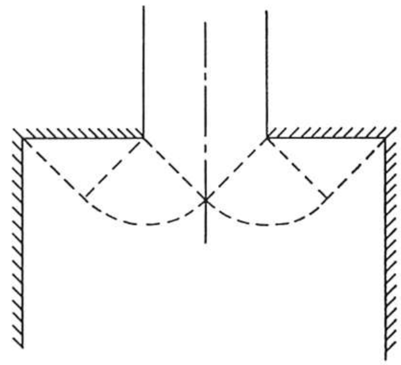

Fig. 9

according to these plane-strain solutions, will take place without a dead-metal region.

It is interesting to note that the authors did not observe a dead-metal region in their experiments.

Authors' Closure

The authors wish to thank Professor Shaffer for his discussion of the paper, and agree with him that Fig. 8 requires a minor modification. Lines should intersect the center line of the extrusion at an angle of 45 deg. This requires a slight shifting of a few lines and Fig. 8, except for this modification, is the correct solution of the inverted extrusion problem as given by Hill.

Additional experiments and calculations which have been made by the authors recently, indicate that the solution given in Fig. 9 does not agree with the experimental observations in spite of the fact that this solution permits plastic flow along the die surface. As a matter of fact, they find, that the plastic sector of Fig. 8 is a good first approximation for the maximum shear directions. They also observed that the shear strain rates in the so-called dead-metal region, i.e., outside of the plastic region of the Hill solution, are opposite in sign to those within this sector. These findings will be reported in a paper now in preparation.