to the second and first invariants of the stress tensor, respectively, and are defined as

$$\sigma_e = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$P = -{}^1/_3(\sigma_1 + \sigma_2 + \sigma_3)$$

where σ_1, σ_2, and σ_3 are the principal stresses. The materials investigated in reference [9] indicate that $\sigma_e - P$ diagrams often are of the form shown in Fig. 4. If σ_e and P are plotted to the same scale, then the definitions of these parameters can be used to show that a tension test will generate stress states that lie along a path of slope -3. Likewise, torsion stresses are on a vertical path, compression on a path of $+3$, etc. If these tests are conducted in a fluid pressure environment, the starting point of the stress path is shifted from zero to the value of the fluid pressure.

The test data collected in this program have been plotted on $\sigma_e - P$ coordinates, and the results are shown in Fig. 5. The model shown in Fig. 5 could be made more complete by conducting notched tension and biaxial tension, and by performing uniaxial tension in higher pressure environments. The effective strain (ϵ_e) defined as

$$\epsilon_e = \frac{\sqrt{2}}{3} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]^{1/2}$$

where ϵ_1, ϵ_2, and ϵ_3 are the principal strains, can be used to construct the $\epsilon_e - P$ diagram shown in Fig. 6. Since the tensile test data defines essentially the same yield and fracture lines (these lines become surfaces when mapped into the principal stress space) as the plate bending data, the tension data collected in various pressure environments could be used to define the response of the plate specimen.

As better finite element stress analysis programs become available, both the forming operation (in the project, plate bending) and the testing specimen (tension necking and compression barrelling) can be analyzed to provide more accurate stress and strain distributions that can be used to construct $\sigma_e - P$ and $\epsilon_e - P$ diagrams that are useful for predicting the performance of metalforming operations under pressure.

Conclusions

It has been shown that tension and compression tests can be conducted in controlled pressure environments to generate material response data which is used to construct effective stress-pressure and effective strain-pressure diagrams. These diagrams were, in turn, shown to be suitable for predicting the performance of plate bending in various pressure environments. To generalize this procedure, one would conduct sufficient uniaxial and biaxial tests under pressure to allow for the complete construction of the $\sigma_e - P - \epsilon_e$ model. The critical stresses occurring in a particular forming operation (finite element programs would help here) could be located on the $\sigma_e - P - \epsilon_e$ model which would indicate if the material at that particular point in the workpiece was in an elastic or plastic state, or if fracture would occur.

Future work on this project will include the application of a finite element program for evaluating (more accurately) the stress-strain states in necked tension and barrelled compression specimens.

References

1 Bobrowsky, A., "Metal-Working Under High Static Pressures (Controlled Ductility Forming," Proc. of CIRP International Conference on Manufacturing Technology, ASTM, 1967.

2 Bobrowsky, A., "Pressure Research Notes," Pressure Technology Corporation of America.

3 Bobrowsky, A., "Modern Metal Forming—Metal Working Under High Pressures," MAP-PENNTAP Short Course, 1970.

4 Alexander, J. M., and Lengyel, B., "Semi-Continuous Hydrostatic Extrusion of Wires," *Proc. I. Mech. E.* 1965–1966, p. 180.

5 Pugh, H. Ll. D., "Hydrostatic Extrusion," Ninth Commonwealth Mining and Metallurgical Congress, Paper No. 18, 1969.

6 Green, D., and Slater, H. K., "Augmented Hydrostatic Extrusion of Continuous Bar," "High Pressure Engineering," The Institute of Mechanical Engineers, 1968.

7 Schmehl, G., Western Electric Production Processes. Note from: MAP-PENNTAP Short Course. "Modern Metal Forming—Metal Working Under High Pressure," 1970.

8 Bobrowsky, A., "Description of Some Methods for Deformation of Solids Under Hydrostatic Pressure," ASME Paper No. 64-WA/PT-29.

9 Davis, R. L., "A Macroscopic Model of Material Behavior Under Variable Pressures," *International Colloquium on the Physics of Solids Under Pressure*, Grenoble, France, 1969.

10 Bobrowsky, A., "Formability Criteria for Metals," Russian and American, American Society of Tool and Manufacturing Engineers, Paper No. 792.

11 Girijarallabhan, C. V., and Mehta, K. C., "Stress-Strain Relationship From Compression Tests on Nonlinear Materials," "Finite Element Methods in Structural Mechanics," Vanderbilt University, 1969.

12 Davis, R. L., and Jackson, J. W., "A Plastic Stress Analysis of Cylindrical Wafers Under Elastically Deformable Compression Plates," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 89, No. 3.

DISCUSSION

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The authors are to be congratulated on suitably interpreting and correlating several previously found occurrences. The discusser, as well as Fiorentino, Love, and Meyer, had found bend angle and bend ratio, respectively, to be linear functions of environmental pressure at fracture. The authors employ effective strain at fracture as the common basis for all deformations under pressure, and find that it is linear with pressure for their experiments, hence for others' experiments also.

Although such a result is comforting in that it substitutes a generality for specific data, it is also disturbing since it tends to imply that ductility is a linear function of environmental pressure. This linearity may be true for many metals with cubic crystal structures, but it is not true for many metals with hexagonal close-packed structures over an extensive pressure range. Bending tests of beryllium in the 350,000–550,000 psi pressure range, or of zinc in the 10,000–20,000 psi range might provide critical data on the generality of this linearity.

The authors have taken a large step in attempting to assess stress states in barrelled compression specimens. An alternative assistance is to minimize barrelling experimentally, as by spiral grooving, lubricating, and/or fine finishing, and conical indenting, as suggested by Thomsen, Bridgman, Okhrimenko, and Siebel, respectively. One looks forward to a finite-element stress analysis of the barrelled specimen.

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