Positive Pion Production in Proton-Nucleus Collision

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Positive pion production process on $^{12}$C by 185 MeV proton is studied within the framework of DWBA by adopting the Kisslinger-type optical potential for the final state interaction. The qualitative features of this reaction are explained if the off-shell part of the optical potential is modified by introducing the vertex function. This modification scarcely affects elastic pion-nucleus scattering.

§ 1. Introduction

Pion production process $(p, n^+)$ on nuclei is one of the interesting phenomena in pion-nucleus interactions.$^{11-15}$ As a neutron is transferred into the target nucleus, this reaction is analogous to the ordinary nuclear reactions, such as $(d, p)$ and $(n, \gamma)$. Since the measurements of the cross sections of $^{12}$C$(p, n^+)^{13}$C by the Uppsala group by the use of the 185 MeV proton beams,$^{71-75}$ this reaction has been paid much attention theoretically.$^{71-78}$ Most of the theoretical work was based upon the DWBA theory by adopting the Kisslinger-type optical potential for describing the emitted pion,$^{24,25}$ and the conventional static pion-nucleon interaction for the vertex of pion production. The calculated values of the cross section were shown to be larger than the experimental values by one or two orders of magnitude.$^{77,78}$ In this connection, Miller found the parameter set of the Kisslinger-type optical potential which explains elastic scattering and the $(p, n^+)$ reaction on $^{12}$C.$^{79}$ His parameters are, however, quite different from those commonly used in elastic scattering on $^{12}$C.$^{26,27}$ Recently Miller and Phatak$^{28}$ have calculated the $^{12}$C$(p, n^+)^{13}$C reaction cross section, using the separable pion-nucleon potential proposed by Landau et al.$^{29,31}$ Although the results are in agreement with the experimental data, the range parameters adopted for the nuclear form factor are somewhat too large. Besides, the vertex of pion emission was also investigated by Lee and Pittel,$^{22}$ and by Noble,$^{23}$

From the kinematical consideration the above complicated situations can be ascribed to the off-shell behavior of pion-nucleon interaction, to which elastic pion-nucleus scattering is rather insensitive: If we switch off the initial and final state interactions in the above reaction, the momentum of the transferred neutron is $460 \sim 660 \text{ MeV}/c$, which is far above the Fermi momentum. Therefore, the cross section for this reaction is expected to be very small. However, if we take into
account the final state interaction between the emitted pion and the residual nucleus, it will be possible to emit the pion with high momentum at the pion-nucleon vertex and then we expect a large cross section for this reaction owing to the possible transfer of a neutron with low momentum to the nucleus. We can confirm the above observations from the calculated values of the cross section, for example, by Keating and Wills.\(^{19}\) However, as mentioned above, their theoretical values of the cross section are too large compared with the experimental data. Therefore, it is probable that this failure of the Kisslinger-type optical potential for \((p, \pi^+)\) reaction is due to its wrong off-shell behavior.

The purpose of the present paper is to investigate the effects of pion-nucleon off-shell interaction on the \((p, \pi^+)\) reaction. It is shown that the conventional off-shell extrapolation of the \(p\)-wave pion-nucleon interaction in the Kisslinger-type optical potential is not valid. Introduction of the phenomenological vertex function (cutoff function) can improve the off-shell behavior of the Kisslinger-type optical potential without affecting elastic pion-nucleus scattering, and appreciably reduce the absolute values of \((p, \pi^+)\) reaction cross section in conformity with the experimental data.

In § 2, the formulas for the differential cross section of the \((p, \pi^+)\) reaction are derived. In § 3, we shall describe the proton and pion optical potentials adopted in our calculations, and we propose a modification of the off-shell extrapolation of the Kisslinger-type optical potential. The results of the numerical calculations are given in § 4 and are discussed in § 5. Throughout the paper we adopt the natural units \(\hbar = c = 1\).

§ 2. Distorted wave Born approximation to \((p, \pi^+)\) reaction

Suppose that the incident proton with momentum \(p\) is captured by the target nucleus with spin \(I\), and the positive pion with momentum \(q\) is emitted leaving the final nucleus in the state with spin \(I'\) (see Fig. 1). The relevant transition matrix element \(T\) is written under the distorted wave Born approximation as

\[
T = i\sqrt{2} f \int \frac{d^3q'}{(2\pi)^3} \frac{1}{\mu} \phi_{q'}^{(-)}(q') F_p(q') dq'
\]

(2.1)

where we assume the \(ps-p\bar{n}\) coupling for pion-nucleon interaction \((f^2/4\pi = 0.081)\). The function \(\phi_{q'}^{(-)}(q')\) is a pion distorted wave in the momentum space, \(v(q')\) is a vertex function of pion-nucleon interaction and \(\mu\) is a pion mass. \(F_p(q')\) is

![Fig. 1. Kinematics of the \((p, \pi^+)\) reaction.](https://academic.oup.com/ptp/article-abstract/58/2/575/1866743)
a nuclear form factor which can be given by

$$F_p(q') = \sum_{l'f'p'm} S[l'f',l,f] \left( \langle l'f'm|l,f \rangle \right) \int \psi_{l'f'M}(p') \sigma \cdot \left( q' - \frac{p'}{M} \right) \psi_p(p') \, dp' \, . \quad (2.2)$$

Here, $\psi_p(p')$ is a proton distorted wave with spin orientation $\lambda$ in the momentum space. $\psi_{l'f'M}$ and $\psi_{l'f'M}$ are the wave functions of the target and the residual nuclei respectively. ($I_e$ and $I'$ denote the z-components of the angular momenta.) The operators $\sigma^{(0)}$ and $\tau^{(0)}$ are those of spin and isospin of the $s$-th nucleon, $\omega_{l'}$ ($= \sqrt{q'^2 + m^2}$) is a pion energy and $M$ is a proton mass.

The nuclear form factor (2.2) consists of two types of diagrams as shown in Fig. 2. Diagram 2(a) shows a pion emission from the incoming proton and diagram 2(b) a pion emission from the target nucleus. The contribution of the latter diagram can be neglected for the heavy target nucleus compared with the former contribution, since the recoil energy of the target is very small. In what follows, we shall retain diagram 2(a) by neglecting the recoil effects of the target nucleus. Then we obtain a simple expression for Eq. (2.2) by adopting the nuclear shell model:

$$F_p(q') = \sum_{l'f'p'm} S[l'f',l,f] \left( \langle l'f'm|l,f \rangle \right) \int \psi_{l'f'M}(p') \sigma \cdot \left( q' - \frac{p'}{M} \right) \psi_p(p') \, dp' \, , \quad (2.3)$$

where $\psi_{l'f'M}$ is a single particle wave function of neutron with total (orbital) angular momentum $j_B(l_B)$ and its z-component $m_B$. The factor $S$ is defined as

$$S[l'f',l,f] = \sqrt{A+1} \phi_{l'f'M} \left( \Phi_l \bigotimes \psi_p \right) \left( \gamma_{l'f'M} \right) \quad (2.4)$$

By expanding the wave functions of pion and nucleons in the polar coordinate as

$$\psi_{l'f'M}(p') = \sum_{l'f'p'm} Y^*_{l'f'M} q' \psi_{l'f'M}(q'), \quad (2.5a)$$

$$\psi_p(p') = \sum_{l'f'p'm} Y^*_{l'f'M} q' \psi_{l'f'M}(p'), \quad (2.5b)$$

Fig. 2. Schematic diagrams of the ($p, \pi^+$) reaction. Diagram 2(a) shows a pion emission from the incident proton, and diagram 2(b) a pion emission from the target nucleus.

* We adopt the abbreviation $[U_{l',f'}T_{l'l}]_{m'} = \sum_{m} \langle l'm'|LM \rangle U_{l'm}T_{m'}$. The spin wave function is denoted as $\zeta_{l'f'}$. 

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and

$$\psi_{n'j'B}^{nB}(p' - q') = [Y_{1B}(p' - q') \otimes \xi_{1/2}^{nB}](j'p') u_{n'j'B}(p' - q'),$$  \hspace{1cm} (2.5c)

we rewrite the matrix element (2.1) as

$$T = i2\sqrt{2} \left( \frac{f}{\mu} \right) \sum_{nB,J} S[(lB)_{jB}, I; I'] \sum_{jB,J} \sqrt{(2L+1)(2jB+1)}/(2jB+1)$$

$$\times W \left( l_{jB}, jB \frac{1}{2} ; Lj_{jB} \right) \left( LM \frac{1}{2} \right) (2LmB) (II_{jB}mB | I'_{jB}) [Y_{1B}(q) \otimes Y_{1B}(p)]_{jB}^{n'j'B}$$

$$\times \int p'^2 d\rho' q'^2 d\rho' v(q') [u_{n'j'B}(|p' - q'|) \phi_{ljB}(-q')] [u_{n'j'B}^(+) \rho')$$

$$\times \left( [Y_{1B}(p' - q') \otimes \xi_{1/2}^{nB}]_{jB} \right) \left[ q' - \frac{\omega q'}{M} \rho' \right] [Y_{1B}(q') \otimes [Y_{1B}(p')]$$

$$\otimes \xi_{1/2}^{nB}]_{jB}^{j'B} \right).$$  \hspace{1cm} (2.6)

Then the differential cross section is given as follows:

$$\frac{d\sigma}{d\Omega_q} = \frac{E_p \omega q}{p} \frac{1}{2(2I + 1)} \sum_{j'B,jB} |T|^2$$

$$= \frac{1}{(2\pi)^2} \left( \frac{f}{\mu} \right) E_p \omega q \frac{2I + 1}{2(2I + 1)} \sum_k P_k(\hat{p}q) \cdot \left[ \sum_{L_{jB,j'}j} \sum_{L_{jB,j'}} \sum_{jB,j'} (-1)^{L+k} \right. $$

$$\times (2L+1) \left( l_{jB}l_{jB} | 0 \right) \left( l_{jB}l_{jB} | 0 \right) W \left( l_{jB}, l_{jB}; kL \right)$$

$$\times \sum_{jB,j'} \frac{1}{2(jB+1)} \left| S[(lB)_{jB}, I; I'] \right|^2 \left[ U_{j}^{jB} U_{j'}^{jB} \right]$$  \hspace{1cm} (2.7)

with

$$U_{j}^{jB} = \sqrt{(2L+1)(2jB+1)} \left( l_{jB}, jB \frac{1}{2} ; Lj_{jB} \right)$$

$$\times \sum_{jB,j'} \int p'^2 d\rho' q'^2 d\rho' v(q') [u_{n'j'B}(|p' - q'|) \phi_{ljB}(-q')] [u_{n'j'B}^(+) \rho')$$

$$\times \left( [Y_{1B}(p' - q') \otimes \xi_{1/2}^{nB}]_{jB} \right) \left[ q' - \frac{\omega q'}{M} \rho' \right] [Y_{1B}(q') \otimes [Y_{1B}(p')]$$

$$\otimes \xi_{1/2}^{nB}]_{jB}^{j'B} \right).$$  \hspace{1cm} (2.8)

§ 3. Distorted waves and optical potentials

3.1. Distorted wave of pion

For describing the pion distorted waves, we adopt the conventional optical potential of the Kisslinger-type, which provides a simple parametrization of the optical potential and has successfully described elastic pion-nucleus scattering. The
pion distorted waves $\phi_q(q')$ obey the Klein-Gordon equation with Coulomb and optical potentials, $V_c(q', q'')$, $V_q(q', q'')$, as

$$ (q^2 - q'^2) \phi_q(q') = 2\omega_q \int [V_c(q', q'') + V_q(q', q'')] \phi_q(q'') dq'' \quad (3.1) $$

with

$$ V_c(q', q'') = \frac{\epsilon^2}{4\pi} \tilde{\rho}_c(q' - q'') / (q' - q'')^2 \quad (3.2) $$

and

$$ V_q(q', q'') = -b_c(q) q^2 \tilde{\rho}(q' - q'') + b_i(q) v(q') q' \cdot q'' v(q'') \tilde{\rho}(q' - q'') \quad (3.3) $$

where $\tilde{\rho}_c(q' - q'')$ is the nuclear charge form factor and $\tilde{\rho}(q' - q'')$ is the Fourier transform of the nuclear density $\rho(r)$, which is assumed to be

$$ \rho(r) = \frac{24}{(2 + 3\omega \sqrt{\pi b})} [1 + \omega (r/b)^2] \exp[-(r/b)^2]. \quad (3.4) $$

(The harmonic oscillator model corresponds to $\omega = 4/3$.) The coefficients $b_c$ and $b_i$ in the optical potential are left as free parameters which can be determined by the elastic scattering data.

Here it should be noticed that the vertex function $v(q')$, which has rarely been mentioned, is introduced in the optical potential (3.3). This type of the vertex function plays an important role in a description of the off-shell components of pion distorted wave, while it has negligible effects on elastic scattering (as will be shown in Fig. 3). Since the elastic scattering data on $^{12}$C are not available, we adopt the coefficients $b_c$ and $b_i$ determined by Marshall et al. from elastic scattering of pions on $^{12}$C at kinetic energy 30.2 MeV, which is very close to the energy of the emitted pion in $(p, \pi^+)$ reactions.

By adopting the parameters as $b_c = -4.41 + 0.14i \text{ fm}^3$, $b_i = 5.26 + 0.18i \text{ fm}^3$, the differential cross section of elastic scattering of $\pi^+ - ^{12}$C. The curves $a$, $b$ and $c$ correspond to the cutoff parameters $\Lambda = \infty$, 1.0 and 0.7 GeV, respectively. The harmonic oscillator model is adopted for the nuclear density in the Kisslinger-type potential. The experimental data are taken from Refs. 27) and 28).
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\[ b = 1.64 \text{ fm and } \omega = 4/3, \text{ and by assuming the vertex function as}^9 \]

\[ v(q') = \exp\left[ -\frac{(q'^2 - q^2)}{4}\right], \quad (3.5) \]

we obtain the distorted waves of pion by solving the Klein-Gordon equation (3.1).

In Fig. 3, we show the calculated cross sections of elastic scattering on \(^{12}\text{C}\) for the cutoff parameters \(A = 0.7, 1.0\) and \(\infty\) GeV, respectively.

3.2. Distorted wave of proton

We shall obtain the proton distorted waves by solving the non-relativistic Schrödinger equation with the phenomenological optical potential given in the coordinate space by

\[ V(r) = U_1 f_1(r) + i U_2 f_2(r) + \frac{1}{\mu r^2} \left[ U_3 \frac{d}{dr} f_3(r) + i U_4 \frac{d}{dr} f_4(r) \right] \mathbf{L} \cdot \mathbf{\sigma} \quad (3.6) \]

with

\[ f_i(r) = \frac{1}{[1 + \exp((r - R_i)/a_i)]}, \quad (3.7) \]

where \(U_i, a_i\) and \(R_i\) are the depth-, diffuseness- and range-parameters. We use the numerical values for these parameters which were determined by Johansson et al. from the chi-square fitting to the angular distribution of elastic scattering of 180 MeV proton on \(^{12}\text{C}\) and also to the polarization data of 155 and 173 MeV protons. They are summarized in Table I. The effects of the Coulomb potential are neglected because of high-energy scattering. In the numerical calculations of \((p, \pi^+)\) reaction cross sections, we use the proton distorted waves described by the optical potential (3.6).

### Table I. Parameters of the proton optical potential.

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<td>0.5</td>
<td>0.5</td>
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See Eqs. (3.6) and (3.7) in the text.

§4. Differential cross sections of \(^{12}\text{C}(p, \pi^+)^{13}\text{C}\)

By using the formulas in §2, we calculate the differential cross sections for the transitions to the \(1/2^+\) (gr.), \(1/2^+\) (3.09 MeV) and \(5/2^+\) (6.87 MeV) states. For spinless target nuclei, the angular momentum of single particle \(j_B(I_B)\), which appears in the summation of Eq. (2.7), is fixed as \(j_B = I\), and the spectroscopic factor \([S^2(I_B; j_B, 0; j_B)]^2\) can be factorized out. At present, we take this factor

\[^9\text{ There is no a priori reason to assume the same form of the vertex function in Eq. (3.5) as in Eq. (2.1).}\]
to be unity in the computation, and later the effects of this factor will be discussed separately. The relevant matrix elements $U_{LP}^{3p}$ with pion angular momenta up to $L_z=7$ are kept in the numerical calculations. The distorted waves of the proton and pion are taken from the previous section.

In order to see the effects of the vertex function in the pion optical potential, the theoretical values of the $(p,\pi^+)$ differential cross section for the cutoff parameters $A=0.7, 1.0$ and $\infty$ GeV are shown in Fig. 4. Here we assume the harmonic oscillator model for the neutron wave function and adopt the oscillator parameter $b=1.64$ fm.

Next, in order to see how much the reaction cross sections are affected by the choice of the neutron wave function we adopt the model of the Woods-Saxon potential. Its range and diffuseness are taken as 2.75 and 0.65 fm, respectively. The spin-orbit force is fixed to be 6 MeV, and the depth of the central potential is adjusted so as to reproduce the experimental single particle energy.\(^\text{39}\) The cutoff parameter $A$ is fixed to be 1 GeV. The calculated values of the $(p,\pi^+)$ cross section are shown in Fig. 5. For comparison, the results for the pion optical potential with the nuclear density parameters, $\omega=1$ and $b=1.72$ fm in Eq. (3·4), are also shown in the same figure.

![Fig 4](https://academic.oup.com/ptp/article-abstract/58/2/575/1866743)

**Fig 4.** Cutoff parameter dependences of $^{12}$C$(p,\pi^+)^{14}$C reaction cross sections. Figures 4(a), 4(b) and 4(c) are the cross sections for the transitions to the ground, first $1/2^+$ and second $5/2^+$ excited states of $^{12}$C, respectively. The curves $a$, $b$ and $c$ in each figure correspond to the cutoff parameters $A=\infty$, 1.0 and 0.7 GeV, respectively. The harmonic oscillator model is assumed for the single particle wave function of the transferred neutron and also the nuclear density in the Kisslinger-type optical potential. The experimental data are taken from Ref. 10).
Fig. 5. Effects of neutron wave function on $^{12}\text{C}(p, \pi^+)^{13}\text{C}$ reaction cross sections. Figures 5(a) and 5(b) are the cross sections for the transitions to the ground, and the first $1/2^+$ excited states of $^{13}\text{C}$, respectively. The cutoff parameter is $\Lambda=1.0$ GeV. The Woods-Saxon potential is adopted for the single particle wave function of the transferred neutron. The curve a corresponds to the nuclear density parameters, $w=4/3$ and $b=1.64$ fm, and the curve b, $w=1$ and $b=1.72$ fm. See Eq. (3.4) and the captions of Fig. 4.

§ 5. Discussion and conclusion

As is shown in Fig. 4, the $(p, \pi^+)$ reaction cross section is reduced appreciably by introducing the vertex function $\nu(q')$, while the elastic scattering cross section is varied little by the vertex function, as shown in Fig. 3. This result suggests that the original Kisslinger-type optical potential predicts the correct on-shell behavior of the pion wave function and can be applied to elastic scattering, while the off-shell behavior of the $p$-wave part in the optical potential will not correctly describe the high-momentum component of the pion wave function. In order to see the effect of the vertex function, we show the real part of the pion wave function $\Re \phi_i^p(q')$ with $s$- and $p$-waves at kinetic energy 30.2 MeV in Fig. 6. It is clearly seen that, by reducing the off-shell interaction, the high-momentum component of the pion wave function is suppressed, while the on-shell part ($q=q'$) still remains almost unchanged. As a consequence, the experimental data on $(p, \pi^+)$ cross sections for the transitions to ground and first $1/2^+$ excited states are reproduced by choosing the cutoff parameter $\Lambda$ as a nucleon mass which is commonly accepted.\cite{46,47} It is noticed that the separable pion-nucleon potential of Landau and Tabakin provides similar properties of cutting off the high-momentum component of pion,\cite{29,30,31} which leads to the $(p, \pi^+)$ cross sections in agreement with the experimental data.\cite{30,22,23}

So far we have assumed the spectroscopic factor $|S[(L_B)j_B, 0;j_B]|^2$ to be unity.
The spectroscopic factors for the ground and $1/2^+$ states are slightly less than unity, as shown by Cohen and Kurath. Therefore they are irrelevant to the above discussion. On the other hand, the spectroscopic factor for the second $5/2^+$ excited state cannot be neglected since it is expected to be very small; the dominant configurations of this level are the states of $1d_{3/2}$ and $2s_{1/2}$ particles coupled to the collective $2^+$ (4.43 MeV) state of $^{12}$C. If we assume the spectroscopic factor to be about $10^{-3}$, which was obtained by Meder and Purcell, the calculated values of the reaction cross section are too small to explain the experimental data. In this case, it will be important to take into account the higher-order processes, like the two-step reaction in which the proton excite the ground state of $^{12}$C to the $2^+$ state and then a neutron is transferred into the state of $2s_{1/2}$ and $1d_{5/2}$ orbits coupled to the $2^+$ state of $^{12}$C. It should be noticed that except such accidentally unfavoured transitions, the contributions from the higher-order processes will not be important. In fact the experimental data show that the $(p, \pi^+)$ reaction occurs by magnitude one order stronger than the $(p, \pi^-)$ reaction which is only possible through the higher-order processes.

Next we shall examine to what extent the reaction cross section is dependent upon the neutron and the pion wave functions. The curve $a$ in Fig. 5 shows that if we adopt the Woods-Saxon potential for the neutron wave function, the calculated cross section for the ground state is reduced appreciably at large angle. On the other hand, in the case of the $1/2^+$ excited state, the backward cross section is appreciably enhanced and also the second dip appears at around $115^\circ$. Although the higher-order processes may become non-negligible at large angle, the qualitative features of the angular distributions are explained by the present DWBA calculations. Comparison of the curves $a$, $b$ in Fig. 5 shows that the reaction cross section depends little upon detail of the nuclear form factor $\phi(q' - q)$. Here, it should be mentioned that the angular distributions are affected by the choice of the neutron single particle wave function, especially at large angle, but the absolute values are almost unaltered.

As a conclusion, in the framework of DWBA, the qualitative features of the
$^{12}$C($p, \pi^-$)$^{14}$C reaction cross section are explained if the off-shell part of the optical potential of Kisslinger type is modified by introducing the vertex function. This modification scarcely affects the elastic scattering cross section. In order to proceed more detailed arguments of the ($p, \pi^-$) reaction, it will be necessary to take into account the effects of the higher-order processes and also the nuclear state-dependences of the pion-nucleus optical potential. Such investigations are under way.

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