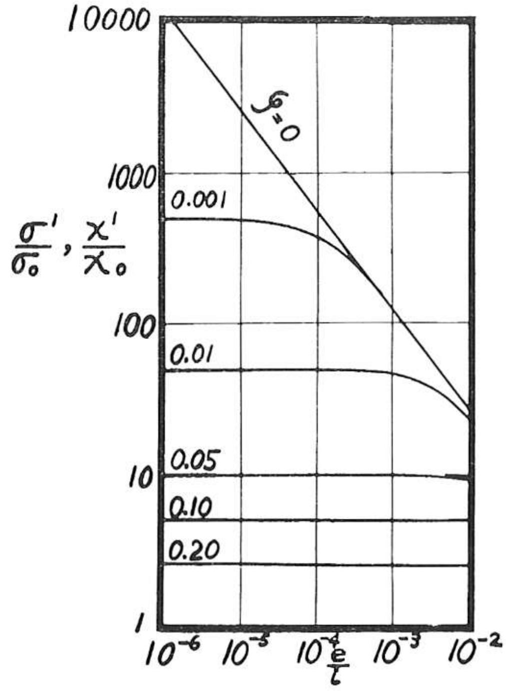

Fig. 5 Maximum stress and maximum deflection, referred to σ_0 and x_0

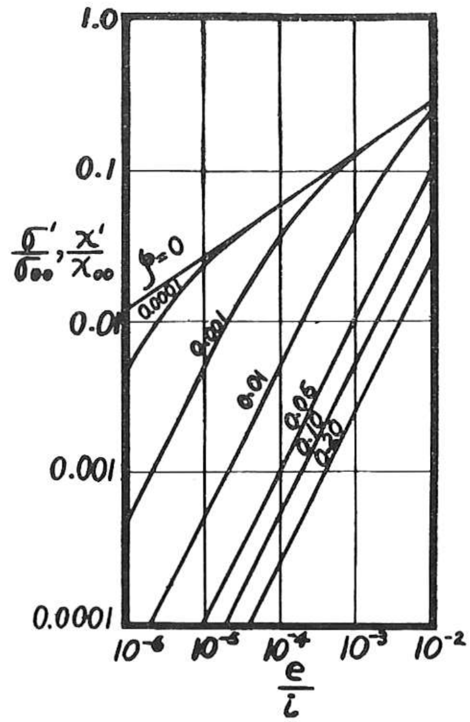

Fig. 6 Maximum stress and maximum deflection, referred to σ_{00} and x_{00}

$$x_{00} = \beta^2 i = \left(\frac{e}{i}\right)^{-1} x_0 \qquad (22a)$$

and a reference stress

$$\sigma_{00} = Kx_{00} \qquad (22b)$$

each of which is independent of e.

From (22*a*), (22*b*), and (17)

$$\frac{\sigma'}{\sigma_{00}} = \frac{x'}{x_{00}} = \mu' \frac{e}{i} \qquad (23)$$

Equation (23), plotted in Fig. 6, for the case of $\beta = \beta'$, illustrates the complete variation of σ' and x' with e/i, hence with e. The importance of careful balancing is clearly seen here, as well as the importance of damping. Care should be used, however, in drawing conclusions from Fig. 6, and one should not be hasty, since σ_{00} and x_{00} are very large quantities. Fig. 6 also illustrates that σ' and x' remain finite, even when $\zeta = 0$. Throughout the design range, both ζ and e/i, i.e., both damping and balance, are important. The benefits of large damping and/or small unbalance are clearly apparent.

V. Acknowledgments

The author is grateful to Messrs. M. L. Land of Worthington Corporation, A. M. G. Moody of the Trane Company, A. H. Raye of the Boeing Company, and R. A. Riester of Carrier Corporation for providing values of e/i which are representative of present-day industrial practice. The cooperation of Keck-Craig Associates in the publication of this paper is gratefully acknowledged.

References

1 F. M. Lewis, "Vibration During Acceleration Through a Critical Speed," Trans. ASME (APM section), vol. 54, 1932, pp. 253–261.

2 D. Macchia, "Acceleration of an Unbalanced Rotor Through the Critical Speed," ASME Paper No. 63—WA-9.

3 J. G. Baker, "Mathematical-Machine Determination of the Vibration of Accelerated Unbalanced Rotor," *Journal of Applied Mechanics*, vol. 6, Trans. ASME, vol. 61, 1939, pp. A-145–A-150.

4 G. D. McCann, Jr., and R. R. Bennett, "Vibration of Multifrequency Systems During Acceleration Through Critical Speeds," *Journal of Applied Mechanics*, vol. 16, Trans. ASME, vol. 71, 1949, pp. 375–382.

5 F. Howitt, "Accelerating a Rotor Through a Critical Speed," *The Engineer*, October 27, 1961, pp. 691–692.

6 J. P. den Hartog, *Mechanical Vibrations*, McGraw-Hill Book Company, Inc., New York, N. Y., third edition, 1948.

7 Private communications from Messrs. M. L. Land, A. M. G. Moody, A. H. Raye, and R. A. Riester.

DISCUSSION

F. M. Lewis[2]

A rotating system which is slowed down by a very small damping force so that its rate of deceleration is infinitesimal will eventually arrive at the point $\beta_{\min}$ on the steady-state resonance curve. In passing through a critical, rotational energy is in part lost to the small rotational damping and is in part converted to vibrational energy. Considering the case of zero vibrational damping, all of the vibrational energy is reconverted to rotational. There is no reason to believe that the maximum vibrational energy occurs at the point β' on the steady-state resonance curve. The maximum vibrational energy could just as well be at any point on the horizontal through $\beta_{\min}$. The manipulation of steady-state formulas in an attempt to solve a transient problem is extremely dubious mathematics. I do not believe that this problem is to be solved by the methods of this paper. I believe that the subject has been treated by a Russian author, but I am unable to cite the reference at this time.

Author's Closure

Professor Lewis has raised a very worthwhile question. As pointed out in the last sentence of section III, the details of the processes occurring during the resonant jump are not evaluated in the present paper; however they would, in themselves, constitute an interesting paper.

[2] Professor, Department of Naval Architecture and Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

The statement, "considering the case of zero vibrational damping, all of the vibrational energy is reconverted to rotational," is surprising to the author—in fact, the statement is incorrect.

Professor Lewis states that "there is no reason to believe that the maximum vibrational energy occurs at the point β' on the steady-state resonance curve." It is necessarily true, however, that in the region $\beta' \leq \beta \leq \beta_{\min}$ the total energy of the system is constant in the zero damping case, or decreasing slightly with increasing time in the finite damping case. Since the rotational energy, by definition, decreases as β decreases, the energy available for vibration necessarily increases in the zero damping case and also does so for the finite damping case provided the damping is small enough. This leads to the maximum amount of vibrational energy in the condition in which the rotational energy is a minimum. This occurs at the smallest value of β, which is β' in the region under consideration here. The two parts of the energy—kinetic and potential—are both dependent on shaft deflection; hence they will increase together. These results are consequences of the energy relations, and must hold regardless of the details of the vibration calculations. Since the shaft stress is related to the shaft deflection through the laws of elasticity without regard to questions of vibration, the shaft stress must also have its maximum value at β'.

In the limiting case of zero vibrational damping, the vibrational pattern established in the resonant jump will persist in the region $\beta \leq \beta'$. In the real case of finite damping—no matter how small—these transients will damp out and the system will pass through a succession of steady states in the region $\beta \leq \beta'$. If the deceleration (i.e., the reduction of β with increase of time) is small enough, the vibration will attenuate to the steady state while β decreases a negligible amount. In the limiting case of this attenuation occurring at β', the deflection and stress will be as indicated in section IV of the paper. In the actual case of β decreasing below β', the deflection and stress will be less than the values given.

Based on the reasoning detailed above, the results presented in the paper are offered as working relations; they are approximations. The author will be interested in seeing the results of an analysis of the details of the resonant jump, if and when this is solved. He would be especially pleased if Professor Lewis could find time to carry out this analysis; although more complicated, it is a logical sequel to his 1932 paper [1]. If such results do become available, a quantitative check on the energy-based approximations offered in the present paper can be made.