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A Dimensionless Solution of Lag Time for Diverging Surface

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Using the kinematic wave theory, a dimensionless expression is obtained, and solved numerically in order to estimate the lag time for diverging surfaces. The findings show that the dimensionless lag time, which is the ratio of lag time for diverging surface to that of plane surface, ranges from 1 to 0.5 depending on the degree of divergence and friction formula used. The results can be used easily and reliably for all ranges of the geometric parameter for urban and rural catchments.

Introduction

The concept of surface water lag time proposed by Overton is used in this study. In his concept, lag time is the lapse between the times of occurrence of 50% rainfall and 50% runoff volume (Overton and Meadows 1976).

The flow equations for a catchment can be derived from the kinematic wave theory, which is applicable to most hydrologically significant cases of overland flow. According to this theory, the lag time is related to the representation of the catchment geometry, catchment length, slope, roughness, and rainfall excess.

Overton (1971) has investigated lag time for a plane surface while Singh and Ağiralioğlu (1982) have considered lag time for a diverging surface. Ağiralioğlu (1985) made a comparison between lag times for plane and converging surfaces.

In this study, a general expression is obtained in order to estimate the lag time for diverging surface. This expression is made dimensionless by using the lag time for a plane surface assuming that some physical characteristics such as catchment length, slope, roughness and rainfall excess are the same for both surfaces. The solution of this expression can be used for all ranges of geometric parameter.

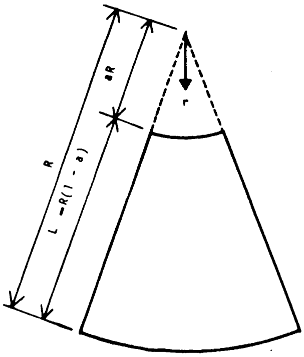

Fig. 1. Geometric representation of diverging surface.

Lag Time for Diverging Surface

The diverging surface is shown in Fig. 1 where R – radius of flow region, a – a parameter relating to the degree of divergence, and L – length of catchment. In many catchments a is vanishingly small, and this is the extreme case of the diverging configuration. As a approaches unity, the diverging geometry transforms to a plane one.

For a diverging surface the basic equations of flow, based on kinematic wave theory, can be written on a unit width basis as (Singh and Ağiralioğlu 1982)

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial r} = i - \frac{Q}{r} \tag{1}$$

$$Q = \alpha h^{\beta} \tag{2}$$

where h – local depth of flow, Q – rate of outflow per unit width, r – space coordinate, t – time coordinate, α – parameter related to slope and roughness, and β – an exponent related to friction formula.

At equilibrium condition the transient term in Eq. (1) will vanish. Subject to the condition, $Q = 0$ at $r = a$R, the solution of the new equation, which is based on Eq. (1), can be obtained. By eliminating Q from that solution and by considering Eq. (2), the depth at equilibrium can be written as

$$h = \left[\frac{i\,(r^2 - a^2R^2)}{2\alpha r}\right]^{1/\beta} \tag{3}$$

The catchment storage S_e is

$$S_e = \frac{1}{R(1-a)} \int_{aR}^{R} h\,dr \tag{4}$$

Since $t_L = S_e/i$, where t_L – lag time, the lag time can be written substituting Eq. (3) into Eq. (4) as

$$t_L = \frac{1}{R(1-a)}\,(i)^{(1-\beta)/\beta} \left(\frac{1}{2\alpha}\right)^{1/\beta} \int_{aR}^{R} \left[\frac{r^2 - a^2R^2}{r}\right]^{1/\beta} dr \tag{5}$$

A Dimensionless Solution

The expression of Eq. (5) can be made dimensionless using the lag time for a rectangular plane surface. In this procedure, it is assumed that diverging and plane representations have the same catchment length, slope, roughness, and rainfall excess. In other words, L, α and i are the same for two representations.

For a rectangular plane surface, the lag time, based on kinematic wave theory, can be written as (Overton and Meadows 1976)

$$t_L = \frac{\beta}{\beta+1}\left(\frac{L\, i^{1-\beta}}{\alpha}\right)^{1/\beta} \tag{6}$$

Using $L = R$ (1-a), the dimensionless solution, the ratio of the lag time for diverging surface to the lag time for plane surface, can be obtained from Eq. (5) and Eq. (6)

$$\tau_L \equiv \left(\frac{\beta+1}{\beta}\right)\left(\frac{1}{L}\right)^{(1+\beta)/\beta}\left(\frac{1}{2}\right)^{1/\beta}\left(\frac{1}{1-a}\right)^{2/\beta}\int_{aL/(1-a)}^{L/(1-a)}\left[\frac{r^2(1-a)^2-a^2L^2}{r}\right]^{1/\beta} dr \tag{7}$$

In order to eliminate L in Eq. (7), a change of a variable is considered as $r = u\, L/(1\text{-}a)$. Then Eq. (7) becomes

$$\tau_L \equiv \left(\frac{\beta+1}{\beta}\right)\left(\frac{1}{2}\right)^{1/\beta}\left(\frac{1}{1-a}\right)^{(\beta+1)/\beta}\int_a^1\left[\frac{u}{u^2-a^2}\right]^{-1/\beta} du \tag{8}$$

In Eq. (8), the integrant becomes infinite as $u = a$, the lower limit of integration. In order to solve Eq. (8) numerically, it can be considered a change of the variable as $u = (y^{\beta/(\beta+1)} + a^2)^{1/2}$. Then Eq. (8) becomes

$$\tau_L \doteq \left[\frac{1}{2(1-a)}\right]^{(\beta+1)/\beta}\int_0^{(1-a^2)^{(\beta+1)/\beta}}\frac{1}{[y^{\beta/(\beta+1)}+a^2]^{(\beta+1)/(2\beta)}}\, dy \tag{9}$$

In Eq. (9) when $a = 1$, there is no solution. If we use $y = (1\text{-}a^2)^{(\beta+1)/\beta} z$ as another change of the variable, Eq. (9) yields

$$\tau_L \equiv \left[\frac{1+a}{2}\right]^{(\beta+1)/\beta}\int_0^1\frac{1}{[z^{\beta/(\beta+1)}(1-a^2)+a^2]^{(\beta+1/(2\beta)}}\, dz \tag{10}$$

Eq. (10) can be solved numerically for different values of β and a. Since parameter β is only related to the choice of friction formula, some formulas widely used in practice are examined herein. For Darcy-Weisbach, Manning, and Chezy formulas, $\beta = 3$, $\beta = 5/3$, and $\beta = 3/2$, respectively.

Values of a vary from 0 to 1 depending on catchment geometry for diverging surface. If a approaches 1, which means that the diverging representation transforms to the rectangular plane, Eq. (10) yields $\tau_L = 1.0$ for all values of β as expected. If $a = 0$, which is the extreme case of diverging representation, $\tau_L =$

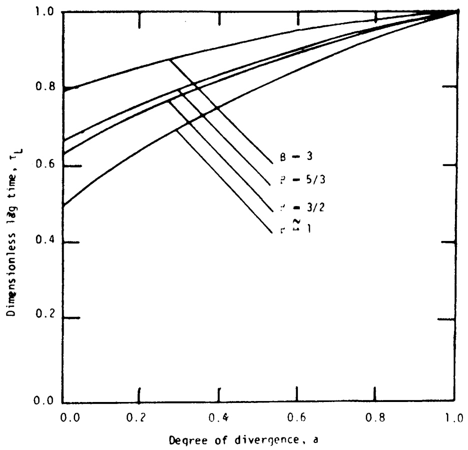

Fig. 2. Dimensionless lag time for diverging surface.

$0.5^{1/\beta}$ as an exact analytical solution.

The dimensionless expression is solved numerically by Simpson's method for different values of *a* and β. The results of the solution are evaluated graphically in Fig. 2 for β = 3, β = 5/3, β = 3/2, and β = 1. As seen from this figure, the diverging representation yields smaller lag time than that of rectangular one. Dimensionless lag time increases while parameter β increases.

Conclusion

Based on kinematic wave theory, a dimensionless solution of the lag time for diverging surface is obtained comparing the lag time of rectangular surface. The findings show that the dimensionless lag time varies from 1.0 to 0.5 depending on the degree of divergence and friction formula used. The results are evaluated graphically for application.

References

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