Nuclear Disintegration caused by Cosmic-Rays

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The diffusion of the nucleon component through the atmosphere is investigated. They are classified into three groups according to their energy, and the various properties of their nuclear interactions are discussed.

§ 1. Introduction

During last several years, various authors investigated the nuclear interactions caused by cosmic-rays, using Wilson chambers, ionization chambers, or G-M counters. Although they brought to light some properties of such nuclear events, the complete understanding has not yet been obtained. The photographic experiments with highly sensitive emulsion have become very powerful on this subject, by which Bristol group has recently carried out detailed investigations. Their results make us possible to clarify the various properties of nuclear interactions more directly in wider range of energies than ever, and thus we have now somewhat ample knowledges on them.

These nuclear events by cosmic rays are the only available data on high energy mesonic phenomena, which we have now at hand. In order to facilitate our understanding on these problems, we must, first of all, build a picture or a model for cosmic-ray nuclear interactions from various experimental data. Here, we shall be concerned with this preliminary analysis, and ask the kinds, the energies and the genetic relations of agents of these events. Its result will be discussed in more detail in a separate paper, comparing with the calculation on meson theories.

Firstly, we describe a survey of the experimental results of Bristol group, including the definition of some standards of energy for later use (§ 2). According to these standards of energy, we can classify the nuclear events and the nucleon component as tentatively named A, B and C (§ 3). The genetic relations between these components are discussed by solving a simplified diffusion problem, although there remains some ambiguity if we wish to get forth into quantitative points (§ 4). The radiationless process of B-component is, however, treated more unambiguously and we can get some informations complementary to the accelerator experiments (§ 5).

§ 2. Preliminaries

In the photographic experiment of Bristol group, they classified the tracks
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of nuclear events into the following three groups according to their grain density; thin tracks of grain density $\lesssim 1.5 g_m$, grey tracks of $1.5 - 5 g_m$, and black tracks of $\gtrsim 5 g_m$, where $g_m$ is the minimum observed value of the grain density. As they pointed out, these three groups can be interpreted as follows. i) The thin group consists of fast mesons and fast protons. Incident protons of the events are also included in it. ii) The main part of the grey group are protons with energies 25–330 MeV, and the rest will be deuterons, slow mesons and others. iii) Most of the evaporated protons, α-particles and other heavier nuclear fragments have rather low energies, and they make the black group.

Using the above classification of tracks, they specified the observed nuclear events by the number $n$ of their thin secondary tracks (shower tracks) and the number $N$ of the grey and black tracks (heavy tracks). Furthermore, the suffix $n$ or $p$ is attached, according as the agent is neutral or charged.

These $n$ and $N$ are closely related to the mechanism of nuclear interactions and their available energies. To see this, we suppose that one cosmic-ray nucleon hits a Ag or Br nucleus in emulsion. It will make successive collisions with several nucleons in the nucleus. If it has a sufficiently large energy, a number of mesons will be produced, and we shall observe them as thin tracks. At the same time, collided nucleons may receive a considerable amount of recoil energy. They will also make several collisions in the nucleus, and finally be stopped, or run away from the nucleus and be detected as thin or grey tracks. The residual nucleus will be in a highly excited state due to these disturbances, and then it will evaporate protons, neutrons, α-particles and other nuclear fragments, which are recorded as black tracks.

If we could make an analogy between the electromagnetic and the mesonic interaction, the meson production might correspond to the bremsstrahlung and the knocked-out protons to the knocked-on electrons. Then a certain energy $E_e$ can be introduced in the same way as the critical energy in the cascade theory, though it may have less meaning than in the latter case.

First, we consider the case of an incident energy larger than the critical energy $E_e$ of nuclear collision. As most of thin tracks can be regarded as mesons, their number $n$ is closely related to the number of collisions with the available energy $\gtrsim E_e$. For example, their relation is linear, if mesons are always produced singly. In the case of genuine multiple production, it becomes more complicated.

When an incident energy is below $E_e$, the elastic nucleon-nucleon scattering will be more predominant than the meson production. In this case, we are mainly concerned with the grey and the black group, since most of knocked-out protons are classified as grey tracks. We shall be able to obtain various informations on nuclear transparency, by examining their relations.

Thus the determination of the critical energy $E_e$ of nuclear collision becomes very important for us to interpret these nuclear events. For this purpose, we shall introduce further two standards for the energy of the nucleon component,
and determine their values comparing with each other.

One standard is the geomagnetic cut-off energy $E_{cut}$ (or the knee) of the primary proton spectrum. At the top of the atmosphere, the energy spectrum of nucleons will have a remarkable discontinuity at $E_{cut}$, although contributions from primary $\alpha$-particles may smear it out in some respects. Therefore, we can expect that the same circumstance will occur in the size-distribution curve of nuclear events observed in the stratosphere.

The other standard is the critical energy of the diffusion through the atmosphere. Protons of the nucleon component suffer ionization loss, whereas neutrons do not. From this ionization and the mean free path for the nuclear interaction, we can define the critical energy $E_j$ of diffusion. Protons with energies $\gtrsim E_j$ make nuclear interactions, on an average, before they are stopped by ionization loss, while those with energies $\lesssim E_j$ are stopped before nuclear interactions. Therefore, we shall find below $E_j$ considerable difference between the energy spectra of protons and neutrons. This will result in the different amount of contributions of neutrons and protons to the nuclear events of smaller size.

Furthermore, the comparison of the absolute intensity of nucleon component and the absolute frequency of nuclear events will give us another information on relations of the size of events and the energy of agents. For this purpose, we need some knowledges on the cross-section, which can be obtained from other experimental data.

§ 3. Classification of nuclear events and of nucleon component

From the considerations in § 2, we classify the nucleon component into the following three groups;

A-component with energies $E \gtrsim E_{cut}$. The ratio of numbers of neutrons and protons (we call it simply as the $n$-$p$ ratio) of this component will be considerably smaller than unity at the top of the atmosphere, and will tend to unity at lower altitudes.

B-component with energies $E_{cut} \gtrsim E \gtrsim E_j$. They are produced in collisions of A-nucleons with air nuclei. Their $n$-$p$ ratio will always be nearly 1, since protons and neutrons will be produced equally and the ionization loss does not play an important role in this case.

C-component with energies $E \lesssim E_j$. They are produced by A and B-component in nuclear collisions. Their $n$-$p$ ratio will be considerably larger than 1 due to the ionization loss.

The geomagnetic cut-off energy $E_{cut}$ will be $\sim 2$ BeV at the experimental station of Bristol group.* The critical energy $E_j$ of diffusion depends on an assumed mean free path for collisions, but we may put $E_j \sim 1$ BeV without large error.

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* Bristol group did not write in their paper the station of their experiment in the stratosphere. Here we suppose it will be near Jungfraujoch, where they performed their first experiment.
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Next, we examine the nuclear events and determine which correspond to the above three groups of nucleons. The following table shows the \( n/p \) ratio of their agents calculated from the experimental data of Bristol group.\(^1\) \(^2\)

<table>
<thead>
<tr>
<th>Atmospheric Depth</th>
<th>( n/p ) Ratio of Agents</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 g/cm(^2)</td>
<td>5.45 ± 0.87</td>
</tr>
<tr>
<td>690 g/cm(^2)</td>
<td>7.10 ± 0.43</td>
</tr>
</tbody>
</table>

We see from this table, that nuclear events of \( n_\geq n \geq 2 \) (we denote it simply as \( n_\geq n \geq 2 \)) are mainly caused by \( A \)-component, \( 1 \) by \( B \), and \( 0 \) by \( C \)-component.

Now, we can find the following interpretation for these nuclear events,

\( 0 \): \((n \text{ or } p) \rightarrow 0, \text{ or } \rightarrow n,\)

\( 1 \): \((n \text{ or } p) \rightarrow p, \text{ or } \rightarrow (p+n), (\pi+n),\)

\( 2 \): \((n \text{ or } p) \rightarrow (p+p), (p+\pi) \text{ or } \rightarrow (\pi+\pi+n), (p+\pi+n),\)

Here \( p \) or \( n \) means a proton or a neutron with energy in thin track region, and \( 0 \) indicates a final state without such nucleons. \( \pi \) is a fast charged \( \pi \)-meson.

It can be shown that the events of the first mode are most frequent, as was pointed out by Bristol group.\(^2\) (See also § 5.) The second mode may take place at most 20–30\%. Strictly speaking, therefore, we must improve the above correspondence in the following ways,

\( A \)-component \( \rightarrow \geq 2_+, \) (the second mode of \( 1_+ \)),

\( B \)-component \( \rightarrow (\text{the first mode of } 1_+ \) \( + (\text{the second mode of } 0_+) \),

\( C \)-component \( \rightarrow (\text{the first mode of } 0_+) \).

This remark is important for the comparison of their absolute frequency (cf. § 4).

Furthermore, we see that the critical energy \( E_c \) of nuclear collision will lie between the energies of agents of \( 1_+ \) and \( 2_+ \). From the above consideration, we can suppose that \( E_c \) is between \( E_{\text{min}} \sim 2 \text{ BeV} \) and \( E_1 \sim 1 \text{ BeV} \). Later, we shall discuss on them in more detail.

§ 4. Diffusion of the nucleon component

To make our discussion more quantitatively, we treat the diffusion of nucleon components through the atmosphere and ascertain our classification and correspondence on them.

In the course of calculation, we assume that the absorption mean free path
the collision mean free path $L_{\text{col}}$ for nuclear interactions, and the probability $k$ for charge exchange in one collision are all constant for each nucleon component. We tentatively take their numerical values as follows,

\[ L_{\text{abs}} \approx 125 \text{ g/cm}^2 \text{ air}, \]
\[ L_{\text{col}} \approx \text{geometrical one, i.e. } 100 \text{ g/cm}^2 \text{ in emulsion or } 65 \text{ g/cm}^2 \text{ in air}, \]
\[ k \approx 1/2. \]

These are approximately verified by various experiments.

**Diffusion of A-component**  
Neglecting the ionization loss, the vertical intensity $A(t)$ of A-component can be expressed simply as,

\[ A(t) = a \exp \left( -\frac{t}{L_{\text{abs}}} \right), \tag{2} \]

where $a$ is the primary intensity at the top of the atmosphere. Its total intensity $\tilde{A}(t)$ is, integrating over all directions,

\[ \tilde{A}(t) = \int A(t/\cos \theta) d\Omega = 2\pi a \left[ \exp \left( -\frac{t}{L_{\text{abs}}} \right) + \left( \frac{t}{L_{\text{abs}}} \right) \text{Ei} \left( -\frac{t}{L_{\text{abs}}} \right) \right]. \tag{3} \]

If we distinguish the proton intensity $\rho(t)$ and the neutron intensity $n(t)$, they are given by,

\[ \rho(t) = \left( \frac{a}{2} \right) \left[ \exp \left( -\frac{t}{L_{\text{abs}}} \right) + \exp \left[ - \left( \frac{t}{L_{\text{abs}}} \right) - \left( 2k/L_{\text{col}} \right) \right] \right], \]
\[ n(t) = \left( \frac{a}{2} \right) \left[ \exp \left( -\frac{t}{L_{\text{abs}}} \right) - \exp \left[ - \left( \frac{t}{L_{\text{abs}}} \right) - \left( 2k/L_{\text{col}} \right) \right] \right], \tag{4} \]

under the initial conditions,

\[ \rho(0) = a, \quad n(0) = 0. \]

Their total intensities $\tilde{\rho}(t)$ and $\tilde{n}(t)$ are also easily obtained.

Next we shall compare them with the experimental data. Table II shows the absolute frequency of nuclear events measured at two altitudes by Bristol group. Following the consideration in § 3, we can estimate from them the absolute frequency of nuclear events which are produced by A, B and C-component (we call them simply A, B and C-stars). We show in Table III their frequency obtained under the assumption that protons and neutrons are equally produced in a nuclear collision, i.e.

\[ A\text{-stars} \approx (\geq 2) + 2 : (n \text{ or } \rho) \rightarrow (\rho + \rho \text{ or } \rho + n), \ (\pi + \rho \text{ or } \pi + n), \]
\[ B\text{-stars} \approx 2 \times (1, -2) : (n \text{ or } \rho) \rightarrow (n \text{ or } \rho), \tag{5} \]
\[ C\text{-stars} \approx 0, - (1, -2) : (n \text{ or } \rho) \rightarrow 0. \]

1) Stratosphere (45 g/cm²). Assuming the numerical values (1) for the parameter, the $n$-$\rho$ ratio for A-component is expected to be,
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Table II.
Absolute frequency (per 1 day, 1 g emulsion) of nuclear events measured by Bristol group.1,2)

<table>
<thead>
<tr>
<th>atmospheric depth</th>
<th>type</th>
<th>0_\text{n}</th>
<th>0_\text{p}</th>
<th>1_\text{s}</th>
<th>\geq 2_\text{s}</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 g/cm^2</td>
<td></td>
<td>167 ± 7.9</td>
<td>30.7 ± 3.4</td>
<td>46.5 ± 4.2</td>
<td>27.0 ± 3.2</td>
</tr>
<tr>
<td>690 g/cm^2</td>
<td></td>
<td>2.05 ± 0.03</td>
<td>0.288±0.013</td>
<td>0.245±0.012</td>
<td>0.123±0.008</td>
</tr>
</tbody>
</table>

Table III.
Estimated absolute frequency of nuclear events produced by A, B and C-component.

<table>
<thead>
<tr>
<th>atmospheric depth</th>
<th>classification</th>
<th>A-star</th>
<th>B-star</th>
<th>C_\text{n}-star*</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 g/cm^2</td>
<td></td>
<td>39.</td>
<td>70.</td>
<td>148.</td>
</tr>
<tr>
<td>690 g/cm^2</td>
<td></td>
<td>0.19</td>
<td>0.35</td>
<td>1.95</td>
</tr>
</tbody>
</table>

* C_\text{n}-star means the nuclear event produced by C-neutrons.
C_\text{p}-stars are discussed later (cf. § 5).

\[ \frac{\overline{n}}{\overline{p}} = 0.52 \] at \( t = 45 \text{ g/cm}^2 \),
which is nearly in agreement with the observed one, \( 0.57 \pm 0.19 \) for \( \geq 2_\text{s} \) (cf. Table I). As for the absolute intensity, we take the data obtained by the rocket experiments,4)

\[ a = 0.12/\text{sec. cm}^2 \text{ sterad.} \] (6)

Then, we have the total intensity \( \overline{A}(\tau) \) at this altitude, as,

\[ \overline{A}(45 \text{ g/cm}^2) = 2.67 \times 10^4/\text{cm}^2 \text{ day}, \] (7)

which cause the nuclear events with the frequency,

\[ A\text{-star at 45 g/cm}^2 = 2.67 \times 10^2/\text{g. day}. \] (8)

This should be compared with the observed frequency of \( A\)-stars, 39/g. day, which is much smaller than the expected one (8) (cf. Table III). We can find some possible explanations to account for this large discrepancy.

a) The assumed values (1) of \( l_{\text{abs}} \) and \( l_{\text{col}} \) may be wrong. But, as will be discussed later concerning the altitude dependence, we can not take a smaller value for \( l_{\text{abs}} \). While, as is seen, the geometrical mean free path (1) is the minimum estimation for \( l_{\text{col}} \), and its maximum estimation will be \( \approx l_{\text{abs}} \), i.e. twice of the geometrical one. If we take \( l_{\text{col}} \approx l_{\text{abs}} \), this modification decreases the expected frequency of \( A\)-stars by a factor \( \approx 2 \) and somewhat reduces the above discrepancy.
But, at the same time, it makes smaller the $n$-$p$ ratio to 0.28, which again contradicts with the observed value. Therefore, we can say that the allowable maximum value for $l_{\text{eff}}$ will be 1.5-times the geometrical one.

b) Recent experiments show that 20～30% of primary particles are $\alpha$-particles or heavier nuclei, which have much larger absorption coefficient than protons. Therefore, we should equate $a$ in (2) to the primary proton intensity, and not to the total intensity of primary rays.

c) As is seen from Table I, the $n$-$p$ ratio of events $I_*$ may be somewhat smaller than 1. If this is really the case, we must attribute some of $I_*$ of the first mode to $A$-stars. From its $n$-$p$ ratio, at most $1/3$ of $I_*$ of the first mode can be regarded as $A$-stars, and thus we have the estimated maximum frequency of $A$-stars as follows (cf. Table III),

$$A\text{-stars} = 39 + 70/3 = 62/\text{g. day}. \quad (9)$$

In this case, the frequency of $B$-stars becomes $2/3$ of the previous value, i.e. $47/\text{g. day}$.

d) Furthermore, we should not overlook the possibility that the considerable amount of back rays may exist at the top of the atmosphere, and may increase the apparent primary intensity. The maximum estimation of the intensity of back-rays can be nearly the same order of magnitude as the primary rays.

e) Bristol group observed the nuclear events with $N^2_\text{e} \geq 3$. Therefore, we must add the frequency of missed events to their experimental value. Extrapolating their $N^2_\text{e}$-spectrum, we can estimate the contribution from these small stars may be 10%.

Taking into account the remarks a) and b), we have the minimum expected frequency of $A$-stars as,

$$(8) \times \frac{1}{1.5} \times 0.7 \approx 125/\text{g. day}, \quad (10)$$

while, the maximum estimation of its observed frequency is, from the remarks c) and e),

$$62 \times 1.1 = 68/\text{g. day}. \quad (11)$$

Furthermore, if we regard this somewhat still larger expected value as due to the back-rays (cf. remark d)), the discrepancy will not be so large as stated before.

2) Mountain altitude (690 g/cm$^2$). We may tentatively take (9) as a standard for the intensity of $A$-component, since we do not know the precise value for the primary intensity. Then we obtain the expected frequency of $A$-stars at this altitude as follows, assuming the absorption mean free path $l_{\text{abs}}$ to be 125 g/cm$^2$,

$$A\text{-star at 690 g/cm}^2 = 0.062/\text{g. day}. \quad (12)$$

It is nearly one third of the observed frequency of $A$-stars, 0.19/g. day (cf. Table III). If we adopt $l_{\text{abs}} \sim 150$ g/cm$^2$ instead of (1), i.e. 125 g/cm$^2$, we can get the-
correct expected value equal to the observed one. This rather large absorption mean free path may be due to the difference in the energy spectra of $A$-component at two altitudes, as is seen from the comparison of $n_x$-spectra.*

The $n-\rho$ ratio of this component is expected to be 1 at this altitude. The experimental value, $0.76 \pm 0.14$, is somewhat smaller than 1. If this slight difference really exists, we must take at most $k \sim 0.3$, which again destroys the agreement obtained for the stratosphere data. Large statistical error prevents us to get a definite conclusion.

We have regarded that in the stratosphere a part of $I$, of the first mode were due to $A$-component, whereas at the mountain altitude $A$-component responded only to $\geq 2\sigma$. It may look rather strange, but we can suppose that the different magnitude of contributions from $B$-component will result in such correspondences (cf. later discussion).

**Diffusion of $B$-component**

If we assume that a direction of motion is conserved in $A-B$ transmutations, we can easily calculate the intensity of $B$-component. Neglecting the ionization loss, we get its vertical intensity $B(t)$ at atmospheric depth $t$ as follows,

$$B(t) = \alpha v(t/l_{\text{rel}}) \exp(-t/l_{\text{abs}}),$$

where $\nu$ is the number of $B$-nucleons produced in one collision. Its total intensity $\bar{B}(t)$ is, integrating over all directions,

$$\bar{B}(t) = 2\pi \alpha v(t/l_{\text{rel}})[ -Ei(-t/l_{\text{abs}})].$$

Comparing it with (3), we get the ratio of both components,

$$\frac{\bar{B}(t)}{\bar{A}(t)} = \nu \langle t \rangle / I_{\text{rel}},$$

$$\langle t \rangle = \frac{-tEi(-t/l_{\text{abs}})}{\exp(-t/l_{\text{abs}}) + (t/l_{\text{abs}})Ei(-t/l_{\text{abs}})} = \left\{ \begin{array}{ll} 85 \text{ g/cm}^2 & \text{for } t = 45 \text{ g/cm}^2, \\
870 \text{ g/cm}^2 & \text{for } t = 690 \text{ g/cm}^2. \end{array} \right.$$  

While, we can determine this ratio from the experimental data. Equating both figures, we determine the unknown parameter $\nu$.

1) Stratosphere. From the remark c), we estimate the frequency of $B$-stars as,

$$B-\text{star} = 47 \text{ g/day}.$$  

Thus their ratio is,

* The absorption mean free path $l_{\text{abs}} \sim 125 \text{ g/cm}^2$ was commonly ascertained by the experiments on hard showers and bursts, agents of which are supposed to be high energy $A$-component. The integral energy spectrum of nucleon-component will be, as is discussed later, nearly $1/E^2$ at lower altitudes, whereas its primary spectrum is less steep than this power law in lower energy region ($\geq 5 \text{ BeV}$). Therefore, we can suppose that the intensity of $A$-component can not be expressed by a simple exponential function as (2), but its lower energy part (2--5 BeV) may show somewhat less steep altitude dependence (cf. discussion on $B$-component).
\[
\frac{B}{A} = (16)/(9) = 47/62 = 0.76.
\]

Therefore, comparing it with (15), we have
\[
\nu \left( \frac{l_{\text{abs}}}{l_{\text{coul}}} \right) = 0.76/0.68 = 1.12.
\]

2) Mountain altitude. In this case, we have from Table III,
\[
B\text{-star} = 0.35/\text{g. day.}
\]

In the same way as above, we can estimate as,
\[
\frac{B}{A} = (19)/\text{the value in Table III} = 0.35/0.19 = 1.84,
\]
and
\[
\nu \left( \frac{l_{\text{abs}}}{l_{\text{coul}}} \right) = 1.84/6.2 = 0.30.
\]

We should not directly compare these two values of \( \nu \), (18) and (21), for the following reasons. First, as we discussed above, the energy spectrum of \( A \)-component may be somewhat different at the two altitudes. Therefore, the comparison of mean values of \( \nu \) above obtained will have little meaning, and it seems necessary to treat more precisely taking into account the energy dependence of the multiplicity \( \nu \). Qualitatively, \( \nu \) in (21) will be a mean value with larger weight for low energy \( A \)-component, since we adopted there the value in Table III as the frequency of \( A \)-stars. If we use (12) for its frequency at the lower altitude, we get the three times (21) for \( \nu \), which will be, on the contrary to the above, a mean value with larger weight for high energy \( A \)-component. These two determination of \( \nu \) at the lower altitude will be its maximum and minimum estimation. We find somewhat still larger \( \nu \) at the higher altitude.

Furthermore, contributions from heavier primaries should not be overlooked and can be estimated as follows. Here we denote the quantities concerning with heavier primaries by asterisk *. From the experiment of Bradt and Peters,5) we may take into account only primary \( \alpha \)-particles, the intensity of which can be written as,
\[
A^* (t) = a^* \exp \left( -t/l_{\text{abs}}^* \right),
\]

where
\[
a^* \approx 0.1a, ^{1}\)
\[
l_{\text{abs}}^* \approx 40 \text{ g/cm}^2.
\]

The intensity \( B^* (t) \) of \( B \)-nucleons produced by heavier primaries is obtained as,
\[
B^* (t) = a^* \nu^* \left[ \frac{l_{\text{abs}}}{l_{\text{abs}}^*} \right] \left[ \exp \left( -t/l_{\text{abs}} \right) - \exp \left( -t/l_{\text{abs}}^* \right) \right]
\]

\(^{1}\) The total intensity of heavier primaries is 20%~30% of that of protons, whereas the geomagnetic cut-off energy per nucleon for the former is a half of that for the latter. A maximum estimation of \( a^* \) from heavier primaries is 20%~30% of \( a \), and its minimum one is 5%~7.5% which is obtained from the energy spectrum \( \sim 1/E^2 \) at higher energy region. We have adopted there a mean value, \( a^* \approx 10\% \) of \( a \).
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\[ 1.47a^* \nu^* \left[ \exp(-t/l_{abs}) - \exp(-3.1t/l_{abs}) \right], \]

where \( \nu^* \) is the multiplicity of \( B \)-nucleons in one collision, and we have put \( l_{col} \approx l_{abs} \).

Therefore, the ratio of \( B^* \) to \( A \) is,

\[ B^*/A = 0.15 \nu^* \left[ 1 - \exp(-2.1t/l_{abs}) \right], \]

and

\[ \bar{B}^*/\bar{A} = \begin{cases} 
0.12 \nu^* & \text{at } t = 45 \text{ g/cm}^2, \\
0.15 \nu^* & \text{at } t = 690 \text{ g/cm}^2. 
\end{cases} \]

Taking this into account, we should add it to the above obtained \( \bar{B}/\bar{A} \). This modifies (18) and (21) as follows. Assuming \( \nu^* \sim 4 \nu \), we have,

\[ \nu = \begin{cases} 
1.12/[\langle l_{abs}/l_{col} \rangle + 0.71] & \text{at } t = 45 \text{ g/cm}^2, \\
0.30/[\langle l_{abs}/l_{col} \rangle + 0.1] & \text{at } t = 690 \text{ g/cm}^2. 
\end{cases} \]

If we assume \( \langle l_{abs}/l_{col} \rangle \approx 2 \), this results in \( \nu = 0.41 \) or 0.14, and for \( \langle l_{abs}/l_{col} \rangle \approx 1.5 \), \( \nu \) is \( = 0.51 \) or 0.19. We see that the contributions from heavier primaries should not be neglected especially in the stratosphere, and they diminish the discrepancy in \( \nu \) stated before.

**Diffusion of C-component** The treatment of C-component is not so simple as the above cases, since they are secondary or tertiary products. First, we shall be concerned only with C-neutrons, and later discuss C-protons taking into account the ionization loss. Furthermore, for simplicity, we assume that a direction is conserved in \( A-C \) or \( B-C \) transmutations, though this approximation will be worse than in the former case, i.e. \( A-B \) transmutations. Then we can get the intensity \( C_A(t) \) of C-component produced by A-component in the same way as above,

\[ C_A(t) = \left( \mu_A/\nu \right) B(t). \]

\( \mu_A \) denotes the multiplicity in \( A-C \) transmutation. The contribution from B-component is also obtained. We have, denoting as \( \mu_B \) the multiplicity in \( B-C \) transmutation,

\[ C_B(t) = \alpha \nu \mu_B \cdot (\langle t^2/l_{col}^2 \rangle) \exp(-t/l_{abs}). \]

Thus its total intensity is,

\[ \bar{C}_B(t) = 2\pi \alpha \nu \nu \mu_B (l_{col}) \cdot (\langle t^2/l_{col} \rangle) \exp(-t/l_{abs}) \]

\[ = (\mu_B l_{abs}/l_{col}) (\langle t^2/l_{abs} \rangle) \bar{B}(t), \]

where

\[ \langle t^2/l_{abs} \rangle = \exp(-t/l_{abs}) \frac{\exp(-t/l_{abs})}{-2Ei(-t/l_{abs})} = \begin{cases} 
0.90 & \text{at } t = 45 \text{ g/cm}^2, \\
6.3 & \text{at } t = 690 \text{ g/cm}^2. 
\end{cases} \]
Summing up both contributions, we have

$$\bar{C}(t)/\bar{B}(t) = (\mu_A/\nu) + \mu_B(t)/l_{col}.$$  \hspace{1cm} (24)

If we assume, as another limiting approximation, that angular divergence in $A-C$ or $B-C$ transmutation is isotropic, $\bar{C}_A(t)$ and $\bar{C}_B(t)$ can be obtained as follows,

$$\bar{C}_A(t) = (\mu_A/l_{col}) \left\{ \int_0^t dt' \bar{A}(t-t') \left[ \exp(-t'/l_{abs}) + (t'/l_{abs}) \text{Ei}(-t'/l_{abs}) \right] + \int_t^\infty dt' \bar{A}(t+t') \left[ \exp(-t'/l_{abs}) + (t'/l_{abs}) \text{Ei}(-t'/l_{abs}) \right] \right\},$$

and

$$\bar{C}_B(t) = (\mu_B/l_{col}) \left\{ \int_0^t dt' \bar{B}(t-t') \left[ \exp(-t'/l_{abs}) + (t'/l_{abs}) \text{Ei}(-t'/l_{abs}) \right] + \int_t^\infty dt' \bar{B}(t+t') \left[ \exp(-t'/l_{abs}) + (t'/l_{abs}) \text{Ei}(-t'/l_{abs}) \right] \right\}.$$  \hspace{1cm} (25)

Integrating them numerically, we have,

$$\bar{C}(t)/\bar{B}(t) = \begin{cases} 0.88 (\mu_A/\nu) + 0.60 \mu_B(l_{abs}/l_{col}) & \text{at } t = 45 \text{ g/cm}^2, \\ 0.49 (\mu_A/\nu) + 3.1 \mu_B(l_{abs}/l_{col}) & \text{at } t = 690 \text{ g/cm}^2. \end{cases}$$  \hspace{1cm} (27)

Comparing (24) with (27), we find no serious difference between these two approximations.

Now, the experimental value of $\bar{C}/\bar{B}$ can be obtained from Table III, and remark c):

$$\bar{C}/\bar{B} = \begin{cases} 148/47 = 3.2 & \text{in the stratosphere,} \\ 1.96/0.35 = 5.6 & \text{at the mountain altitude.} \end{cases}$$  \hspace{1cm} (28)

Equating them to (24), we have

$$\mu_A/\nu = 2.8$$

$$\mu_B(l_{abs}/l_{col}) = 0.45$$

(direction is conserved).  \hspace{1cm} (29)

If we use (27) instead of (24), they become

$$\mu_A/\nu = 2.7$$

$$\mu_B(l_{abs}/l_{col}) = 1.4$$

(isotropic).  \hspace{1cm} (30)

Assuming $l_{abs} \approx 1.5 l_{col}$ and $\nu \approx 0.3$, the multiplicity $\mu_A$ and $\mu_B$ are,

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>$\mu_A$</th>
<th>$\mu_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction is conserved</td>
<td>0.87</td>
<td>0.30</td>
</tr>
<tr>
<td>Isotropic</td>
<td>0.84</td>
<td>0.93</td>
</tr>
</tbody>
</table>
We can suppose thus $\mu_A \sim 0.8$ and $\mu_A \sim 0.6$, though we can not get a definite conclusion due to the crudeness of our treatment. Nearly the same number of $C$-protons are supposed to be emitted at the same time.

In order to treat the diffusion of $C$-protons and get the $n-p$ ratio of $C$-component, we must take into account the ionization loss. It will be discussed later, comparing of its energy spectrum.

§ 5. Nuclear scattering

Fig. 1 shows integral $N_A$-spectrum of events $1_n$, $1_p$, $0_p$, and $0_n$ at different two altitudes obtained by Bristol group. As is seen at once, in the stratosphere three curves for $1_n$, $1_p$, $0_p$ resemble with each other. This shows, that these events are essentially the same phenomena, i.e. the penetration of fast nucleons through the nucleus, and the charge exchange probability $k$ is $\sim 1/2$. At the lower altitude, the frequency of $0_p$-stars becomes more than that of $1_n$ and $1_p$. This is of course due to the increase of slow protons, which can not penetrate the nucleus.

Now if the meson is not produced, the incident nucleon loses its energy through secondary nucleons (black and grey nucleons). Powell and his cowor-
Y. Fujimoto and S. Hayakawa

No. of events kers estimated the energies of black and grey nucleons and find the relation between \( N_a \) and energy loss \( \varepsilon \). As is seen from Fig. 1, the incident energies for \( 0_n \) stars are in most cases so low that the agent can not penetrate through the nucleus, or even if it penetrates its energy is degraded into grey region. Therefore we can obtain the spectrum of incident energy \( E \) for \( 0_n \) stars by identifying \( E \) with the loss \( \varepsilon \) itself. The curve shown in Fig. 1 is obtained in this way from the data at mountain altitude. It is well approximated by \( E^{-2} \) and ranges from 200 MeV \( \sim \) 1 BeV. This supports our views on \( C \)-component.

If we apply the same method for \( I_s \)-curve, we can not get the power spectrum for loss \( \varepsilon \) as seen from Fig. 2. It is flatter in lower energy region and tends to \( E^{-2} \) in higher energy regions, 700 MeV \( \sim \) 1.3 BeV. This will be due to wrong \( N_a-\varepsilon \) relation, which was made to fit for \( 0_n \) stars. We can suppose that the above relations are considerably lower estimation for \( \varepsilon \) of small \( I_s \) stars, and perhaps corrected spectrum will be \( \sim 1/E^2 \) as illustrated by dashed line, in Fig. 2. If this is the case, and assuming \( \varepsilon/E\sim 1/3 \), the incident energy spectrum may be \( 1/E^2 \) and ranges 0.9 BeV \( \sim \) 3.9 BeV.

These considerations on \( I_p, I_n \)-stars are supported by the following scattering data. Bristol group investigated the distribution of angle \( \delta \) between the incident direction and the secondary thin tracks. In the case of single nucleon-nucleon scattering, the scattering angle \( \delta \) is connected with the incident and final energies \( E_0 \) and \( E \), in following way

\[
\cos \delta = \left( \frac{1 + \frac{2M}{E_0}}{1 + \frac{2M}{E}} \right)^{1/2}.
\]

This relation holds approximately even in plural scattering inside a nucleus.

The experimental values of angle \( \delta \) are in most cases confined between 10° \( \sim \) 50°, and its distribution curve has a maximum at \( \sim 30° \). Large angle scatterings are seen in few cases. Lying aside these few large angle cases, which will be discussed later, the mean value for \( \delta \) is

\[
\bar{\delta} = 33.6° = 0.59, \quad \cos \bar{\delta} = 0.78.
\]

Now we shall apply the formula (31). If we assume \( E \approx E_0/2 \), \( \cos \delta \) becomes
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\[
\cos \delta = 0.78 \text{ or } 0.83, \quad \text{for } E_0 = 1 \text{ or } 2 \text{ BeV. (33)}
\]

They are well in agreement with the experimental one (32). However, one may doubt that the assumed fractional loss 1/2 is too large. But we can interpret the case of the larger values of \( \delta \) than in (33) as the one in which the incident nucleon changes into a neutron and it accompanies a fast proton of smaller energy. Therefore the case of the larger \( \delta \) will correspond to the smaller fractional loss than 1/2.

From the definition of thin tracks, the secondary proton must have larger energy than \( \geq 330 \text{ MeV} \). This determines the maximum value \( \delta_{\text{max}} \) for the scattering angle. On the other hand, we can estimate the minimum energy loss in penetration through the nucleus as \( \sim 200 \text{ MeV} \) from the condition \( N_\alpha \geq 3 \). This determines the minimum value \( \delta_{\text{min}} \). Applying (31), we calculate \( \delta_{\text{min}} \) and \( \delta_{\text{max}} \) as shown in Table IV.

<table>
<thead>
<tr>
<th>incident energy (BeV)</th>
<th>( \delta_{\text{min}} )</th>
<th>( \delta_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19°</td>
<td>42°</td>
</tr>
<tr>
<td>2</td>
<td>9°</td>
<td>57°</td>
</tr>
<tr>
<td>4</td>
<td>3°</td>
<td>62°</td>
</tr>
<tr>
<td>experimental value</td>
<td>( \sim 10° )</td>
<td>( \sim 50° )</td>
</tr>
</tbody>
</table>

The result of Table IV supports our views on \( B \)-component.

The large angle scattering \( \delta = 50° \sim 90° \) and the back scattering were found in several cases in the experiment. They amount nearly 40% of the small angle scatterings \( \delta = 0° \sim 50° \) which we analyzed above. We are hard to explain these cases as nuclear scattering and suppose that they are meson production. Although there are some ambiguities in distinguishing between large and small scattering, we may conclude nearly 30% of \( L \)-stars are associated with meson production.

Furthermore, we have some information on meson production in \( L \)-stars, from its \( N_\alpha \)-distribution. Comparing precisely the \( N_\alpha \)-spectrum of \( 0_\alpha \) and \( 1_\alpha \) stars in the stratosphere, we find that the frequency of \( 0_\alpha \)-event is larger than that of \( 1_\alpha \) stars for \( N_\alpha \geq 9 \), while it becomes smaller for \( N_\alpha \leq 9 \). The large \( N_\alpha \) corresponds to the nuclear event with large energy. This slight difference \( \sim 20\% \) in large \( N_\alpha \) seems to be due to the meson production.

These estimation on meson production in \( L \)-stars should be only qualitative. The large statistical error and the ambiguities of interpretation prevent us from further quantitative discussions. More direct experiments are hoped on this point.

If we know the energy spectrum of nucleon component, we can get the \( n-p \)-ratio of \( C \)-component. Assuming the integral spectrum as \( E^{-2} \), which was ascertained above, we calculate them for an examples. The experimental values are shown in Table V.
Table V. \(n/p\) ratio of \(0^0\) stars at mountain altitude.

<table>
<thead>
<tr>
<th>(N_h)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7~8</th>
<th>9~12</th>
<th>(\geq 13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n/p) ratio</td>
<td>27±14.3</td>
<td>13±6.2</td>
<td>5.3±2.0</td>
<td>3.6±1.5</td>
<td>2.5±0.93</td>
<td>2.4±0.98</td>
<td>1.45±0.90</td>
</tr>
</tbody>
</table>

Example \(N_h=5\). Incident energy is estimated as \(\sim 280\) MeV. The proton is assumed to cause nuclear event, after traversing \(l_{\text{col}}\). If we take \(l_{\text{col}}=1.5 \times l_{\text{sm}}\), the energy of proton at its birth is \(\sim 560\) MeV. Then we have

\[
n/p = \left(\frac{560\ \text{MeV}}{289\ \text{MeV}}\right)^2 = 8.
\]

As is seen, this \(n/p\) ratio is very sensitive to the value of \(l_{\text{col}}\). If we wish \(n/p\) to be exactly 5.3, we have only to put \(l_{\text{col}} = 1.15 \times l_{\text{sm}}\).

§ 6. Concluding remarks

We have analyzed the photographic data on nuclear events caused by high energy cosmic-rays, classifying the nucleon component into three groups according to their energies. Following characteristic features are found.

1) The cross-section for the nuclear interaction is nearly equal to the geometrical cross-section of the collided nucleus.

2) The integral energy spectrum of nucleon component is \(\sim 1/E^2\) at lower altitudes.

3) For incident nucleons with energies below 2 MeV, nucleon-nucleon collisions are almost elastic, and collisions associated with meson production will be \(\sim 20\%\).

4) The fractional energy loss in one nuclear collision is \(\sim 1/3\) and it is nearly independent of incident energies.

5) The probability for charge exchange in one nuclear collision is \(\sim 1/2\).

These are ascertained in the region of energies from several hundred MeV to several BeV, and allow us to infer the nuclear interaction at high energies. A main difficulty of our analysis is the inconsistency between the primary intensity of cosmic-rays and the absolute frequency of nuclear events observed in stratosphere, but later experiment\(^9\) shows smaller primary intensity than before, and the discrepancy will not be so large as supposed at first. Contributions from heavier primaries also introduce various ambiguities. It seems necessary to analyze more precisely taking into account the energy spectrum of nucleons, when we will compare these results with the theoretical predictions. Also more detailed experiments are required for this purpose.

Previously, we calculated the problem on the penetration of fast nucleons through the nuclear matter.\(^{10}\) Assuming the current nuclear potential which is well fitted to the neutron-proton scattering and deuteron binding energy, we find the cross-section which is nearly proportional to \(e^2/c^2\) and tends to \(\sim 5 \times 10^{-27}\).
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111 cm² per one nucleon at high energies. This result does not seem to be able to account for the observed nuclear events, and some new type of nuclear forces may be necessitated in high energy regions. We shall discuss on these problems in a separate paper.

References

2) U. Camerini, T. Coor, J. H. Davies, P. M. Fowler, W. O. Lock, H. Mairhead and N. Tobin, 40 (1949), 1073. American authors also investigated the nuclear events in the stratosphere using the photographic method. Cf. J. J. Lord and M. Schein, Phys. Rev. 77 (1950), 19. As their results are almost identical with those of Bristol group, we shall refer mainly to the latter.
3) P. H. Fowler, Phil. Mag. 41 (1950), 169.
6) Cf. the appendix of B. Rossi, Rev. Mod. Phys. 20 (1948), 537.
8) Detailed studies on the stars and the nucleon component were carried out by Italian authors. Cf. G. Bernardini, G. Cortini and A. Manfredini, Phys. Rev. 76 (1949), 1792 and in press. We are grateful to Prof. Bernardini for sending us their manuscript before publication.

Addendum

1) If the absolute intensity of primary rays is revised as

\[ a = 0.070 \text{sec. cm}^2 \text{sterad}, \] (6')

following the recent estimation of Van Allen et al.\(^9\), or

\[ a = 0.093 \text{sec. cm}^2 \text{sterad}, \] (6'')

considering that in higher latitude where the experiment may be carried out, then some figures in § 4 are modified as follows (figures based on the latter estimation (6'') are represented in the brace).

\[ \bar{A}(45 \text{ g/cm}^2) = 1.54(2.08) \times 10^4 \text{cm}^2 \text{day}, \] (7')

\[ A\text{-stars at } 45 \text{ g/cm}^2 = 1.54(2.08) \times 10^5 \text{g. day}, \] (8')

the minimum expected frequency of \( A\)-stars \( \approx 72(97)/\text{g. day}. \) (10')

2) Taking into account the existence of \( \alpha \)-particles in primaries, the diffusion of \( A\)-component is somewhat modified. The number of \( A\)-protons or -neutrons given rise from \( \alpha \)-particles is evaluated by

\[ \rho(t) = \frac{1}{l_{abs}} \int_0^t 2a^* \exp(-t/l_{abs}^*) \rho(t-t')dt', \]

where \( \rho(t) \) is the solution of (4) with initial condition \( \rho(0) = 0. \) Assuming \( n^* = 0.1a, \) we obtain

\[ \rho^* = 0.094, \quad \bar{n} = 0.30 \]

and

\[ \bar{A}/\bar{A}(45 \text{ g/cm}^2) = 0.14. \]

Accounting for the above, (10') must be modified as

\[ 82(111)/\text{g. day}. \]

The \( n^*/\bar{n} \) ratio at the stratosphere increses to 0.37 from 0.31, the latter being derived from the proton primary hypothesis and \( l_{col} = 1.5l_{geo}. \)