Remarks on the Viscosity of Liquid Metals

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The state of knowledge, both experimental and theoretical, of the viscosity of liquid metals is characterized more by the difficulties besetting the theory of liquids rather than by the success of the Ziman theory of the electronic properties. Where electronic processes dominate, such as in electrical and thermal conductivity, the latter theory has been very successful.

On the other hand, for the viscosity the ionic processes dominate. The theory of the viscosity of liquid metals has, for a long time, been characterized by fairly accurate but quasi-empirical formulae such as that of Andrade,\(^5\) and by the simple kinetic theory of Longuet-Higgins and Pope\(^2\) based on a rigid sphere fluid, which deviates from experiment by factors of three or more. The most ambitious calculations have been those of Chapman\(^3\) based on the Kirkwood-Eisenschitz\(^6,5\) theory of liquids, who obtained a universal curve for the viscosity of 21 metals by assuming a law of corresponding states. His formulae require only a knowledge of three parameters: the density as a function of temperature, the interatomic close-packed spacing at 0\(^\circ\)K and the melting temperature. Since his results agree with experiment to within \(\sim\) ten percent, we were motivated to supplement that work by considering the contribution of the electrons to the viscosity. Most of the experimental data on viscosities
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have been reviewed by Wilson and in Refs. 7 and 8).

The electronic contribution to the viscosity of liquid metals is easily calculated and we expect formulæ akin to the Ziman result for other transport coefficients, expressed in terms of form factor and the structure factor. We may follow the method of Rice, as employed in the calculation of the transport coefficients of a paramagnetic Fermi liquid in which nearly free electrons were scattered by local fluctuations. This relies on the variational solution to the Boltzmann equation for the electron distribution function. We shall not go into the details of the calculation, as the procedure is by now well-known. One obtains the following variational formula for the viscosity:

\[
\eta_{el} = \frac{2m\sigma}{15e^2} \int_0^1 a(q) |v(q)|^2 q^3 dq
\]

where \(\sigma\) is the electrical conductivity, \(a(q)\) is the structure factor, \(q = k/2k_F\), \(k\) the momentum transfer and \(v(q)\) is the pseudopotential.

Our modest aim is simply to estimate \(\eta_{el}\) for some liquid metals, by utilizing accurately known data for structure factors and the well-known empty-core pseudo potential theory of Ashcroft.

Fortunately, there are now very accurate and detailed data for several elements. We shall now list the sources utilized in the evaluation of the above formula. Thus for Na and K the X-ray experiment of Greenfield et al. are the most accurate. For Mg, Al and Pb the extensive work of Waseda et al. is the most reliable. For Zn we used the data of Wingfield et al. The functional form of \(v(q)\) is taken from Cohen and Heine. The empirical form factors of Wiser and Kaveh were used for interpolation.

In Table we present \(\eta_{el}\), together with experimental results taken from Wilson, and the ionic viscosity contribution from the theory of liquids, evaluated at the melting temperatures. One may summarize the state of affairs as expressed by Table by saying that the relation \(\eta_{exp} = \eta_{ion} + \eta_{el}\) is quite well obeyed, although this statement implies an excessive reliance on the theory of ionic viscosity expressed in terms of a law of corresponding states as in Ref. 3). In fact one sees that the latter is not reliable for Al and Cs. On the other hand, the values of \(\eta_{el}\) may be accepted as having the accuracy of the current nearly free electron theory of liquid metals.

I thank Professor K. Wiser and Professor A. Greenfield for discussions and for supplying relevant data, respectively.

1) E. N. da C. Andrade, Endeavor 13 (1954), 117.

Table I.

<table>
<thead>
<tr>
<th></th>
<th>Al</th>
<th>Cs</th>
<th>K</th>
<th>Li</th>
<th>Mg</th>
<th>Na</th>
<th>Pb</th>
<th>Rb</th>
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</thead>
<tbody>
<tr>
<td>(\eta_{exp} \times 10^{-3}) poise</td>
<td>13.5</td>
<td>6.3</td>
<td>5.2</td>
<td>5.9</td>
<td>13.1</td>
<td>6.7</td>
<td>22</td>
<td>6.7</td>
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<tr>
<td>(\eta_{el} \times 10^{-3}) poise</td>
<td>2.76</td>
<td>0.36</td>
<td>0.28</td>
<td>0.31</td>
<td>1.09</td>
<td>0.31</td>
<td>0.57</td>
<td>0.38</td>
</tr>
<tr>
<td>(\eta_{ion} \times 10^{-3}) poise</td>
<td>11.91</td>
<td>5.93</td>
<td>4.9</td>
<td>5.59</td>
<td>10.08</td>
<td>6.37</td>
<td>21.2</td>
<td>5.66</td>
</tr>
<tr>
<td>(\frac{\eta_{el} + \eta_{ion}}{\eta_{exp}} \times 100%)</td>
<td>108</td>
<td>79</td>
<td>99</td>
<td>100</td>
<td>85</td>
<td>99</td>
<td>99</td>
<td>90</td>
</tr>
</tbody>
</table>
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6) J. R. Wilson, Metall. Rev. 10 (1965), 381.