evaluate maximum longitudinal growth of a NDE missed flaw under fatigue loads. Axial length determinations provide input to leak-before-break condition considerations.

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DISCUSSION

C. W. Smith[2]

The author has presented a technique for use by the analyst in dealing with a class of problems for which analytical methods, except for pioneering studies of Sommer, et al. [6], are not generally available in the open literature, namely fatigue crack growth in three dimensional (3D) cracked body problems. Moreover, by assuming the material parameters in the Paris crack growth law to be applicable to the three dimensional case, the approach allows predictions of subsequent flaw shapes for non-self-similar flaw growth. Thus, the paper appears to be an important contribution to the fracture mechanics literature and should serve as a first step towards the goal of achieving accurate predictions of stress intensity distributions and corresponding flaw shapes in three dimensional cracked body problems.

In this connection, it should be noted that most solutions to bounded 3D cracked body problems are numerical and may involve assumptions pertaining to flaw shapes and growth characteristics which are not, in fact, in agreement with reality [6–13]. These studies suggest that, in complex 3D problems, a stress induced effect exists which produces a variation in flaw growth resistance [6, 8] or constraint [11, 12, 13] along the flaw border, implying that the two dimensional material parameters in the crack growth law may not be appropriate to the three dimensional problem. Moreover, such factors as stress level and cyclic to mean stress ratio may be significant [9].

Nevertheless the above paper constitutes an important contribution and should point the way towards attacking an important class of problems, and the author is to be congratulated.

Additional References

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Author's Closure

The author agrees with Professor Smith that numerical solutions for 3D cracked geometries with various assumptions can lead to misleading interpretations of the Stress Intensity (K_I) variation along the crack front. If an adequate resolution of stress intensity can be made, then the technique presented in the paper will have a wider applicability. The paper presented results for structures with uniform material properties. The model could be easily extended to non-uniform materials as well, i.e.,

For the Paris crack growth law

$$\frac{bd}{dN} = C_1(\Delta K_b)^{n_1} - \text{growth along } b\text{-axis} \tag{1}$$

and

$$\frac{da}{dN} = C_2(\Delta K_a)^{n_2} - \text{growth along } a\text{-axis} \tag{2}$$

By dividing equation (1) by equation (2)

$$\frac{db}{da} = \frac{C_1}{C_2}\frac{(\Delta K_b)^{n_1}}{(\Delta K_a)^{n_2}} \tag{3}$$

For cases where Paris law is inadequate to represent a structural material response to fatigue, representation similar to equation (3) can be generated. Where fatigue crack growth can be represented by a product of parametric effects, i.e.,

$$\frac{db}{dN} = C_1 f_{11}(\sigma_m) f_{12}(x_1) f_{13}(x_2)\ldots(\Delta K_b)^{n_1} \text{ for } b \text{ axis growth} \tag{4}$$

and

$$\frac{da}{dN} = C_2 f_{21}(\sigma_m) f_{22}(x_1) f_{23}(x_2)\ldots(\Delta K_a)^{n_2} \text{ for } a \text{ axis growth} \tag{5}$$

where

$f_{11}(\sigma_m)$ and $f_{21}(\sigma_m)$ = functions dependent on mean stress

and

$$\left.\begin{matrix} f_{12}(x_1),\ f_{13}(x_2) \\ f_{22}(x_1),\ f_{23}(x_2) \end{matrix}\right\} = \text{other functional parameters}$$

It can be shown that for uniform material properties an expression identical to equation (6) of the paper can be obtained namely,

$$\frac{db}{da} = \left(\frac{\Delta K_b}{\Delta K_a}\right)^n \tag{6}$$

The main problem in complex 3-D structures would be to obtain an adequate functional relationship for $\Delta K_b/\Delta K_a$, especially if the crack front does not maintain a general elliptic shape with crack growth.