Neutrino-Nucleon Scatterings under Scaling Violation

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Neutrino-nucleon scatterings are discussed within the framework of the Wilson operator product expansion assuming that observed scaling violations are due to the field theoretical origin. It is shown that the total cross sections rise linearly with initial neutrino energy and the neutrino total cross section becomes equal to the antineutrino cross section at sufficiently high energy. The numerical value of muon inelasticity is estimated to be 9/16. The average value of $q^2$ is also discussed.

§ 1. Introduction

Recently, the ratio of antineutrino to neutrino charged current cross sections up to 100 GeV was measured. Above 50 GeV, the total cross section ratio shows a significant departure from 0.35 which coincides with the value predicted by the naive quark parton model. This experimental result seems to indicate the scaling violation. The deep inelastic $\mu-N$ scattering experiments have also evidently indicated the scaling violation in comparison with the lower energy data in SLAC. Many theorists have attempted to explain the patterns of the observed scaling violation from various standpoints. In this paper, we take a standpoint that the observed scaling violation is due to the field theoretical origin.

The scaling invariance guarantees that the total cross sections of $\nu-N$ scatterings rise linearly with the initial neutrino energy. Thus, it is interesting to investigate the total cross sections of $\nu-N$ scatterings under the scaling violation. Although this problem has been studied by several authors, we shall attack this problem by using the formula proposed by Parisi. In § 2, we shall discuss the total cross section of $\nu-N$ scattering and the $\gamma$-anomaly. In § 3, the muon inelasticity and the average value of $q^2$ will be investigated.

§ 2. The total cross section of $\nu-N$ scattering

The inelastic $\nu^\pm-N$ cross section $d^2\sigma/dydx$ may be written as

$$
\frac{d^2\sigma^\pm}{dydx} = \frac{mG^2E_\nu}{\pi} \left\{ (1-y)F_1(x, q^2) + \frac{y^2}{2}xF_1(x, q^2) + \frac{y(2-y)}{2}xF_3(x, q^2) \right\}, \quad (1)
$$

where $E_\nu$ is the initial neutrino energy. In Eq. (1), we neglect the non-leading terms in $E_\nu$. $x$ and $y$ are connected with the variables $q^2$ and $\nu$ in the following:
\[ x = \frac{q^2}{2m
u}, \quad y = \frac{\nu}{E_\nu}, \quad (2) \]

where \( x \) is the ordinary scaling variable. From Eq. (2), \( q^2 \) is given by \( x, y \) and \( E_\nu \):

\[ q^2 = 2mxyE_\nu. \quad (3) \]

Note that, when we perform the \( x \) and \( y \) integrations in Eq. (1), the parameter \( q^2 \) appearing in the structure functions varies according to Eq. (3). Hence, it is impossible to perform the \( x \) and \( y \) integrations without knowing the \( q^2 \) dependence of the structure functions. To do this we may use the following formula derived by Parisi,\(^8\)

\[ F_2(x, q^2) = \int_0^1 F_2(x', q_0^2) \left\{ \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\pi}} \frac{x'^n}{x^{n+1}} \frac{E_{2n}(q^2)}{E_{2n}(q_0^2)} \, dn \right\} \, dx', \quad (4) \]

where \( q_0^2 \) is a constant, \( E_{2n}(q^2) \) is given by the Mellin transformation of \( F_2(x, q^2) \) and \( \varepsilon \) is a infinitesimal constant:

\[ E_{2n}(q^2) = \int_0^1 x^n F_2(x, q^2) \, dx, \quad (5) \]

which can be estimated\(^9\) by the Wilson expansion of current product \( j_\mu^+(x)j_\nu^- (0) \). In the conventional field theory,\(^9\)

\[ E_{2n}(q^2) = b_{2n}(q^2)^{-\gamma_{2n}}, \quad (6) \]

where \( \gamma_{2n} \) is the anomalous dimension of the \( n + 2 \)-rank tensor operator appearing in the Wilson expansion and \( b_{2n} \) is a constant. By the insertion of Eq. (6) into Eq. (4),

\[ F_2(x, q^2) = \int_0^1 F_2(x', q_0^2) \left\{ \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\pi}} \frac{x'^n}{x^{n+1}} \left( \frac{q_0^2}{q^2} \right)^{-\gamma_{2n}} \, dn \right\} \, dx'. \quad (7) \]

In order to integrate Eq. (1) in the variable \( x \), we must estimate the following integration:

\[ \int_0^1 F_2(x, q^2 = 2mE_nxy) \, dx \]

\[ = \int_0^1 dx \int_0^1 F_2(x', q_0^2) \left\{ \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\pi}} \frac{x'^n}{x^{n+1}} \left( \frac{2mE_nxy}{q_0^2} \right)^{-\gamma_{2n}} \, dn \right\} \, dx'. \quad (8) \]

In Eq. (8) we exchange the order of the integrations. Thus,

\[ \int_0^1 F_2(x, q^2 = 2mE_nxy) \, dx \]

\[ = \int_0^1 F_2(x', q_0^2) \left\{ \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\pi}} x'^n \left( \frac{2mE_nxy}{q_0^2} \right)^{-\gamma_{2n}} \left( \int_0^1 x^{-n-\gamma_{2n}} \, dx \right) \, dn \right\} \, dx'. \quad (9) \]

\(^{a)}\) For simplicity, only the singlet part is considered. In the case that the non-singlet part exists, we can consider similarly.
The $x$-integration can be easily performed, where it becomes necessary to estimate the following $n$-integration.

$$I_n = \frac{1}{2\pi i} \int_{-\infty}^{\infty} x'^n \left( \frac{2mE_{xy}}{q_0^2} \right)^{-\gamma_{2n}} \frac{1}{n + \gamma_{2n}} dn.$$ \hspace{1cm} (10)

In Eq. (10), we should remark that $\gamma_{2n}$ must be zero at $n=0$ because the energy momentum tensor enters the Wilson expansion at $n=0$ and it must not have the anomalous dimension. Thus, $\gamma_{2n}$ can be written as

$$\gamma_{2n} = n \alpha_2(n),$$ \hspace{1cm} (11)

where $\alpha_2(n)$ is a regular function of $n$. In Eq. (11), we have assumed that $\gamma_{2n}$ is analytic at $n=0$. Some model calculations satisfy this assumption. According to Nachtmann's positivity condition, $\alpha_2(n)$ is positive for $n>0$.

With these considerations, we see that the integrand in Eq. (10) has a pole at $n=0$, and none at $n>0$. Therefore,

$$I_n = \frac{1}{1 + \alpha_2(0)} + \text{(non-leading terms in $E_0$)},$$ \hspace{1cm} (12)

where the non-leading terms come from the poles at $n<0$. Inserting Eq. (12) into Eq. (9), we can obtain,

$$\int_0^1 F_2(x, q^2 = 2mE_{xy}) dx = \frac{1}{1 + \alpha_2(0)} \int_0^1 F_2(x', q_0^2) dx',$$ \hspace{1cm} (13)

which is independent of $E_0$ in high energy scatterings.

We can obtain an equation similar to Eq. (13) for the structure function $F_1(x, q^2)$.

$$\int_0^1 xF_1(x, q^2 = 2mE_{xy}) dx = \frac{1}{1 + \alpha_1(0)} \int_0^1 x'F_1(x', q_0^2) dx'.$$ \hspace{1cm} (14)

On the other hand, we obtain the following equation for $F_3(x, q^2)$ in the same approximation as in Eqs. (13) and (14).

$$\int_0^1 xF_3(x, q^2 = 2mE_{xy}) dx = 0,$$ \hspace{1cm} (15)

because the energy momentum tensor cannot couple to the vector and axial vector interference term and so the corresponding integrand for $F_3(x, q^2)$ has not a pole at $n=0$.

With these results, we can calculate the total cross sections from Eq. (1), and the differential cross section $d\sigma^\pm/d\theta$.

$$\frac{d\sigma^\pm}{d\theta} = \frac{mg^2 E_0}{\pi} \left\{ (1 - \gamma) \frac{1}{1 + \alpha_2(0)} \int_0^1 F_2(x', q_0^2) dx' ight\}$$

$$+ \frac{\gamma^2}{2} \frac{1}{1 + \alpha_1(0)} \int_0^1 x'F_1(x', q_0^2) dx' \right\},$$
\[ \sigma^\pm = \frac{mG^2E_\nu}{\pi} \left\{ \frac{1}{2(1+\alpha_2(0))} \int_0^1 F_2(x', q_s^2) \, dx' \right. \\
+ \left. \frac{1}{6(1+\alpha_3(0))} \int_0^1 x'F_1(x', q_s^2) \, dx' \right\}, \tag{16} \]

which lead us to the following conclusions:

1. The \( \nu^-N \) total cross sections rise linearly with the initial energy \( E_\nu \), even when the scaling violation exists.

2. The neutrino total cross section \( \sigma^+ \) becomes equal to the antineutrino total cross section \( \sigma^- \) in sufficiently large \( E_\nu \). Thus, in intermediate energy region \( \sigma^+ \) approaches to \( \sigma^- \), which explains the experimental result.\footnote{\textsuperscript{11}}

3. If we assume the Callan-Gross relation and \( \alpha_2(0) = \alpha_3(0) \),

\[ \frac{d\sigma^\pm}{dy} = \frac{mG^2E_\nu}{\pi(1+\alpha_2(0))} \left( 1 - y + \frac{y^3}{2} \right) \int_0^1 F_2(x', q_s^2) \, dx', \]

which exhibit the \( y \)-anomaly for neutrino and anti-neutrino scatterings. Above results were obtained for the conventional field theory.

For the asymptotically free gauge theory, Eq. (6) is replaced by

\[ E_{2n}(q^2) = b_{2n}(\log q^2)^{-\gamma_{2n}}. \]

The integral corresponding to Eq. (10) is written as

\[ I_n' = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^n (\log q_s^2)^{\gamma_{2n}} \left\{ \int_0^1 x^{-n-1} (\log 2mE_\nu + \log x)^{-\gamma_{2n}} \, dx \right\} \, dn, \]

in which the \( x \)-integration may not be easily performed. We, however, expect an appearance of a simple pole in \( n \) at \( n=0 \) by the same reasoning as before. Hence the same results as before may be derived. We do not investigate this case further.

\section*{§ 3. Muon inelasticity}

Several years ago, the present author calculated\textsuperscript{11} the muon inelasticity in the deep inelastic region of \( \nu-N \) scattering assuming the Bjorken scaling. In this section we reconsider the muon inelasticity and the average of \( q^2 \) under the scaling violation.

\[ \varepsilon^\pm = \frac{\langle E_\mu \rangle}{E_\mu} = \int_0^1 \int_0^1 \frac{E_\mu}{E_\mu} (d^2\sigma^\pm/dxdy) \, dx \, dy, \tag{17} \]

where \( \langle E_\mu \rangle \) is the average of the muon energy. With the relation \( E_\mu = E_\nu - v \), \( \varepsilon^\pm \) can be rewritten as

\[ \varepsilon^\pm = 1 - \langle y \rangle = 1 - \int_0^1 \int_0^1 y (d^2\sigma^+/dx\,dy) \, dx \, dy. \tag{18} \]
According to Eq. (1) and Eqs. (13)~(16), we can easily estimate the muon inelasticity.

\[
\varepsilon = \frac{1}{3(1+\alpha_2(0))} \int x F_2(x', q_0^2) dx' + \frac{1}{24} \frac{1}{1+\alpha_1(0)} \int x F_3(x', q_0^2) dx'.
\]

(19)

If we assume the Callan-Gross relation

\[ F_2(x', q_0^2) = x' F_1(x', q_0^2) \]

(20)

and \([\alpha_2(0) = \alpha_1(0)]\), we obtain

\[ \varepsilon^+ = \varepsilon^- = \frac{9}{16}. \]

(21)

It should be stressed here that the muon inelasticities \(\varepsilon^n\) are independent of the initial neutrino energy and \(\varepsilon^+\) is equal to \(\varepsilon^+\). In Ref. 1) the present author obtained the numerical value 9/16 for the muon inelasticity in the case that the Pomeranchuk contribution to charged current-nucleon scatterings is dominant. The reason for this was that for both cases the \(x\)-integrals of the structure function like Eq. (13) are independent of \(E\), and the vector-axial vector interference term \(F_3\) vanishes.

In the similar manner, we can estimate the average of \(y^n\),

\[ \langle y^n \rangle = \frac{3}{2(n+1)(n+2)} + \frac{3}{4(n+3)}, \]

which is also independent of \(E\).

In the following, we consider the average of \(q^2\). From Eqs. (1) and (3) we find

\[ \langle q^2 \rangle = \int q^2 (d^2 \sigma^+/dxdy) dxdy \]

\[ = \frac{m^2 E}{\pi \sigma^+} \int_0^1 dx \int_0^1 dy 2mxyE \left\{ (1-y) F_2(x, q^2 = 2mxyE) \right\} \]

\[ + \frac{q^2}{2} x F_1(x, q^2 = 2mxyE) \right\} + \frac{y(2-y)}{2} x F_3(x, q^2 = 2mxyF_3), \]

(23)

from which we extract the \(x\)-integral for \(F_2(x, q^2)\),

\[ J_2 = \int_0^1 dx x F_2(x, q^2 = 2mxyE). \]

(24)

According to Eq. (7), \(J_2\) can be written as

\[ J_2 = \int_0^1 dx \int_0^1 dx' F_1(x', q_0^2) \left\{ \frac{1}{2\pi i} \int_{\epsilon-i\alpha}^{\epsilon+i\alpha} x^n \left( \frac{2mxyE}{q_0^2} \right)^{-y} dy \right\}, \]

(25)

where we can easily perform the \(x\)-integration,
\[ J_z = \int_0^1 dx' F_2(x', q_s^a) \left\{ \frac{1}{2\pi i} \int_{e^{-i\pi}}^{e^{i\pi}} x'^n \left( \frac{2mE_n}{q_s^a} \right)^{-1/n} \frac{-1}{n + \frac{1}{2} - 1} dn \right\}. \] (26)

Considering the positivity of \( \gamma_{2n} \), we see that the integrand under the \( n \)-integration has a simple pole between \( n = 0 \) and \( n = 1 \). If we denote it \( n_0 \), we get

\[ J_z = \left( \frac{2mE_n}{q_s^a} \right)^{n_0 - 1} \int_0^1 x'^{n_0} F_2(x', q_s^a) dx'. \] (27)

From Eqs. (27) and (23) we expect the large \( E_n \)-behaviour of \( \langle q^2 \rangle \) such that

\[ \langle q^2 \rangle \sim E_n^{n_0}. \] (28)

Here the following inequality holds for the value of \( n_0 \).

\[ 0 < n_0 < 1. \]

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**References**

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