On the Absorption of the Negative $\pi$-meson by Deuteron

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We have analyzed the various processes which are induced by the absorption of the negative $\pi$-meson by the deuteron. The negative $\pi$-meson is assumed to be bound in $K$-orbit. The results indicate that, as for the model of meson, the vector and the pseudovector types are not favourable. The $\pi$-meson of pseudoscalar type is most favourable.

§ 1. Introduction

In the analysis of the processes in which $\pi$-mesons take part, it is generally difficult to support the validity of the obtained results because of the complicated correlations among the meson model, the approximation method and the theory in itself. However, we would rather accept such standpoint that, in the concrete analysis of the various processes guided by the present theory, we search the key perspecting the characters of the difficulties. The problem here treated is one of the works with such intentions.

On the $\pi$-meson production by the nucleon- and photon-nucleon collision various analysis have been done. By these works, the pseudoscalar $\pi$-meson theory is shown to be advantageous. Previously, we have studied the absorption of the negative $\pi$-meson by the proton and found that the pseudoscalar theory was also favourable in this case. But because of the arbitrariness of the coupling constants of the $\pi$-meson with the nucleon, we could not give any affirmative conclusion.

It must be noted that in the deuteron case, here treated, the obtained results do not depend upon the magnitude of the coupling constant, but do only upon the type of the interaction between the meson and the nucleon.

With absorption process of the $\pi$-meson in $K$-orbit, we can expect the following three processes tentatively,

\[ \pi^- + D \rightarrow N+N+\gamma \]  
\[ N+N+\pi^0 \]

(i)

(ii)

(iii).

(i) is the process in which the deuteron disintegrates into two neutrons, getting the rest energy of the $\pi^-$-meson, and (ii) is the one which accompanies the photon emission. The process (iii) is allowed only if the rest energy of the neutral $\pi$-
meson is less than that of the negative $\pi$-meson minus the binding energy of the deuteron and the neutron-proton mass difference. Recent experiment shows that $m_\pi - m_\pi^0$ is almost of ten times of the electron mass, so the process (iii) is not always forbidden. But, on account of the smallness of $m_\pi - m_\pi^0$, the momentum of the emitted $\pi^0$-meson becomes small. Accordingly, the contribution to the probability of the transition (iii) from the final state density comes out to be smaller compared with that in the case (ii). So, we shall not perform the exact calculation on the process (iii), but only give the order estimation about the relative magnitude of the probability of process (iii), comparing with that of (ii).

The experiments on these processes have been done by Panofsky et al.\(^6\) The roughness of the obtained results does not enable us to affirm the existence of the process (iii). And it has been known that the processes (i) and (ii) occur with the comparable magnitude.*

Before entering into the calculation, we give some preliminary discussions. The perturbation method is used. Process (i) is of the first order and the processes (ii) and (iii) are of the second. Moreover, for the process (i), the effects of the third order correction are studied. Our aim is pointed to the analysis of the $\pi$-meson model, that is, of the meson-nucleon interaction, and so the relativistic treatment becomes necessary. However, the initial state—deuteron in our problem—is the bound one, so that such a treatment becomes seriously difficult. So, we treat the states of the nucleons in Pauli's approximation. Considering the energy of the associated nucleon to be at most one half of the rest energy of the $\pi$-meson, we may expect that such a treatment is valid. The state of the nucleon is expressed by

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \overline{\psi} = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix},$$

then, we can write in Pauli's approximation,

$$\overline{\psi} = (-i/2M) (\sigma \text{ grad}) \psi; \quad M; \text{ nucleon mass.}$$

Some remarks are needed concerning to the wave function of the deuteron and of the two neutrons after disintegration. In fact, it has become clear that the assumption upon the type of the internucleonic potential bring rather large influence on the results, from the theoretical analysis on the $\gamma$-disintegration of the deuteron.\(^5\) On the other hand, by the analysis of the nucleon-nucleon scattering, it has been clear out that the long-tailed potential give good results.\(^6\) We take, accordingly, the solution for Serber-Hulthen potential as the ground state of the deuteron.

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* Private conversation with Prof. H. Yukawa, in August 1950.
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Because of the comparatively small associated energy, the state of the neutrons after disintegration cannot be treated as free. Any accurate knowledge, however, has not yet been obtained on the neutron-neutron potential. Accordingly, from the analogy with the results on the analysis of low energy ($\lesssim 10$ Mev) nucleon-nucleon scattering,—charge independence of the nuclear potential,—we assume the neutron-neutron potential to be also of Serber-Hulthen type. There is, of course, no support on that assumption up to the energy treated here ($\sim 100$ Mev). But when we study the relative probability of the processes like (1), the small variations of the nuclear potential may result in small correction. The states of the neutrons after disintegration are separated into the states $^1S, ^3P, ^1D$, etc. which are the solutions in the above potential. And we perform the calculation only about the allowed transition to the lowest level, whose probability takes the most part of the total transition probability. The error caused by this simplification is of some 10% in magnitude.

§ 2. $\pi^- + D \rightarrow N^+ + N$; First order process

As the nucleon-$\pi^-$-meson interaction, we take only scalar, vector, pseudoscalar and pseudovector meson theory, respectively. For instance, for the pseudoscalar theory, we take

$$H_{\pi^s} = f \phi \cdot \sigma \cdot \nabla \psi .$$

According to Pauli's approximation, it results in

$$H_{\pi^s}' = f \{ \phi \psi - \phi \psi^* \} = f \{ \phi \psi (i/2M) (\sigma \cdot \nabla) \psi - (i/2M) (\nabla \psi^* \sigma) \phi \psi \} .$$

As equivalent form, we take the following

$$H_{\pi^s}' = f (i/2M) \phi \psi (\sigma \cdot \nabla) \psi . \quad (4)$$

Similarly, for other meson types we obtain the followings,

$$H_{\pi^v} = f \phi \cdot \sigma \cdot U \psi , \quad H_{\pi^v}' = (if/M) \phi \sigma \cdot U \nabla \psi \cdot \psi . \quad (5)$$

$\phi$ and $U$ are the wave functions of $\pi$-meson of scalar type and of vector type, respectively.

From the facts that the deuteron is in $^3S$ state and that two neutrons exist in the final state, there occur some restrictions on the transition allowed by the interaction (4) and (5). In the scalar theory, for instance, the interaction having no $\sigma$-dependent term, only the transition to the triplet state is to be allowed. If the final state is a triplet one, its parity must be odd. Considering that the parity of the deuteron is even and $\pi$-meson has a spherical-symmetric wavefunction, such a transition must be forbidden. Allowed transition to the lowest level are as follows,

- scalar: $^3S \rightarrow no$
- vector: $^3S \rightarrow ^3P$
pseudoscalar : $^3S ightarrow ^3P$, pseudovector : $^3S ightarrow ^1S$.

To calculate the matrix element, we separate the wave function of the nucleon into two parts, that is, into $\sigma$-space and into the ordinary space, letting it to be $\phi = \chi(\sigma) u(r)$. ($r$ is relative coordinate.) The matrix elements result in

$$H_{p,s} = f(2\pi/x) \frac{1}{2} i(\hbar/2M) \int_0^\infty v_0 v_x r^2 dr,$$

$$H'_{p,s} = f(2\pi/x) \frac{1}{2} i(\hbar/2M) \int_0^\infty v_0 v_x r^2 dr,$$

$$H_{p,v} = f(2\pi/x) \frac{1}{2} i(\hbar/2M) \int_0^\infty v_0 v_x r^2 dr.$$

(7)

Here, we have neglected the binding energy of the $\pi$-meson compared with its rest energy. And we have replaced $\phi$ by $\phi_0$ (value of $\phi$ at the origin), for $\phi$ is almost constant in the region of the deuteron's extent. $K$ is the relative momentum of the neutron after disintegration. $\sigma_k$ and $\sigma_\sigma$ are the $k$- and $U$-component of the spin vector $\sigma$. For convenience of the calculation of the matrix in $\sigma$-space, the projection operators which select the triplet and the singlet are introduced,

$$A_\pi = (1 - \sigma^{(1)} \sigma^{(2)}) / 4, \quad A_\pi = (3 + \sigma^{(1)} \sigma^{(2)}) / 4.$$

(8)

Using $H'$ in (7), the probability can be obtained as

$$W = 2|H'|^2 \rho_{\pi}, \quad \rho_{\pi} = \frac{k}{\sqrt{M^2 + k^2}} / 2(2\pi)^3 d\Omega,$$

$$k = [M(x - \epsilon - \Delta M)]^{1/2}.$$

(9)

$\epsilon$ and $\Delta M$ are the binding energy of the deuteron and the neutron-proton mass difference, respectively. $M$ is the neutron mass.

If we assume the mass of the $\pi^-$-meson to be 278 electron mass, two neutrons whose kinetic energy is about 11 MeV, are to be emitted simultaneously in the opposite directions in the above process.

§ 3. $\pi^- + D \rightarrow N + N + \gamma$

In this case, the transitions are allowed for all meson types and the lowest final state is $^1S$. The process is of the second order and the explicit virtual states are expressed as,

$$D + \pi^- \rightarrow \begin{cases} P' + \gamma + \pi^- + N \\ [P] + N' + N + P \end{cases} \quad (1) \rightarrow N + N + \gamma.$$

(10)
The dash means the particles which take part in the emission and the absorption of the proton in the virtual state, and [ ] means a hole of the negative state. In the virtual state, we treat the nucleon as free. Our processes are associated with the absorption of the $\pi^-$-meson, so we cannot separate the photon-nucleon interaction into the dipole, quadrupole ... interaction as in the case of the photo-disintegration. For instance, in the pseudoscalar theory, the contribution from the negative state nucleon becomes distinctly large. We should treat the process one-body-problematically.

On the scalar type meson theory, the interaction in the transition (12) is,

$$H' = eF^\ast aA F + \frac{ie}{4\pi} A \text{ grad} \phi + \mu_2 F^\ast \sigma H(x_2) F + \mu_3 F^\ast \sigma H(x_3) F + \nu \nu^\ast \nu \phi,$$

$$O = \beta: \text{ for scalar, } O = \beta \gamma_5: \text{ for pseudoscalar.} \quad (11)$$

$x_1$ and $x_3$ are the position of the proton and of the neutron, respectively. We have introduced the anomalous magnetic moment of the nucleon, phenomenologically, indicating it as $\mu$. Process (3) in (12) is of little contributions to the probability, because of the small binding of the $\pi^-$-meson. On the vector type meson theory, however, though the above discussion is not valid, we might be able to neglect the process (3) according to the reason discussed below. On one hand, if the $\pi^-$-meson is on the orbit after radiating $\gamma$-ray in the virtual state, the contribution of the space integral to the matrix element becomes quite small. And on the other hand, if the $\pi^-$-meson is out of orbit in that case, the density of the $\pi^-$-meson's wave function comes out to be small, effecting to make the absorption of the $\pi^-$-meson by the proton difficult. Though we cannot examine such an effect quantitatively, it may be not far from the reality.

We can calculate the matrix element of the interaction (11) explicitly. For a example, we take the pseudoscalar case. In this case, the process (1) in (12) gives the largest contribution. Hereafter, we shall assume $2 \gg x/M$, and we neglect $O(x/M)$ compared with unity. In this approximation, only the process (1) is effective. Noting the initial, virtual and final states of the proton, as $\Psi_i$, $\Psi_v$ and $\Psi_f$, respectively, we set similarly to the treatment in § 2,

$$\Psi_i = \sum_p \chi_p \nu'_p, \quad \Psi_v = \sum_p \chi_p ^\ast \nu'_p = \chi'_p \exp (ik'x_1),$$

$$\Psi_f = \sum_p \chi_p \nu'_p, \quad \rho = 1, 2, 3, 4. \quad (12)$$

Then, the matrix element is

$$H' = e(2\pi/x)^{1/2} (2\pi/k)^{1/2} (\chi'_p \gamma'_5 \cdot \chi''_p \nu'_p) \phi_0 \int \nu e^{ik'r} dr.$$
\( |k| = k \) is the energy of the photon in the final state and \( q \) is the momentum of the proton in the relative coordinate. \( E_{|k-q|} \) is the proton's energy in the virtual state, including its sign. We replace \( \phi \) by \( \phi_0 \) according to the same reason as discussed in § 2. This simplification may be valid on the assumption that the various transitions in the virtual state occur in the region of the \( K \)-shell of the \( \pi^- \)-meson.*

Further approximation is made. Space function \( \nu_f \) can be replaced by the plane wave assigned to the momentum \( q \) except in the nearest neighbour of the origin where the nuclear force has a strong effect. Accordingly, the most contribution of the first integral of (13) comes from the region \( |k'| \sim |q| \). And so we can take the denominator in (13) to be nearly constant in the integration on \( k' \). It gives simply \( \delta (r-r') \). The calculation in the spin space is as follows;

\[
\langle \chi' \beta \cdot \chi'' \cdot \chi a \chi' \rangle \frac{1}{E_0 + E_p} = \langle \chi' \beta \cdot \chi_0 \rangle \delta(\rho - \beta M) \hat{a} \chi' \times (1/E_0 - E_p^2) \\
\cong (-1/2M) (\gamma a \chi_0),
\]

where \( E_0 = x - \sqrt{M^2 + (q-k/2)^2} \), \( E_p = E_{|k-q|} \).

Here, we have used the approximation \( 1 \gg (x/M) \). \( \sigma_0 \) is the component of the spin vector \( \sigma \) in the polarization direction of the emitted photon. In the last line of (14), \( \chi \) is 2-dimensional spinor in Pauli's spin space. Thus, the interaction becomes,

\[
H_{ps'} = e(2\pi/x)^{1/2} (2\pi/k)^{1/2} \langle -1/2M \rangle (\chi \sigma \chi_0) \int u_0(q) e^{-ikr/2} \nu \, dr
\]

\( v_0 \) and \( u_0 \) are the wave function of the ground state of the deuteron and the \( S \)-wave function of the wave number \( q \).

The probability is

\[
dW = 2\pi |H'|^2 \nu \nu' .
\]

The density of the final states is,

\[
\rho \, d^3K \, |(2\pi)| \cdot d\Omega = \frac{q^2 \, d\Omega}{(2\pi)^3} \frac{\partial \Omega}{\partial E_p} ,
\]

\[
E_p = E_0 + x = \sqrt{M^2 + (q-k/2)^2} + \sqrt{M^2 + (q+k/2)^2} + k.
\]

\( q-k/2 \) and \( -q-k2 \) are the final momenta of the two neutrons in the center of mass system (in this case, laboratory system).

* Strictly speaking, it is the three-body problem. Because of the difficulty in such problem, we rather depend on the physical picture.
§ 4. Numerical estimations and the process accompanied with the \( \pi^0 \)-emission

I) We shall begin with numerical analysis of the results obtained in the foregoing paragraphs. The spectra of the \( \gamma \)-ray emitted in the process \( D + \pi^- \rightarrow N + N + \gamma \) are presented in Fig. I, being compared with Panofsky's results. Taking into account the uncertainty of the experiments, the consistency is satisfactory. Still more a few remarks on the spectra are given. The spectra of the emitted \( \gamma \)-ray accompanied with the absorption of scalar \( \pi^- \)-meson and of pseudovector \( \pi^- \)-meson are of same type. Concerning to vector and pseudoscalar \( \pi \)-meson, the situations are similar. The former spectrum differs from the latter only within the experimental errors. From Fig. I, it is clear that the majority of the emitted \( \gamma \)-rays have the energy greater than 110 Mev. Accordingly, the probability that the neutron is emitted with the energy more than 30 Mev is only of magnitude less than 1%. This situation is favourable to discriminate the above process from the process \( \pi^- + D \rightarrow N + N \) in the experimental researches.

The ratios of \( W_{\pi N} \) (probability of the process \( \pi^- + D \rightarrow N + N \)) to \( W_{\pi^0 N} \) (probab. of the process \( \pi^- + D \rightarrow N + N + \gamma \)) are presented in Table I. In these calculations, the coupling constants of the \( \pi \)-meson with the nucleon cancel out. Accordingly, the indefinitness caused by the magnitude of the coupling constants vanishes. As is mentioned in the introduction, these discussions, of course, is valid only in the region in which the value of the coupling constant allows the perturbation method. From Table I, we can see that the results for any mesonic type are inconsistent with the experimental result which indicates \( W_{\pi N}/W_{\pi^0 N}\sim 1 \). The following considerations, however, may be needed. Considering the coupling constants only, we may expect \( W_{\pi N}/W_{\pi^0 N}\sim 1/e^2 = 137 \). The situations that \( W_{\pi N}/W_{\pi^0 N} \) deviates far from \( 10^2 \) are accidental, being caused by the peculiarity of the interaction of the \( \pi \)-meson with the nucleon. For

<table>
<thead>
<tr>
<th>Meson Type</th>
<th>Ratio ( W_{\pi N}/W_{\pi^0 N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.</td>
<td>0</td>
</tr>
<tr>
<td>P.S.</td>
<td>0.02</td>
</tr>
<tr>
<td>V.</td>
<td>5.102</td>
</tr>
<tr>
<td>P.V.</td>
<td>7.106</td>
</tr>
</tbody>
</table>

Table I.
instance, in the case of pseudoscalar $\pi^-$-meson, the operator $\hat{\gamma}_5$, which is included in the 1st approximation makes the value of $W_{\pi^-}$ quite small. In this case, higher order correction to the process $\pi^- + D \rightarrow N + N$ may be needed and be expected to contribute considerably. For the case of scalar $\pi^-$-meson, similar affairs are expected. In the following paragraph, we shall analyse the 3rd order correction, which does not include such peculiarity as in the 1st approximation. From the viewpoint of the perturbation method, these considerations are unnecessary for the $\pi^-$-meson of vector type. And so, for the vector type meson, the consistency with the experiment are not expected.

Further, the ratios of $W_{D,T}$ to $W_{P,T}$ (probab. of the process $P + \pi^- \rightarrow N + \gamma$) are given in Table II. It is to be noted that the values given in Table II must not be compared with the ratio of the process $D + \pi^- \rightarrow N + N + \gamma$ to the process $P + \pi^- \rightarrow N + \gamma$ given by Panofsky's result. Because Panofsky's result remains unchanged, even if the probabilities of all transitions caused by the absorption of $\pi^-$-meson by the proton increase by an arbitrary factor on the whole.

Further, the ratios of $W_{D,T}$ to $W_{P,T}$ (probab. of the process $P + \pi^- \rightarrow N + \gamma$) are given in Table II. It is to be noted that the values given in Table II must not be compared with the ratio of the process $D + \pi^- \rightarrow N + N + \gamma$ to the process $P + \pi^- \rightarrow N + \gamma$ given by Panofsky's result. Because Panofsky's result remains unchanged, even if the probabilities of all transitions caused by the absorption of $\pi^-$-meson by the proton increase by an arbitrary factor on the whole.

II) We have mentioned only that the process $D + \pi^- \rightarrow N + N + \pi^0$ is allowed energetically. Here, we estimate its probability relative to $W_{D,T}$ based on the density of the final states. In the calculation of $W_{D,T}$, the density of the final states is given by (20). And most contribution to the probability comes from the region $k \sim q \sim x$. On the other hand, the density of the final states in the transition $D + \pi^- \rightarrow N + N + \pi^0$ becomes

$$Pd\Omega/(2\pi)^3 \cdot d\Omega_1 \cdot g^2 d\Omega_2/(2\pi)^3 \cdot (\partial\Omega/\partial E).$$

(18)

$I$ is the momentum of the emitted $\pi^-$-meson. Now, the mass difference of $\pi^-$-meson and $\pi^0$-meson is $\sim 10 m_e$ ($m_e$: electron mass). Taking into account of the proton-neutron mass difference and the binding energy of the deuteron, the available kinetic energy of the emitted $\pi^0$-meson is only $\sim 4 m_e$. This indicates $t_{\text{max}} \sim \sqrt{8x_0 m_e}$ ($x_0$: $\pi^0$-meson mass). Accordingly,

$$[\rho_\pi(\pi^- + D \rightarrow N + N + \pi^0)/\rho_\pi(\pi^- + D \rightarrow N + N + \gamma)] < (t_{\text{max}}/x)^4 \sim 0.1\%.$$  

(19)

The results in (19) support the discussions in §1. on the process or $\pi^- + D \rightarrow N + N + \pi^0$. Hitherto we have not treated the matrix element in detail. The contribution of the process $\pi^- + D \rightarrow N + N + \pi^0$, however, is of magnitude less than 10% compared with that of $\pi^- + D \rightarrow N + N + \gamma$, even if we take the coupling constant $g_\pi$ of the $\pi^0$-meson with the nucleon such as $g_\pi^2 \sim 1$.

§ 5. $\pi^- + D \rightarrow N + N$: 3rd order correction

As mentioned above, the fact that the probability of the process $\pi^- + D \rightarrow N + N$ calculated in the 1st order becomes unexpectedly small in the case of the scalar type meson, is accidentally caused by the character of the interaction
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treated in the 1st order approximation, and so we may need to calculate the 3rd order correction to this process. We treat this problem in this paragraph.

The ordinary perturbation method is generally not so perspective in the treatments of the higher order correction. We follows Feynman-Dyson formalism. $F-D$ formalism, however, is based on the treatments of the free field. Accordingly, its application to the problem which includes the transition from the bound states such as deuteron may lack some strictness. However, we may expect the validity of this procedure on the point of the qualitative discussions. We give only the order estimation of the result, even if quantitative estimation is carried out.

The lowest corrections to the process that one of two nucleons system absorbs the $\pi^-$-meson are expressed by Fig. II.

Fig. II.

We dropped the figures which give the mass term correction in Fig. II. $\phi_1$ and $\phi_2$ are the wave functions of neutron and proton in the initial states respectively. If the initial and final wave functions of the nucleons are free, the calculation can be performed strictly. In our problem, initial state is bound one. However, the effect of the binding to such correction is expected to be small. In fact, the binding energy of the deuteron is quite small compared with the rest energy of the nucleon. The main part of the 3rd order correction may be obtained from the diagrams in Fig. II.

Process (a): This process, as is well known, belongs to Lamb shift type. The corrected interaction caused by this process is,

$$ H_a = \frac{i}{8} \sum_{n} \int dx_1 dx_2 \phi^*_n(x_1) \omega S_p(x_1 - x_2) \phi_p(x_2) $$

$$ + \omega = \gamma_5 \text{ for pseudoscalar, } \omega = 1 \text{ for scalar,} \quad (20) $$

the derivation of which follows the neglec of the motion of the neutron in the deuteron. Then, the calculation is straightforward and is presented only its results.

$$ H_a = i \frac{\bar{f} \cdot f^*}{4 \pi} \sum_{n} \int \phi^*_n(x_1) \omega \frac{\phi_p(x_2)}{F(M, x, x_0)} + O(|x^2/M^3|)^* \quad (21) $$

$F$ is the numerical constant which includes logarithmic divergence, and is neglected for the coupling renormalization. The operator $\Box - x^2/M^2$, being separated from

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* $\bar{f} = (f_2 \cdot f_4^p)/2$, $\Delta f = (f_2^p - f_4^p)/2$. $f^a$ is the coupling constant of $\pi^a$-meson with the nucleon.
time component, reduces to $A/M^2$ which operates on the wave function of the $\pi^-$-meson. In our problem the binding of the $\pi^-$-meson is so small that the contribution (24) is quite small.

Process (b): From the view point of the character of the corrected interaction, this diagram is equivalent with (c) and (d). And we pick up this diagram as a sample and calculate it. The $S$-matrix for this process is,

$$S_b = \frac{i \int \frac{d^4 \phi - \phi^0}{4}}{d^4 x_0} \int d\phi \bar{\phi} \phi S_{x_0}(x_0 - x_1) \times \psi \psi(x_1 - x_2) \psi(x_0) \phi(x_0). \quad (22)$$

In order to express the initial and final state not so explicitly, we take the following approximation. The special variations of $\bar{\phi} \phi$ are replaced by the deuteron state,* whose explicit expression is unnecessary. Then, time integral is performed. In the neglect of the binding energy, it results in,

$$S_b \approx \frac{i \int \frac{d^4 \phi - \phi^0}{4(2\pi)^2}}{d^4 x_0} \int d\phi \bar{\phi} \phi (r_1 - r_2) \beta' \omega' \exp \left[ ik(r_0 - r_2) \right] \times \beta' \omega' \frac{M(1 - \beta) + x}{2Mx} \beta' \omega' \phi(r_0, r_2). \quad (23)$$

As is well known, $\int \beta' \omega' \frac{\exp [ik(r_0 - r_2)]}{k^2 + 3/4x^2} \beta' \omega' d^4 k$ is the nuclear force potential caused by the exchange of $\pi$-meson in the 2nd approximation. We represent it by $V(r_1 - r_2)$. Taking out the interaction from (7), we obtain,

$$^{3}H_{p\sigma} = \frac{\pi}{2} \int \frac{d^4 \phi - \phi^0}{4(a^2 \sigma \frac{i}{2M} - \frac{k^2}{x} \frac{1}{x})} \exp \left[ ik(r_0 - r_2) \right] \beta' \omega' \phi(r_0, r_2). \quad (24)$$

Pauli's approximation (3) is used in the derivation of (24). For the 3rd order correction of the absorption process of scalar $\pi^-$-meson, we take into account only the correction due to the exchange of scalar $\pi^-$-meson. For the pseudoscalar case, it is the same. As is seen from (24), the correction in the case of scalar $\pi^-$-meson, does not include any $\sigma$-dependent term. Accordingly, for the scalar $\pi^-$-meson, the process $\pi^- + D \rightarrow N + N$ is forbidden even in the 3rd order correction.

For the case of the pseudoscalar $\pi^-$-meson, in (24) $\nabla \phi$ appears in the place of $\nabla \phi$ in (4). The fact that the contribution of the process $\pi^- + D \rightarrow N + N$ in the first approximation becomes quite uneffective is caused by the lax binding of the $\pi^-$-meson. The breadth of the wave function $\phi$ of the bound $\pi^-$-

* Such procedure may be not taken as the precision of the approximation. In fact, for the process which includes the bound state, the reconstruction of $F-D$ formalism is heeded. The above procedure is valid only as an approximation.
meson is about compton wave length of $\pi^-$-meson multiplied by $1/\varepsilon^2 \sim 100$. Taking into account that $V/\pi$ spreads in the region of compton wave length of the $\pi^-$-meson, $H_P^\prime/\sigma(H_P) \lesssim (f_N^2)/\varepsilon^2$. Accordingly, if $(f_N^2)^2 \sim 0.1$, the value of $W_{3N}/W_{D\pi}$ can be consistent with the experiment.

In the calculation of the 3rd order correction above, we have assumed that the absorbed $\pi$-meson is of identical type to the exchanged one. This assumption is not always necessary. In the absorption process of the scalar $\pi^-$-meson, the correction due to the exchange of another type meson, for instance, of $\tau$-meson may occur. If the obtained $V$ includes the operator* not commutable with $0'(1)0'(2)$, the process $\pi^- + D \rightarrow N + N$ is allowed for the case of the scalar $\pi^-$-meson in the 3rd order transition.

§ 6. Summary

From the analysis above, we can see that the cases of scalar and pseudoscalar meson may be consistent with the experiment by Panofsky et al. Especially, the pseudoscalar meson is most favourable because of its consistency by itself.

Our conclusion, however, may be restricted by the validity of the perturbation method. The method indicates some doubtness in the application to the problems which correlate to the structure of the elementary particle, for instance, to the problem of the anomalous magnetic moment of the nucleon. On the other hand, in the process in which real mesons are emitted and absorbed, the analysis based on the perturbation method gives not always unreasonable results. The problem of the meson production by the $\gamma$-ray may seem to support this method. Accordingly, we may expect that our conclusion is not far from reality.

In conclusion, we should like to express our deep gratitude to Prof. S. Sakata for his valuable suggestions. And we are also indebted to Mr. Yamaguchi and Mr. Fujimoto for their kind discussions.

References

4) A. Amoldt, J. Hadley and W. Panofsky, Phys. Rev. 80 (1950), 282.

* Such operators make the transition triplet $\pm$ singlet possible.