Progress of Theoretical Physics, Vol. 58, No. 6, December 1977

Cosmological Constraints on the Mass and the Number of Heavy Lepton Neutrinos

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(Received May 23, 1977)

If the neutrinos associated with the heavy leptons have the masses, they may decay into the lower mass neutrinos. We discuss implications of the decays of those massive neutrinos in the standard big bang model of the universe and derive the constraints on their masses and the number from the cosmological observations, i.e., 1) the lower limit of the age of the universe and the upper limit of deceleration parameter, 2) the upper limit of the observed cosmic background radiation and 3) the upper limit of the primordial abundance of ^{4}He. The following results are then obtained: 1) No neutrinos should exist in the mass range 70 eV$<m_\nu<$23 MeV. 2) If the muon and electron neutrinos are also massive, number of neutrinos lighter than 70 eV should be less than four. 3) A limit to the number of the neutrinos heavier than 23 MeV but lighter than 50 MeV is obtained as a function of the mass of neutrinos.

§ 1. Introduction

Anomalous μe events in the e^+e^- annihilation process[1] strongly suggest the existence of the heavy lepton. They are most naturally explained through the scheme of the sequential heavy leptons which implies the existence of a pair of the heavy leptons and the associated neutrinos. Many questions then arise: How many leptons are there? Are all the associated neutrinos massless? If they are massive, what masses do they have, and does Cabibbo-like mixing exist in the weak currents of leptons? Especially the last question has attracted some attention inspired by the recent experimental results on the $\mu\to e\gamma$ process.[2]

The purposes of the present paper are first to give the cosmological constraints on the masses of the heavy lepton neutrinos and second to obtain the limit to the number of massive neutrinos. Recently Steigman et al.[3] discussed the cosmological limit to the number of neutrinos and gave the constraint on the number of leptons. However they assumed that all neutrinos are massless. In this paper we present more general arguments. When neutrinos are massive and Cabibbo-like mixing takes place among them, some of the neutrinos become unstable through decays for example, $\nu\to\nu'\gamma$. To make the discussion definite, throughout this paper, we take the Weinberg-Salam model and assume that all left (right)-handed components are transformed as doublets (singlets) under the weak $SU(2)$ transformation. We make no assumption on the number of leptons and permit the most general mixing. Precise definition of the model and the estimate of lifetimes of neutrinos are given in § 2. We discuss the role of the massive neutrinos and their decays in the hot big bang model of the universe and give the constraints on the mass and the limit to the number in § 3. In § 4, the other constraints which come from the astrophysical phenomena are discussed with some remarks.

§ 2. The lifetimes of the massive neutrinos

As mentioned in the previous section, we consider the weak interactions through the left-handed doublets. More explicitly, they are given as follows: All left-handed components form the following doublets,

$$\begin{pmatrix} \sum_i U_{e\nu_i}\nu_i \\ e \end{pmatrix}_L , \begin{pmatrix} \sum_i U_{\mu\nu_i}\nu_i \\ \mu \end{pmatrix}_L , \cdots \begin{pmatrix} \sum_i U_{l_j\nu_i}\nu_i \\ l_j \end{pmatrix}_L \cdots \tag{2·1}$$

and all right-handed components are singlets. Here ν_i and l_j denote any kind of neutrinos and (negatively) charged leptons, respectively. $U_{l_j\nu_i}$ is the unitary matrix which represents a generalized Cabibbo-like mixing;

$$\sum_i U_{l_j\nu_i} U^*_{l_{j'}\nu_i} = \delta_{jj'} \tag{2·2}$$

and

$$\sum_j U^*_{l_j\nu_i} U_{l_j\nu_{i'}} = \delta_{ii'} , \tag{2·2'}$$

where the summations in i and j are to be taken over all possible neutrinos and charged leptons, respectively. For convenience of the notation, we shall sometimes omit the subscripts l and ν in the suffices. The relevant weak interaction Lagrangian is given by

$$L_{\text{int}} = \frac{g}{2\sqrt{2}} \sum_{ij} \bar{l}_j \gamma_\mu (1-\gamma_5) U_{ji} \nu_i \cdot W_\mu + \text{h.c.} , \tag{2·3}$$

where we have quoted only the part of the interaction with the charged weak boson W_μ because the neutral currents are completely diagonal in this scheme and they play no roles in the following discussion. The coupling strength g is related to the usual Fermi constant G_F as follows:

$$G_F = g^2/8M_W^2 . \tag{2·4}$$

Strictly speaking, this is not correct because in the present scheme, the coupling strength for $\mu \to e\nu\nu$ decay is not exactly $g^2/8M_W^2$ due to the mixing. However, this difference is not so large.[4] Thus the relation (2·4) is sufficient for the following discussion: For the conceptual convenience, we name those two neutrinos which mainly couple to the electron and the muon as electron type and muon type neutrinos, ν_e and ν_μ, respectively, and the other neutrinos as the heavy lepton neutrinos, ν_h.

Now we estimate the lifetime of a neutrino ν_i. First, suppose that its mass is

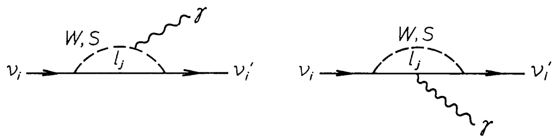

Fig. 1. The dashed line denotes the weak boson (W) and the Higgs field corresponding to the Goldstone mode (S).

smaller than the twice of the electron mass, i.e., $m_{\nu_i}<2m_e$. Then, (provided it is not the lowest one) ν_i can decay into the photon and the other lighter neutrino ν_i' through the graph of Fig. 1:

$$\nu_i \to \nu_i' \gamma\ .$$

The width for the above decay mode is calculated[5] as

$$\Gamma_{\nu_i'\gamma}=\frac{G_F{}^2 m_{\nu_i}^5}{128\pi^3}\frac{\alpha}{\pi}\frac{9}{16}\left(\sum_j U_{ji}U_{ji'}^*\frac{m_{l_j}^2}{M_W{}^2}\right)^2, \tag{2·5}$$

where α is the fine structure constant and it is assumed that the charged leptons are much lighter than the weak boson, i.e., $m_{l_j}\ll M_W$. We have neglected the mass of $\nu_{i'}$ since we are interested in the upper bound of the width. The total width is given by summing up possible ν_i' states. Here we have a useful relation due to the unitarity of U_{ji};

$$\sum_{i'<i}\left(\sum_j U_{ji}U_{ji'}^* m_j{}^2\right)^2 \le \sum_j U_{ij}U_{ij}^* m_j{}^4-\left(\sum_j U_{ij}U_{ij}^* m_j{}^2\right)^2$$
$$\le \frac{1}{4}\operatorname*{Max}_{j,j'}\{(m_j{}^2-m_{j'}^2)^2\}. \tag{2·6}$$

From (2·5) and (2·6), we have

$$\Gamma_i<\frac{G_F{}^2 m_{\nu_i}^5}{128\pi^3}\frac{\alpha}{\pi}\frac{9}{64}\frac{1}{M_W{}^4}\operatorname*{Max}_j[m_{l_j}^4]/M_W{}^4 \tag{2·7}$$

This bound strongly depends on the mass of the heaviest charged lepton. Omitting the factor $\mathrm{Max}_j[m_{l_j}^4]/M_W{}^4$, however, we have a rather loose bound of the lifetime,

$$\tau_j>6.0\times 10^7\ (m_{\nu_i}/1\ \mathrm{MeV})^{-5}\ \mathrm{sec}\,. \tag{2·8}$$

We will also consider a more strict bound by making an assumption on m_{l_j} in connection with the case $m_{\nu_i}>2m_e$.

Next we discuss the lifetime of a neutrino whose mass is greater than the twice of the electron mass but smaller than the muon mass, i.e., $2m_e\le m_{\nu_i}\ll m_\mu$. The most dominant decay mode will be the process of Fig. 2;

$$\nu_i\to\nu_e e^+e^-\ .$$

The width for this process is given by

$$\Gamma=\frac{G_F{}^2 m_{\nu_i}^5}{192\pi^3}|U_{e\nu_i}U_{e\nu_e}^*|^2. \tag{2·9}$$

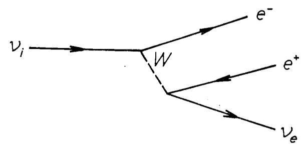

Fig. 2.

In the present mass range of the neutrino, however, the mixing parameter $U_{e\nu_i}$ is strictly limited: The experimental ratio of the decay widths of $\pi\to e\nu$ and $\pi\to\mu\nu$ is consistent with the μ-e universality. On the other hand, the decay process $\pi\to e\nu_i$, if it exists, is extraordinarily enhanced, since the amplitude of the process $\pi\to l\nu$ is proportional to the mass of the produced lepton or neutrino. Taking into account the experimental uncertainty of the μ-e universality, we estimate that the contributions of $\pi\to e\nu_i$ to the total width of $\pi\to e\nu$ is less than 5% and this limit can be described as

$$\sum_{i\neq e}|U_{e\nu_i}|^2(m_{\nu_i}/m_e)^2<0.05\,. \tag{2·10}$$

This imposes the limit $|U_{e\nu_i}|^2<0.05(m_e/m_{\nu_i})^2$ for every heavy lepton neutrino and we obtain the lower bound for the lifetime,

$$\tau>2.2\times10^6\,(m_{\nu_i}/1\,\text{MeV})^{-3}\,\text{sec}\,. \tag{2·11}$$

Since the limitation of (2·10) is so stringent, we must examine the other decay processes carefully. The possible competing ones may be $\nu_i\to\nu_i'\gamma$ and $\nu_i\to\nu_i'e^+e^-$ with the virtual photon exchange (Fig. 3). The partial width for the former one is given by (2·7). Taking into account the fact that this formula is obtained by expanding with respect to $m_{l_j}^2/M_W^2$, we may assume here $m_{l_j}\leq10$ GeV. Then using $M_W\sim60$ GeV, we have

$$\Gamma_\gamma^{-1}>7.5\times10^{10}(m_{\nu_i}/1\,\text{MeV})^{-5}\,\text{sec}\,. \tag{2·12}$$

For the latter process, after some calculation, the partial width is estimated as

$$\Gamma_{e^+e^-}=\frac{G_F^2m_{\nu_i}^5}{192\pi^3}\,\frac{1}{2}\,\frac{\alpha^2}{(3\pi)^2}\,|\sum_{j\neq e}U_{ji}U_{ji'}^*\ln(m_{l_j}^2/M_W^2)|^2, \tag{2·13}$$

where we have used the approximation of $U_{e\nu_i}=0$ taking into account (2·10). Similarly to (2·6), the upper bound will be obtained when only muon and the heaviest charged lepton have non-vanishing contribution to the summation on r.h.s. However, the coupling between the muon and such massive neutrino as ν_i cannot be so strong and estimated to be $|U_{\mu\nu_i}|^2<0.1$. Then we have

$$\Gamma_{e^+e^-}<\frac{G_F^2m_{\nu_i}^5}{192\pi^3}\,\frac{1}{2}\,\frac{\alpha^2}{(3\pi)^2}\,0.1\left(2\ln\frac{10\,\text{GeV}}{m_\mu}\right)^2 \tag{2·14}$$

and

$$\Gamma_{e^+e^-}^{-1}>5.3\times10^9\,(m_{\nu_i}/1\,\text{MeV})^{-5}\,. \tag{2·15}$$

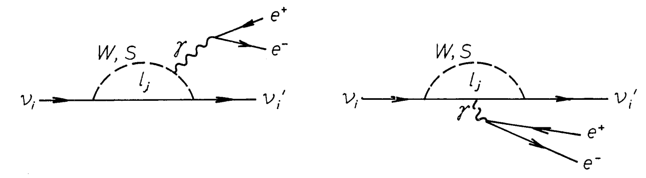

Fig. 3. The same as in Fig. 1.

Comparing (2·11) (2·12) and (2·15), we can conclude that in the range of $2m_e < m_{\nu_i} < 50$ MeV, the lower bound of the lifetime is controlled by (2·11), provided that the charged heavy leptons are lighter than 10 GeV. The last assumption seems to be very plausible one, so that in the following sections, we will discuss mainly this case, which will be referred to as Case A. (In this case, we use the formula (2·12), instead of (2·8), for the range $m_{\nu_i} < 2\,m_e$ in accordance with the assumption on m_{l_j}.) However if there exist charged heavy leptons whose masses are much heavier than 10 GeV and to which relevant neutrinos couple in appreciable strength, the decay $\nu_i \to \nu_i'\gamma$ may be the dominant decay mode even in the mass range $2\,m_e < m_{\nu_i} \ll m_\mu$. So we also discuss this possibility as Case B, in which (2·8) is used for all mass range.

§ 3. Cosmological constraints on the masses and the number of neutrinos

In the very early stage of the universe in the hot big bang model every type of neutrinos is in thermal and chemical equilibrium with matter.[6] However, in the course of the expansion, interactions between the neutrinos and the matter are switched off when the time scale of the annihilation and creation of the neutrinos τ_w becomes equal to the expansion time scale of the universe $\tau_{\rm exp}$. The number ratio of the massive neutrinos ν_i to the photons γ ar this "ν-freezing time" can be estimated roughly from the equality $\tau_w = \tau_{\rm exp}$ as

$$(n_{\nu_i}/n_\gamma)_F \approx 3/4 \cdot {\rm Min} \cdot \{1,\ (10\ {\rm MeV}/m_{\nu_i})^3\}. \qquad (3\cdot1)$$

If there are neither creation nor annihilation processes of both neutrinos and photons in the following stages of the evolution of the universe, this ratio must be conserved. However, as photons are created by e^+e^- annihilation before the nucleosynthesis era, this ratio is modified to

$$n_{\nu_i}(t)/n_\gamma(t) \sim 3/11 \cdot {\rm Min}\{1,\ (10\ {\rm MeV}/m_{\nu_i})^3\}, \qquad (3\cdot2)$$

if the lifetime τ of the neutrino is longer than the cosmic time t, i.e., $\tau > t$.

In the following subsections (1) (2) and (3), we discuss only the constraints on the masses of neutrinos and, to avoid complications, we discuss the case with only one massive neutrino. It is of cource easy to extend this model to a case of many massive neutrinos, provided that the mass spectrum of the neutrinos is known. In the subsection (4), we derive the number limit to the neutrinos independent of the neutrino mass spectrum.

(1) *Hubble constant, deceleration parameter and the age of the universe*

If the lifetime of the massive neutrino is longer than or comparable to the age of the universe t_0, the dynamics of the expansion of the present universe may be governed by the mass density of the neutrinos,[7] instead of the nucleon mass density in the case of standard model. Therefore, mass density of the neutrino should be less than the dynamical energy density of the present universe ρ, which is derived from the observations of the Hubble constant H_0 and the deceleration parameter q_0, i.e.,

$$2\times\frac{3}{11}\,m_\nu n_{\gamma 0}\exp(-t_0/\tau)\cdot\mathrm{Min}\{1,\ (10\ \mathrm{MeV}/m_\nu)^3\}<\rho\ , \qquad (3\cdot3)$$

where

$$\rho=\Omega\rho_c\ , \qquad (3\cdot4)$$

$$\Omega=2q_0 \qquad (3\cdot5)$$

and

$$\rho_c=3H_0^2/8\pi G\ . \qquad (3\cdot6)$$

The photon number density of the present background radiation $n_{\gamma 0}$ is $400\mathrm{cm}^{-3}$ (temperature=2.7K). The upper limit of the density of the present universe is $2.7\times10^{-29}\mathrm{g/cm}^3$ ($H_0=60$ km/s/Mpc, $q_0=2$) which comes from the limit of direct observation of q_0 and the lower limit of the age $t_0>8\times10^9$ y (see Fig. 1 of the paper of Gott et al.[9]). Then we can obtain the following constraint:

$$m_\nu\exp(-t_0/\tau)\,\mathrm{Min}\{1,\ (10\ \mathrm{MeV}/m_\nu)^3\}<70\ \mathrm{eV}\ . \qquad (3\cdot7)$$

This constraint is shown in Fig. 4 and we can conclude that there should be no neutrinos in the mass range

$$70\ \mathrm{eV}<m_\nu<100\ \mathrm{keV}\quad \text{for Case A}\ .$$

$$(70\ \mathrm{eV}<m_\nu<10\ \mathrm{keV}\quad \text{for Case B})$$

(2) *neutrino decay and cosmic background radiations*

If the lifetimes of the neutrinos are shorter than the cosmic age, the photons produced in the decays will be superposed on the cosmic black body radiation and may be observed as the cosmic background radiation. Previously, Sato and Sato[9] discussed the decay of hypothetical small-mass Higgs meson in the hot big bang universe and gave the constraint on its mass from the upper limit of the cosmic background radiation.*) Here we derive the constraint on the mass of massive neutrinos using the same procedure. As is well known, in the evolution of the

*) Recently, Weinberg[11] and Linde[12] gave the more strict constraint on the Higgs meson mass, i.e., $m_\phi>4.9$ GeV.

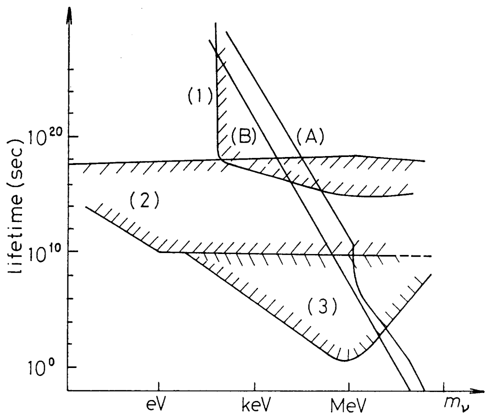

Fig. 4. The forbidden regions (shaded) in the mass-lifetime diagram. (1) Constraint from the age of universe $t_0 > 8\times 10^9$y and the lower limit of the deceleration parameter $q_0 < 2$. (2) Constraint from the background radiation. (3) Constraint from the ^{4}He abundance, where it is assumed that the neutrinos ν_e and ν_μ are also massive ($g=2$). The boundary line between regions (2) and (3) is not so precise. However, it is not necessary to obtain it precisely, because both the sides of this boundary line are anyway forbidden region. The lower limits for the lifetimes of the neutrinos obtained from the theory of weak interaction are shown by the lines (A) and (B) for Case A and Case B, respectively (see §2).

hot big band universe, the energy exchange between thermal photons and electrons via Compton scattering and bremsstrahlung is frozen out at the cosmic time $t_{\rm CB} \approx 10^8 \sim 10^{13}$ sec.[6] If the neutrinos decay sufficiently earlier than this time, emitted photons from the decay $\nu \to \nu' \gamma$ are thermalized and the Planck distribution of the black body radiation is never distorted. But, if they decay around the time, $t_{\rm CB}$, the observed black body radiation may be strongly distorted[10] from the Planck distribution. Moreover if they decay in the later stage when the photons are already free from the energy exchange, the emitted photons may be observed directly as a cosmic background radiation, suffering the red shift due to the cosmic expansion. By taking the Einstein-de Sitter model for simplicity, the energy spectrum of the emitted photons at present can be calculated as

$$Fd\varepsilon = \frac{9}{44\pi} n_{\gamma 0}(t_0/\tau)(\varepsilon/\varepsilon_\nu)^{1.5} \exp\{-(t_0/\tau)(\varepsilon/\varepsilon_\nu)^{1.5}\} d\varepsilon$$

$$\text{for } \varepsilon_\nu > \varepsilon > (t_{\rm CB}/t_0)^{2/3}\varepsilon_\nu , \qquad (3\cdot 8)$$

where

$$\varepsilon_\nu = 0.5\, m_\nu . \qquad (3\cdot 9)$$

The above energy spectrum should be smaller than the upper limit of observed background radiations.[13]

If the decay $\nu \to \nu' e^+ e^-$ is the most dominant process instead of the decay $\nu \to \nu' \gamma$ in the mass range $m_\nu > 1$ MeV, no photons are emitted from the decays. However, even if it is the case, the present discussion holds essentially, because the pairs of the electrons emitted from the decays will be annihilated into two photons and/or will collide with the photons of the cosmic black body radiation (inverse Compton scattering). As the result, the Plank distribution may be distorted and/or the high energy photons created by these processes may be observed as the cosmic X- and γ-ray background radiations.

The residual pairs of the electrons may be observed as cosmic ray electrons and the constraint on the mass and lifetime of the neutrinos can be obtained from the observational upper limit[14] of the cosmic ray electrons, but we do not discuss this problem here.

In Fig. 4, the forbidden region on the mass and lifetime diagram obtained from the upper limit of observed background radiation is shown. The precise computations of the spectrum of background radiations and of the cosmic ray electrons, and their comparisons with the observations will be shown elsewhere. From Fig. 4, we can conclude that the mass region 70 eV$<m_\nu<$1 MeV (70 eV$<m_\nu<$300 keV for Case B) should be ruled out with cooperation of the previous constraint on energy density of the universe.

(3) *^{4}He abundance and nucleosynthesis in the early stage of the universe*

If the massive neutrinos decay and disappear before the nucleosynthesis, no effect can be expected on the synthesized abundance of the elements. On the contrary if they decay after the nucleosynthesis has begun, abundances of elements produced by primordial nucleosynthesis are significantly modified by the two mechanisms. As is well known, produced abundances are essentially controlled by the following two parameters;[15] the one is the parameter h which represents the ratio of nucleons to the photons and the other is the parameter ξ which represents the ratio of the expansion speed to the universe to that of the standard big band model. They are defined as

$$h\equiv\rho_B/T_9^3 \tag{3·10}$$

and

$$\xi\equiv(\partial R/\partial t)/(\partial R/\partial t)_{\text{standard}}=\sqrt{\rho/\rho_{\text{standard}}}\,, \tag{3·11}$$

where ρ_B is the baryon mass density in units of g/cm^3, T_9 radiation temperature in units of 10^9K and R curvature radius of the universe. The energy density ρ is given by

$$\rho=\rho_{\text{standard}}+(g-1)\frac{7}{4}aT_\nu^4+(1.5kT_\nu+m_{\nu_h})n_{\nu_h} \tag{3·12}$$

and

$$\rho_{\text{standard}}=\begin{cases} aT_\gamma^4+\dfrac{7}{4}aT_\nu^4 & \text{for } T_\gamma<m_e\,, \\ \left(1+\dfrac{7}{2}\right)aT_\gamma^4 & \text{for } T_\gamma>m_e\,, \end{cases} \tag{3·13}$$

where g is the statistical weight of electron and muon type neutrinos and if they are massive, $g=2$ (we mainly discuss this case). The ratio of the neutrino temperature T_ν and that of photons T_γ at the era of nucleosynthesis[15] is

$$T_\nu/T_\gamma=(4/11)^{1/3}\,. \tag{3·14}$$

Under the standard model of the big bang universe, the value of the parameter h at the era of the nucleosynthesis h_N is just the same as that of the present observed value h_0, because h is the invariant quantity if no heating source is available for photons. However under the presence of unstable neutrinos, photons produced in the decays strongly heat up the radiation if they decay before the time of the universe $t_{\rm CB}$. As the result, the value of the parameter h decreases. The value of h_N can be calculated easily with the aid of the law of thermodynamics $Tds=dQ$ as

$$h_N=h_0\left[1.0+\frac{3}{4}\left\{\frac{2\cdot f\cdot(1.5kT_\nu+m_{\nu_h})n_{\nu_h}}{aT_\gamma^4}\right\}_D\right], \qquad (3\cdot15)$$

where

$$f=\begin{cases}1/2 & \text{for } \nu_h\to\nu\gamma\,, \\ 2/3 & \text{for } \nu_h\to e^+e^-\nu\,.\end{cases}$$

The subscript D represents the values at the cosmic time $t=\tau$, when neutrinos decay. With the aid of the relation

$$aT_\gamma^4/n_\gamma(T_\gamma)\approx 2.7\,T_\gamma \qquad (3\cdot16)$$

and substituting (3·2) into (3·15), we obtain the following equation:

$$h_N=h_0[1+0.10(m_{\nu_h}/T_{\gamma D})\times{\rm Min}\{1,\ (10\,{\rm MeV}/m_{\nu_h})^3\}. \qquad (3\cdot17)$$

The reasonable lower limit to the present value h_0 is estimated as

$$h_0\geq 1.2\times10^{-5}({\rm g/cm^3})\,(10^9{\rm K})^{-3}\,, \qquad (3\cdot18)$$

where we adopted the constraint on the density parameter $\Omega>0.05$ and the upper limit of the age of the universe $t_0<18\times10^9$y given by Gott et al.[9)]

With the aid of the same procedure, the expansion parameter ξ can be evaluated easily. Substituting (3·1), (3·2), (3·12) (3·14) and (3·16) into (3·11), we obtain the square of ξ

$$\xi^2=1+\begin{pmatrix}0.39\\0.31\end{pmatrix}g+\begin{pmatrix}0.12\\0.14\end{pmatrix}(m_{\nu_h}/T_\gamma)\cdot{\rm Min}\{1,\ (10\,{\rm MeV}/m_{\nu_h})^3\}\quad\text{for}\quad\begin{pmatrix}T_\gamma>m_e\\T_\gamma<m_e\end{pmatrix}. \qquad (3\cdot19)$$

The effective value ξ_N for the nucleosynthesis can be estimated by substituting $T_\gamma=T_{np}$, where T_{np} is the freezing temperature of the reactions $p+e\leftrightarrow n+\nu$, which is given roughly,

$$T_{np}=\xi^{1/3}\,{\rm MeV}\,. \qquad (3\cdot20)$$

Note that the existence of the massive neutrinos always make the values of h_N and ξ_N increase. Moreover we can easily understand that the increase of these values make the synthesized abundance of ^{4}He increase:[15),16)] If the value of h_N becomes larger, the nucleon number density becomes larger and nuclear reactions

Table I. The masses of heavy lepton neutrinos which give the abundance of ^{4}He $Y=0.29$. The values of the parameters h_N and ξ_N are also shown.

g	Interaction model	$h_N[10^{-5}(\mathrm{g/cm^3})(10^9\mathrm{K})^{-3}]$	ξ_N	Mass (MeV)
1	Case A	25.	1.2	17
	Case B	12.	1.2	15
2	Case A	6.4	1.3	23
	Case B	3.7	1.3	19

proceed more rapidly, and ^{4}He is more abundantly synthesized. The speed-up of the expansion of big bang universe at the first step makes the number ratio of neutron to proton n/p increase, because the more rapid expansion of the universe makes the freezing time of the weak interaction shift to the earlier time with the higher n/p ratio. As the result of the increase of the neutrons, ^{4}He is more abundantly synthesized. Wagoner[15] calculated the abundance of ^{4}He numerically and gave the "empirical" formula of the mass fraction Y of ^{4}He as an analytic function of these two parameters ξ_N and h_N;

$$Y=0.333+0.0195\log h_N+0.380\log\xi_N \tag{3·21}$$

for

$$10^{-5}<h_N<10^{-3} \text{ and } 0.5<\xi_N<2\,.$$

Now we may calculate the ^{4}He abundance as a function of the mass of the heavy lepton neutrinos substituting (3·17) and (3·19) into (3·21). If the produced abundance of ^{4}He exceeds the upper limit of the present cosmic abundance, 0.29[3),17)] such neutrino masses should be ruled out, i.e.,

$$Y<0.29\,. \tag{3·22}$$

In Fig. 4, the forbidden region on the mass-lifetime diagram from the ^{4}He abundance is shown. The neutrino masses which give the ^{4}He abundance $Y=0.29$ are also shown in Table I for the various cases. If the neutrino masses are smaller than these values, the abundance exceeds the observed limit. With the cooperation of the constraints of the previous subsections (1) and (2), we can conclude that there should be no neutrinos in the following mass range,

$$\text{Forbidden Range } 70\,\mathrm{eV}<m_\nu<23\,\mathrm{MeV} \text{ for Case A} \tag{3·23}$$

$$(70\,\mathrm{eV}<m_\nu<19\,\mathrm{MeV} \text{ for Case B}). \tag{3·24}$$

(4) *Constraints on the number of massive neutrinos*

If there are many types of neutrinos, their masses should be smaller than 70 eV or greater than 23 MeV (for Case A) as shown in the previous subsections. Here we assume that there exist N_1 types of neutrinos in the mass range $m_{\nu_i}<70$

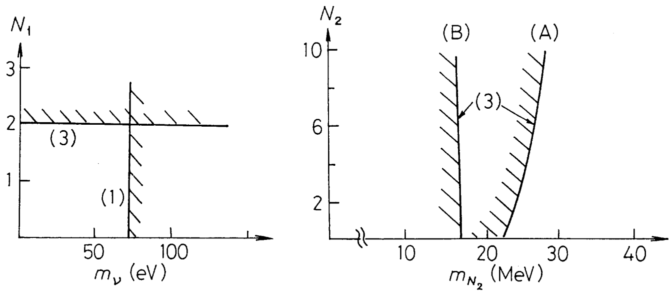

Fig. 5. The constraints on the masses and the number of neutrinos. The ordinates N_1 and N_2 are the numbers of heavy lepton neutrinos (except for ν_e and ν_μ) which have the masses $m_\nu < 70$ eV and 23 MeV$< m_\nu < m_{N_2}$, respectively. It is assumed that the neutrinos ν_e and ν_μ are also massive ($g=2$).

eV except ν_e and ν_μ and N_2 types in the mass range 23 MeV$< m_{\nu_i} < m_{N_2}$, respectively.

First we discuss the number of neutrinos N_1. In this case the expressions of the parameters h_N and ξ_N, (3·18) and (3·19), can be rewritten as $h_N = h_0$ and $\xi_N^2 \leq 1 + 0.39(N_1 + g - 1)$. Obviously this is just the identical case discussed by Steigman et al.[3] except for the statistical weight of the heavy lepton neutrinos, because they assumed that all neutrinos are massless particles. From the bound of $Y < 0.29$ we can easily obtain the approximate limit to N_1 with the aid of (3·21)

$$N_1 < 3.5 - 1.0\,g \leq 4 - g\,. \tag{3·25}$$

Number of heavy lepton neutrinos smaller than 70 eV should be less than or equal to two, $N_1 \leq 2$, if neutrinos ν_e and ν_μ are also massive. On the contrary if neutrinos ν_e and ν_μ are massless, this limit is three, $N_1 \leq 3$.

Next we discuss the constraint on N_2. First we more precisely discuss the lower limit of the lifetime of the neutrino whose lifetime is the longest among the neutrinos with masses 23 MeV$< m_{\nu i} < m_{N_2}$. Let ν_m be the neutrino of the longest lifetime, then the decay width Γ_m is smaller than the mean values of all the N_2 neutrinos, i.e., $\Gamma_m \lesssim (\sum_i^{N_2} \Gamma_i)/N_2$. Substituting (2·9) into this relation, we obtain the following relation,

$$\Gamma_m \leq \frac{G_F^2}{192\pi^3} |U_{e\nu_e}|^2 \frac{1}{N_2} \sum^{N_2} m_{\nu_i}^5 |U_{e\nu_i}|^2 \leq \frac{G_F^2 m_{N_2}^3 m_e^2}{192\pi^3} |U_{e\nu_e}|^2 \frac{1}{N_2} \sum^{N_2} |U_{e\nu_i}|^2 (m_{\nu_i}/m_e)^2. \tag{3·26}$$

With the aid of the constraint (2·10), we can get the lower limit of the lifetime of ν_m which is more stringent than (2·11),

$$\tau_m \geq 2.2 \times 10^6\, N_2 (m_{N_2}/1\,\text{MeV})^{-3}\,\text{sec}. \tag{3·27}$$

Then the number limit of N_2 is obtained from the condition (3·22) as a function of the maximum mass m_{N_2} roughly,

$$N_2 \lesssim (m_{N_2}/23\,\mathrm{MeV})^7\,. \tag{3·28}$$

However, this limit may be applied only for $m_{N_2} < 50\,\mathrm{MeV}$ because neutrinos may decay mainly through other modes than $\nu_i \to \nu_e e^+ e^-$ as discussed in § 2, if the neutrino masses are greater than 50 MeV.

As for the process $\nu_i \to \nu_i' \gamma$ we have no simple constraint between the number and the width as the above one just mentioned. For example, we must take into account the possibility of the cascade decay such as

$$\nu_i \to \nu_i' \gamma$$
$$\hookrightarrow \nu_i'' \gamma$$
$$\hookrightarrow \nu_i''' \gamma$$

which depends on the mass spectrum and turn the problem into a very complicated one. So we do not discuss the constraint on the number of the neutrinos for $m_{\nu_i} \gtrsim 50\,\mathrm{MeV}$ and, for the same reason, for Case B.

§ 4. Remarks and conclusions

(1) *Are there more strict constraints on masses and the number of neutrinos?*

In the present paper, we have derived these constraints from the cosmogony. Can more strict constraints be obtained from other phenomena of the universe? Previously Hayashi et al,[18] showed that the upper limit to the Fermi coupling constant can be obtained from the effects of the neutrino energy loss on the stellar evolution. Later Stothers[19] discussed this method in details and gave the upper limit G_F (universal) $< 10 \times G_F$ (semi leptonic process). This result indicates that the energy loss rate including the heavy lepton neutrinos should be smaller than 100 times the standard ratio of energy loss due to neutrinos in the usual CVC theory, i.e.,

$$R \equiv C_V^2 + (N+1)(C_V - 1)^2 < 100 \tag{4·1}$$

and

$$C_V = 1 + 0.5(4\sin^2\theta_W - 1)\,. \tag{4·2}$$

The ratio R is estimated from the calculation[20] of the neutrino energy loss rate in the Weinberg model and N is the number of the neutrinos whose masses are smaller than the characteristic temperature of the stellar cores ~keV. From this constraint we obtain the upper limit of the number N as

$$N < \{(100 - C_V^2)/(C_V - 1)^2\} - 1 \approx 1574\,, \tag{4·3}$$

where we have assumed the Weinberg angle as $\sin^2\theta_W = 3/8$. Obviously this constraint is very loose and nothing can be said about more massive neutrinos (m_{ν_i}

$\gtrsim$keV). However, these massive neutrinos may be emitted from the supernova cores whose temperature will roughly be 15 MeV. If these neutrinos decay sufficiently fast and the decay products such as the γ-ray can be detected, we can obtain the constraints on their masses and number. But we only know the lower limit of the lifetime, but not the upper limit: If their lifetimes are extremely longer than the lower limit, i.e., the mixing parameters are very small, no neutrinos can decay within the age of the universe. Thus nothing can be said from such discussion. On the contrary, cosmological constraints become more strict and the forbidden mass range increases with the increase of the lifetimes.

(2) *Can supernova explosion be induced by the decay of the massive neutrinos?*

If neutrino-lifetimes are roughly equal to the lower limit derived in § 2, what kind of astrophysical effects can be expected? One of the most desirable effects may be the effect on the supernova explosion. In the current theory of the supernova explosion,[21)~24)] all the imploding stellar cores cannot become the neutron stars, but collapse into the black holes, because it is very difficult to eject the mantle of the stellar cores by neutrino deposition in spite of the discovery of the neutral current interaction.[21),23)] If the massive neutrinos emitted from the hot supernova core of radius $R_c \sim 10^{6.5}$cm decay in the mantle whose radius is $R_m \sim 10^8$cm, the mantle will be expelled and the core may become a stable neutron star. The deposition energy due to the neutrino decays can be roughly estimated as $L \cdot \Delta t(e^{-R_c/v\tau} - e^{-R_m/v\tau})$, where the massive neutrino luminosity is given by $L \approx 2\times 10^{54}(R_c/10^{6.5}\text{cm})^2(T_c/15\text{MeV})^2(m_\nu/15\text{ MeV})e^{-m_\nu/T_c}$ erg/sec, the duration time of the neutrino emission $\Delta t \sim 0.1$ sec, and the diffusion velocity of the neutrinos $v \sim 3\times 10^9(T_c/m_\nu)^{0.5}$cm/sec. If this energy is greater than the gravitational binding energy of the mantle, $GM_cM_m/R_m \sim 3\times 10^{50}(M_c/1M_\odot)(M_m/0.1M_\odot)(R_m/10^8\text{cm})^{-1}$, where M_c and M_m are the masses of the core and the mantle, respectively, we can speculate that the decays of the massive neutrinos will lead to the formation of a neutron star after the supernova explosion. This condition is sufficiently satisfied for the neutrino mass roughly 20 MeV$<m_\nu<$100 MeV for Case A, where the upper limit value is very uncertain because the lifetime of such massive neutrinos is hard to estimate.

(3) *Conclusion*

In the investigation of the present paper, we have obtained the cosmologically forbidden regions of the masses of neutrinos. Within the framework of the sequential heavy leptons, the forbidden range is 70 eV$<m_\nu<$23 MeV, provided that the charged heavy leptons are lighter than 10 GeV (Case A), and without the last assumption the forbidden range is 70 eV$<m_\nu<$19 MeV (Case B). These results seem to be fairly significant, if we take into account, for example, the fact that the mass of the missing neutrino of μ-e events in Ref. 1) is less than 500

MeV. Further we note that to derive the above results, we have allowed "a maximum mixing" in a sense. When we have more information on the mixing of neutrinos, it may be possible to obtain more strict constraints.

In the present paper, we have confined ourselves to a particular model of heavy leptons. It should be noted, however, that the shaded regions in Fig. 4 are forbidden by purely cosmological reasons. Therefore, for any model of the weak interaction of leptons, provided that the products of the main decay mode of neutrinos include the photon and/or the electron, we can make a similar argument by drawing the calculated lifetime in Fig. 4.

Finally we should emphasize that the present discussion is indeed based on the standard hot big bang model, which is the most plausible model of the universe at present. It is very interesting that valuable information on particle physics can be derived from cosmological arguments in spite of large uncertainties inherited there.

Acknowledgements

One of the authors (K.S.) thanks Professor C. Hayashi for continuous encouragement. He also thanks Professor H. Sato, Mr. T. Nakamura and Mr. S. Miyama for valuable discussions. The authors thank Dr. T. Kugo for valuable discussions and Dr. T. Nakano and Dr. C. A. Hoenselaers for reading of the manuscript.

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Note added in proof: Recently Lee and Weinberg[i] discussed the bounds on the mass of heavy lepton neutrino independently of the present work. They assumed, however, that neutrinos are stable. Therefore their discussion corresponds to taking the limit of the infinite lifetime in the subsection (1) of § 3 in the present paper. Dicus et al. [ii] extended their result to the unstable neutrinos and obtained the limit to the lifetime of neutrinos.

In the present paper, we showed only the results in the mass range $m_{\nu_h}<1$ GeV. The cosmologically forbidden region on the mass-lifetime diagram in the wider mass range $m_{\nu_h}<1$ TeV is shown in the talk of I.N.S. symposium.[iii]

Recently Miyama and Sato[iv] computed the big bang nucleosynthesis with unstable neutrinos precisely and improved the present results. They obtained the limit to the lifetime not only from the abundance of ^{4}He but also from the abundances of ^{2}H and ^{7}Li. In particular, they showed that the most stringent limit to the lifetime in the mass ranges $m_{\nu_h}<100$ keV and $m_{\nu_h}>100$ MeV can be obtained from the observed lower limit of ^{2}H abundance. However in the mass range 100 keV$<m_{\nu_h}<$100 MeV, the most stringent limit is obtained from ^{4}He abundance. Therefore the mass limit obtained from ^{4}He abundance in § 3 holds approximately even now.

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