Nuclear Shell Model and the $\beta$-Decay Schemes, I*

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In § 1: the spin-orbit coupling shell model is introduced for nuclei with even mass number besides that for odd mass nuclei.

In § 2: to summarize, the overall agreement between Mayer's shell model and the Fermi theory of $\beta$-decay is very satisfactory. It is also shown that the type of nuclei predicted by the spin-orbit shell theory is useful for the study of analysis of $\beta$-decay schemes and the nuclear spectroscopy.

Introduction

To obtain a definite answer for the check of the Fermi theory of beta decay, exact informations of nuclear states concerned have been desired for a long time. Mayer,1) Feenberg and Hammack,2) and Nordheim3) developed specification of various nuclei by means of their original ideas concerning the nuclear shell model. However, the matter under consideration was essentially the ground state character of nuclei and the magnetic moments, although a discussion was also made on beta decay schemes.

In this paper, in order to provide a basis for analysis of beta decay, consideration will be given to the importance of beta emitters and their daughters with respect to ground states.

§ 1. Shell models

As far as odd mass nuclei are concerned, the general principle of level schemes was already established by Mayer.1) However, as far as even mass nuclei are concerned, it has not been definitely determined. New reasonable assumptions will, therefore, be added in order to complete the level assignments.

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Odd mass nuclei

The following is a summary of Mayer's shell model in view of its bearing upon our researches.

It was first pointed out by Schmidt⁴ that the odd mass nucleus have the following magnetic moment:

\[ \mu_l = g_l + g_s \quad \text{for} \quad J = I + 1/2, \]
\[ \mu_l = g_l \frac{(I+1)(2I-1)}{2I+1} - g_s \frac{2I-1}{2I+1} \quad \text{for} \quad J = I - 1/2, \]

where \( g_p = 1 \) for the proton, \( g_n = 0 \) for the neutron, and \( g_s \) is the intrinsic magnetic moment of the proton or the neutron. Deviations of experimental values from those predicted by Schmidt are found to be considerable. These deviations suggest that the angular momentum \( I \) may not be a good quantum number. In spite of this fact, the general tendencies indicated speak strongly in favor of the proposal that the nuclear configuration can be expressed in terms of the angular momentum.
Fig. 1, b. Magnetic moment of odd neutron nuclei.

Fig. 1 \( \mu \)-values for odd proton nuclei lie along the two lines \( PP \) and \( QQ \), whose angle of inclination is 45°, and \( \mu \)-values for odd neutron nuclei lie along the two horizontal lines \( PP' \) and \( QQ' \). For the nucleus lying along \( PP \) and \( PP' \), the configuration of which has the angular moment \( I=J+\frac{1}{2} \) for the nucleus along \( QQ \) and \( QQ' \), the configuration of which has the angular moment \( I=J-\frac{1}{2} \).

Moreover, it was mentioned that isomers or nuclei which have the same number of neutrons have the same spin value each other with few exceptions of \((\text{Rb}^{86}, \text{Rb}^{87})\), \((\text{Sb}^{121}, \text{Sb}^{125})\), and \((\text{I}^{157}, \text{I}^{159})\). Moreover such nuclei have same configuration with the exceptions of nuclei which have \( N \) or \( Z=51 \sim 75 \) and also \( \text{Rb}^{86} \). In Fig. 2, 3, following facts should be looked at with care. A) Up to the closed shell 20, configurations are completely the same for nuclei with odd protons and for those with odd neutrons. B) The order of levels is \( 1s, 2p, 2s, 3d, 4f, 3p, 5g, \& 4d, 3s, 6h \), etc. This is the order of levels between the oscillator and the square well potentials. C) The levels in which \( J=l+1/2 \) are generally lower than those \( J=l-1/2 \). A strong spin-orbit coupling may provide their explanation. In spite of this, it is looked at with care that there are, on one hand, \( 6 \times 2 \) of \( 3p \) state, \( 36 \) of \( 5g+4d \) state, and on the other hand only \( 4 \times 2 \) of \( 4f \) state and none of \( 4f_{9/2}, 6h_{9/2} \) state. This was a reason why Mayet had to introduce pairing energy.
### Table: 

#### Nuclear Shell Model and the β-Decay Schemes, I

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Mayer’s assumptions are as follows. i) An even number of identical nucleons in any orbit with total angular momentum quantum number $j$ will always couple to give a spin zero and no contribution to the magnetic moment. ii) For a given nucleus, the “pairing energy” of the nucleons in the same orbit will be greater for the orbits with larger $J$. These assumptions should provide a basis for explaining the order of levels, mentioned above. Thus in the region where the number of the protons or the neutrons amounts from 21 to 38, the order of levels is $p_{3/2}, f_{5/2}$. However, with respect to the nuclei which has even number of protons or neutrons in the shell, the level $f_{5/2}$ is lower than $p_{3/2}$. On account of pairing energy, $f_{5/2}$ orbits will therefore, be filled preferentially in pairs. Moreover, $4(3p_{3/2}) + 6(4f_{5/2}) = 10^*$, which is just equal to 38–28. On the other
hand, in the region where 51 to 76 protons are included, the order of levels is
4g_7/2 and 4d_5/2, 6h_11/2. On account of the large pairing energy, the 6h_11/2 orbits
fill preferentially in pairs. Moreover, 12(6h_11/2) + 8(6g_7/2) + 6(4d_5/2) = 26
which is equal to 76-50.

On the case of neutron shell which contains more than 51 neutrons, it is
necessary to investigate in detail some more.

Even mass (odd-odd) nuclei

On the case of those nuclei, nuclei, of which magnetic moment is measured, are
very few. However, there are many data of beta decay which suggest us the
spin parity of those nuclei. Following assumptions are adequate to analyse the
data of β-decay in all.

1) The odd-odd nucleus, the extra proton of which is in j-orbit and the
extra neutron in j'-orbit, will have spin J, where

\[ |j - j'| \leq J \leq |j + j'|. \]

2) Parity of odd-odd nucleus will be equal to the product of the parity
of the orbits which are occupied by the extra proton and that by neutrons.

§ 2. Shell structure and β-decay scheme

As for the study of β-decay itself, only forbidden decays are interested.
The analysis of allowed type β-decays are important only to examine the nuclear
shell model.

I. Nuclear energy levels of N, P in nuclei and types of transitions in
β-decay.

1). The nuclei in which the number of protons up to 20 and level states of
neutrons are below f_7/2. (Namely the mass number is up to about 42.)

i. If two odd nuclei have the same mass and their protons and neutrons
exist up to the same shell (of course, numbers of protons and neutrons in
the last shell are different each other), the mass defect of such nucleus is larger
as the nucleus is rich in neutrons.

Here, (12Mg_27, 15Al_27), (14Si_28, A_28), (16P_31, 15Si_31) are exceptions. 14Si_28 and 15P_31
whose spins are, both 1/2, are a discrepancy for the Mayer’s shell model. 12Mg_27
is expected to be a spin discrepancy for the Mayer’s shell model, since 12Mg_27
has 15 neutrons.

i'. Especially, it is important that, in pairs of mirror nuclei, the mass defect
is larger for neutron-rich nucleus.

ii. If two even mass nuclei are of the same mass number and their pro-
tons and neutrons are even and exists up to the same shell (of course, number
of protons and neutrons in the last shell is different each other), the mass defect
is larger as the nucleus is more rich in neutrons.
Here, \((\text{Be}^{10}, \text{C}^{10})\) are exceptions. Since the mass number of these nuclei is so small, the rule shall not hold with these nuclei.

We can determine the order of energy levels of each shell under the light of features mentioned above (see Fig. 4).

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Fig. 4

From Fig. 4 following facts may be suggested. If the two nucleus have the same mass number and their neutrons are up to \(f_{7/2}\) state and their protons are up to \(d\) state, the mass defect is smaller as nucleus is more rich in neutrons, since the energy level of \(f_{7/2}\) is higher than that of \(d\): Indeed, this is indicated in the case of \((_{12}\text{C}^{16}, _{16}\text{O}^{17})\). Also following things must be noticed in light of Fig. 4. For instance, there is no nucleus in which last neutron exists in \(3d\) state and the last proton exists in \(2p\) shell. Generally there appear no nuclei, in which other level exists between two levels in which last neutron and last proton exists. Since such a nucleus has smaller mass defect than other nucleus which has same mass number and smaller number of nucleons which exist in higher level, such a nucleus is able to be \(N\) or \(P\) emitter.**

On account of these facts nuclei which really exist are limited to one of the following types.*** \((4f, 3d), (3d, 3d), (3d, 2s), (2s, 2s), (2s, 2p), (2p, 2p), (2p, 1s), (1s, 1s)\).

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* In Fig. 4, the spacings between energy levels of the same orbits of \(N\) and \(P\) are exactly determined from mirror nuclei, but the spacing of the energy levels of different orbits of \(N\) and \(P\) may be difficult to be estimated.

** Although we must expect same circumstances for the nuclei, the last \(N\) of which exists in \(d\) (or \(p\)) state and the last \(P\) exists in \(2s\) (or \(1s\)) state, but, since only two nucleons are able to enter into \(s\) state, there are not two nuclei which satisfy such a condition.

*** The nuclei, where neutrons exist up to \(q\) state and protons exist up to \(r\) state, are shown by \((q, r)\).
\(\beta\)-decay schemes then, are limited to one of the following types: (4f/2 \(\rightarrow\) 3d), (3d \(\rightarrow\) 3d), (3d \(\rightarrow\) 2s), (2s \(\rightarrow\) 2s), (2s \(\rightarrow\) 2p), (2p \(\rightarrow\) 2p), (2p \(\rightarrow\) 1s), (1s \(\rightarrow\) 1s):

Thus the analysis of \(\beta\)-decay schemes will do come to examine the transition of the above mentioned types.

**Nuclei, with \(P > 21\)**

There is no mirror nucleus in this region. It is, therefore, difficult to estimate exact energy levels. In this region, nuclei which really exist are limited to one of the following types (see Fig. 6):

\[
\begin{align*}
(4f_{7/2}, 4f_{7/2}), & \quad (3p & 4f_{7/2}, 4f_{7/2}), (3p & 4f_{7/2}, 3p & 4f_{5/2}), \\
(5g & 4d & 3s & 6h_{11/2}, 3p & 4f_{5/2}), & \quad (5g & 4d & 3s & 6h_{11/2}, 5g & 4d_{5/2} & 6h_{11/2}), \\
(6h_{9/2} & 5f & 4p & 7i_{13/2}, 5g & 4d & 3s), & \quad (7i_{11/2} & ... , 4d_{5/2} & 3s) (7i_{11/2} & ... , 6h_{9/2} & 5f & 4p & 7i_{13/2})
\end{align*}
\]

Type of nucleus

\[
\begin{align*}
(4f_{7/2}, 3d) & \quad \text{Type of } \beta\text{-decay} \\
(3d, 3d) & \quad (4f_{7/2}) \\
(3d, 2s) & \quad (4f_{7/2}) \\
(2s, 2s) & \quad (4f_{7/2}) \\
(2s, 2p) & \quad (4f_{7/2}) \\
(2p, 2p) & \quad (4f_{7/2}) \\
(2p, 1s) & \quad (4f_{7/2}) \\
(1s, 1s) & \quad (4f_{7/2})
\end{align*}
\]

From these circumstances, we can roughly estimate the order of levels, which may be shown as Fig. 6.

Thus, there will exist \(\beta\)-decays, types of which may correspond to the above mentioned type of nuclei.

In the present discussion, we confine our attention to nuclei with the neutron number \(N < 41\). As far as nuclei with \(N > 41\) are concerned where the \(g_{9/2}\) state begins to appear the theory remains much to be studied. In this paper, only

\*1) \(\beta\)-decay of nucleus \((y, r)\) is shown by \(y \rightarrow r\). For the odd mass nuclei, this corresponds to the transition from \(y\) to \(s\) state (or contrary). For even mass nuclei this is complicated.
Type of nuclei.

\[
\begin{align*}
(7i_{3/2} \ldots, 6h_{5/2} \ldots) & \quad 7i_{3/2} \rightarrow \ldots, \\
(7i_{3/2} \ldots, 2d_{5/2} \ldots) & \quad 6h_{11/2} \rightarrow 6h_{11/2}, 5f \rightarrow 4p, 4p \rightarrow 3p, \\
(6h_{11/2}, 5f, 4p, 7i_{3/2} \ldots) & \quad 6h_{11/2}, 5f, 4p \rightarrow 3p, 2s, 2s \rightarrow 2s, \\
(5g_{5/2}, 4d_{3/2}, 6h_{11/2} \ldots) & \quad 5g_{5/2}, 4d_{3/2}, 3p, 3s, 4f_{7/2} \rightarrow 3p, 4f_{7/2}, \\
(3p, 4f_{7/2}, 4p, 4f_{7/2}) & \quad 3p, 4f_{7/2}, 4f_{7/2} \rightarrow 3p, 3p, \\
(4f_{7/2}, 3d) & \quad 4f_{7/2} \rightarrow 3d.
\end{align*}
\]

Fig. 6 Type of nuclei and $\beta$-decay.

the transition of type $f_{7/2} \leftrightarrow f_{7/2}, 3p \leftrightarrow f_{7/2}, 3p \leftrightarrow 3p$, will be dealt with.

**II. $\beta$-decay scheme** — up to type $(3p, 3p)$. From the Fermi theory of $\beta$-decay, one can readily ascertain the validity of classification of nuclear states that will be obtained from the theory of shell structure. The Fermi interaction, i.e. $\sigma$ as derived from tensor or axial vector coupling. However, for explaining a certain forbidden spectrum such as RaE, more refined types of coupling may be necessary. Hence, in addition to $\sigma$ all the matrix elements that are actually involved in scalar, polar vector, tensor and pseudoscalar couplings, will tentatively be used for the Fermi interaction* throughout this paper. The results will be indicated for odd mass nuclei and even mass nuclei separately as in Fig. 7 and 8. We shall discuss them in the order of specific transitions. Firstly, $\beta$-decay of the even mass nuclei (Fig. 7) then for the even mass nuclei will be discussed.

**Odd mass nuclei**

(i) As regards to the decay of type $1s \leftrightarrow 1s, 2p \leftrightarrow 2p, 2s \leftrightarrow 2s, 3d \leftrightarrow 3d, 4f_{7/2} \leftrightarrow 4f_{7/2}, 3p \leftrightarrow 3p$, direct transition to ground state (namely the highest group of no $\gamma$ rays) must be the allowed transition.

*) It is interesting to note the availability of pseudoscalar coupling. As will be indicated later, the contribution of $f_{7/2}$ as derived from pseudoscalar coupling is limited only in the case (ii). For in (ii), (ii') the allowed transition due to $f_{7/2}$ are rejected on account of the selection rule, because $f_{7/2}$ require zero spin change. In (iii), (iii'), (iv'), the 1st forbidden transition due to $PS$ interaction (matrix element $f_{7/2}f_{7/2}$) are rejected on account of the selection rule, because $f_{7/2}$ require spin change 0 or ±1.
(ii) As regards the decay of type $4f_{7/2} \rightarrow 3d_{5/2}$, direct transition to ground state (namely, highest group followed by no $\gamma$ ray) must be 1st forbidden.

(ii) As regards the decay of type $2s \rightarrow 2p_{1/2}$, direct transitions to ground
states should be 1st forbidden, provided $PS$ interaction does not exist. If $PS$ interaction exists, allowed transition due to $PS$ interaction (matrix element $\langle f | t \rangle$) should be possible.

(iii) As regards the decay of type $3d_{5/2} \leftrightarrow 2s, 3p_{3/2} \leftrightarrow 4f_{7/2}$, direct transition to the ground state must be 2nd forbidden. Then, beside the particular cases, this transition can not be observed.

Even mass nuclei

(i) As regards the decay of type $(2s, 2s)$, direct transition to the ground state must be the allowed transition.

(ii) As regards the decay of nuclei of type $(2s, 2p_{1/2})$, direct transition to the ground state must be 1st forbidden.

(iii) As regards the decay of nuclei of type $(3d_{3/2}, 2s)$, direct transition to the ground states must be the 2nd forbidden.

(iv) As regards the decay of nuclei of type $(4f_{7/2}, 3d_{5/2})$, $(3p_{3/2}, 4f_{7/2})$ direct transitions to ground states must be the 1st, 2nd or more highly forbidden, respectively.

---

\*1 Especially, if the parent nucleus can not afford sufficient energy for transition to the excited state of the daughter nucleus, direct transition to the ground state will be liable to be observed.
In light of Fig. 7 and 8 we can easily ascertain\(^*\) the above things with few discrepancies.\(^{**}\)

1) Odd mass nuclei

Decay of nuclei of the types \((1s, 1s), (2\rho, 2\rho), (2s, 2s)\):

As above expected, direct transition to the ground state are all allowed transitions.

Decay of nuclei of the type \((2s, 2\rho)\):

\(N^{17}\) decays to the excited state of \(O^{17}\), and direct transition to the ground state is not observed (experiment is not yet definite). If \(PS\) interaction does not exist, this is in agreement with the expectation. On the other hand, when \(PS\) interaction exists, we must understand this as follows, namely allowed transition due to \(PS\) interaction (matrix element \(f'_{s}\)) loses in a competition with that to the excited state of \(O^{17}\) and will not be observed since the matrix element \(f_{s}\) is considerably smaller than \(f_{2}\) and \(f'_{5}\).

Decay of nuclei of type \((3d_{6s}, 2s)\):

\(s^{19}\) is in discrepancy. \(f'_{t}\) value of direct transition to the ground state is too small to be that of second forbidden. No. of neutrons is 11, and this is discrepancy for Mayer's shell model. Thus it may not be surprising to expect this discrepancy. The \(f_{t}\) value is between that of the allowed and the 1st forbidden. If this is allowed, there should exist an \(s\) state above \(2s\) state, since transition to the excited state is also allowed.

Decay of nuclei of type \((3d, 3d)\):

As expected, direct transition to the ground state is all allowed transition but \(Mg^{27}, 16Si^{21}.\)**

As regards \(16Mg^{27}\), since No. 15 is discrepancy for Mayer's shell, \(16Mg^{27}\) shall be discrepancy for Mayer's shell. If this is true, it is explained that direct transition to the ground state is not observed.

As regards \(16Si^{21}\), \(f'_{t}\) value of which is between allowed and 1st forbidden, this decay shall be attended by spin change 1 and no parity change and then allowed, since ground nucleus \(15P^{21}\) is in discrepancy for Mayer's shell model and has \(s\) state.

\(^*\) In spite of this success, some difficulties appear as soon as the number of \(N\) exceeds 41. The regions 41-50 are theoretically expected to be \(g_{7/2}\) state. The difficulties are due to the consequences of the following three facts, namely-(1), in the case of \(N\)-shell, states in the region of over 41, have not yet been determined experimentally, (2) in this region, many states with low spin value came in light, (3) in the region of over 41, decay scheme is very complex on account of small energy space between relevant levels. These shall be treated in the later paper.

\(^{**}\) In the case of \(\beta\)-decay discrepancy, either parent, or daughter nucleus should be discrepancy for Mayer's shell model.

\(^{***}\) Beside these ground nucleus of \(Al^{20}\) decay is \(Si^{20}\) which is discrepancy for Mayer's shell Model. Then \(15Al^{20}\) decay will probably be in discrepancy and be followed by \(\gamma\)-ray, although the evidence is not definite.
Decay of nuclei of type \((4f_{7/2}, 3d_{5/2})\):
As above expected, direct transitions to ground states are all 1st forbidden. Moreover, these transitions require spin change 2 and parity change, then only matrix element \(B_{ij}\) is able to rise these transitions. Then the shape of spectrum of \(\beta\)-decay must be \('a'\)-shape. Indeed \(A^{36}\) has \('a'\)-shape spectrum. This is a very fair success.

Decay of nuclei of type \((3p_{3/2}, 4f_{7/2})\):
As above expected, direct transitions to ground states are all allowed transitions but \(_{28}\)Ca\(^{47}\), \(_{24}\)Cr\(^{49}\).

\(_{28}\)Cr\(^{49}\) may probably be in discrepancy for Mayer's shell model.
It is not yet determined that either of \(_{47}\)Ca\(^{7}\) and \(_{47}\)Sc\(^{7}\) is in discrepancy.

Decay of nuclei of type \((3p_{3/2}, 4f_{7/2})\):
As above expected, \(_{56}\)Fe\(^{58}\) and \(_{56}\)Co\(^{58}\) decay can not go directly to the ground state.

\(_{57}\)Co\(^{58}\) is in discrepancy. Either of \(_{57}\)Ni\(^{57}\) and \(_{57}\)Co\(^{57}\) must be in discrepancy for Mayer's shell.

\(_{57}\)Ni\(^{57}\) should be in discrepancy if this decay is not accompanied by \(\gamma\)-ray.

Decay of nuclei of type \((3p, 3p)\):
As above expected, direct transitions to ground states are allowed transitions with few discrepancies.

\(_{30}\)Zn\(^{66}\), \(_{57}\)Ni\(^{57}\) are in discrepancy. According to Mayer's shell model, \(_{30}\)Cu\(^{64}\) is experimentally known to have \(\rho_{3/2}\) state. Then \(_{30}\)Zn\(^{66}\) and \(_{57}\)Ni\(^{57}\) shall probably be in discrepancy for Mayer's shell model.

(2) \textit{Even mass nuclei}\)

Decay of nuclei of type \((2s, 2s)\):
As above expected, direct transitions to ground states are all allowed.

Decay of nuclei of type \((2s, 2p)\):
As above expected, direct transitions to ground states are all 1st forbidden.

Decay of Nuclei of type \((3d_{3/2}, 2s)\):
As above expected, direct transitions to ground states are not observed.

Decay of nuclei of type \((4f_{7/2}, 3d_{5/2})\):
As above expected, direct transitions to ground states are 1st forbidden \((K^{42}, C1^{38})\) or 3rd forbidden, \((K^{40})\).

Moreover, decay \((4f_{7/2}, 3d_{5/2})\) are attended with more than two spin change. Then direct transitions to the ground state are able to rise only by the matrix element \(B_{ij}\) which belongs to axial vector or tensor interaction. Then the shape of spectrum must take \('a'\)-shape.

\(^*)\) As regards the scheme of \(\beta\)-decay, especially the excited states of daughter nuclei, the nuclei which belong to the same type are very similar to each other (of course, these are completely different from those which lie in the different type). These things will be treated with the analysis of excited states in a later paper.
Indeed, \( ^{42}\text{K} \) and \( ^{32}\text{Cl} \) have the spectrum of shape ' \( a \) '. This is a fair success.

Decay of nuclei of type \((3p_{3/2}, 4f_{7/2})\) :

As above expected, direct transitions to the ground states are not observed.

(3) Estimation of nuclear spin and decay matrix element

Odd mass nuclei.

As regards the odd mass nuclei, it is needless to say, since matrix elements have been determined by spin and parity corresponding to the type of nuclei.

But it is worth while to notice that decay of nuclei of type \((f_{7/2}, 3d_{5/2})\) is accompanied, by 2 spin change and parity change. Indeed, \( ^{39}\text{A} \) is known to have ' \( a \)'-shape spectrum. \( ^{41}\text{A} \) and \( ^{57}\text{S} \), which belong to \((f_{7/2}, 3d_{5/2})\) and whose spectrum are not determined experimentally, must have spectrum of shape ' \( a \)' and arise due to matrix element \( B_{ij} \).

Even mass nuclei.

As previously mentioned, we assumed that the nuclei of type \((q, r)\) have various spin value \( J \) which \( |J_0 - J_R| \leq J \leq J_0 + J_R \) and are not definite. Then, it is necessary to estimate spin and decay matrix element for each nucleus. When nucleus has considerably small \( J \) value, we can estimate spin value \( J \) for each nucleus as follows. Moreover, we can estimate matrix elements pretty well too, in view of the fact that ground nuclei (even-even nuclei) always spin zero.

Decay of nuclei of type \((2s, 2s)\) :

\( ^{18}\text{F} \) is obviously estimated to have spin 1 and decay due to matrix element \( \sigma \).

Decay of nuclei of type \((2s, 2p_{1/2})\) :

This is the decay which is attended by 1 spin change and parity change and is 1st forbidden. Then \( ^{18}\text{N} \) is estimated to have spin 1. (There are several matrix elements corresponding to this decay). (see Konopinski.)

Decay of nuclei of type \((3d_{5/2}, 2s)\) :

Nuclei of this type have spin 2-3. \( ^{20}\text{F} \) is 2nd or more highly forbidden then \( ^{20}\text{F} \) is estimated to have spin \( J=2 \) or 3 (there are several of matrix element corresponding to this decay.)

Decay of nuclei of type \((4f_{7/2}, 3d_{5/2})\) :

According to assumptions previously mentioned as regards the decay of this type, direct transitions to ground states are attended by 2 or more spin change and parity change. Then, direct transitions to ground states which are 1st forbidden, be able to rise only due to matrix element \( B_{ij} \), and must have spectrum of shape ' \( a \)'.

*) Then there are many transitions which are forbidden, but to the higher order, in point of fact, almost all the parent nuclei have sufficient energy for transition to excited state of ground nuclei. Then the direct transition to ground state lose in a competition with transitions to excited state and is not observed in experiment. Merely, direct transitions to ground states are observed. As regards \( ^{13}\text{Be}, \ ^{38}\text{C}, \ ^{33}\text{Cl}, \ ^{19}\text{K} \), since these have not sufficient energy for transitions to excited states of ground nuclei, direct transition to ground state are observed.
This is a very important touchstone! Cl$^{38}$ and K$^{42}$ are 1st forbidden and have spectrum of shape ‘$a$’. Well done! (see also Decay of odd nuclei of this type).

Decay of nuclei of type $(3f_{3/2}, 4f_{5/2})$:

As regards Co$^{60}$, Mn$^{54}$, Co$^{62}$, V$^{52}$, direct transitions to ground states are not yet observed. Then, as above expected these are 2nd or more highly forbidden.

Then these nuclei are estimated to have spin $J$, $5 \geq J \geq 3$.

Decay of nuclei of type $(2p, 2p)$:

According to assumption, the nuclei of this type have spin $J$, $1 \leq J \leq 3$. As regards Li$^6$, Li$^7$ and probably B$^{11}$ too, direct transitions to ground state are allowed transitions, then are estimated to have spin 1 and decay due to matrix element $\{a\}$. But $\{a\}$ requires parity change. This is in contradiction to the assumption that decay of $(3d, 3d)$ type requires no spin change. From these circumstances, although $\{a\}$ value are large, P$^{22}$ and Cl$^{34}$ may be allowed transitions.

As regards Na$^{24}$, Al$^{26}$, K$^{38}$, the direct transitions to ground state are not observed in experiment then are probably 2nd or more highly forbidden. Then P$^{24}$, Al$^{26}$ are estimated to have spin $5 \geq J \geq 2$, and K$^{38}$, $3 \geq J \geq 2$.

Decay of nuclei of type $(4f_{7/2}, 4f_{9/2})$:

According to assumptions, nuclei of this type have spin $J$, $1 \leq J \leq 7$.

As regards Sc$^{44}$ (Fe$^{58}$ decay schemes are not accurate) direct transitions to ground states are allowed transitions. Then Sc$^{44}$ (Fe$^{58}$) are estimated to have spin 1 and decay due to matrix element $\{a\}$.

As regards Sc$^{46}$, V$^{56}$, Mn$^{52}$, Sc$^{44}$ direct transitions to ground state are not observed, then, these are estimated to have spin $J$, $7 \geq J \geq 2$.

Decay of nuclei of type $(3p, 3p)$:

According to assumptions, nuclei of this type have spin $J$, $1 \leq J \leq 3$.

As regards Cu$^{64}$ (Ga$^{68}$ decay scheme is not accurate) direct transitions to ground states is allowed, then Cu$^{64}$ is estimated to have spin 1, and decay due to matrix element $\{a\}$.

As regards Cu$^{60}$, As$^{72}$, Cu$^{60}$, Ga$^{70}$, direct transitions to ground states are not observed, then shall be 2nd or more highly forbidden. Then Cu$^{60}$, As$^{72}$, Cu$^{60}$ Ga$^{70}$ have spin 2 or 3.
III. Selection rules and spectra in the forbidden case

1) 2nd or more highly forbidden case*

In this case, decay due to PS interaction is not observed, since, $n$-th forbidden decay due to PS interaction loses in competition with $n-1$-th forbidden decay due to other interactions.

$^{40}$K; $^{40}$K belongs to ($f_7/2$, $d_3/2$) type, and has spin 4, then $^{40}$K decay must be 3rd forbidden and rise only due to matrix element $S_{ijk}$. Indeed this is the fact. $^{36}$Cl; $^{36}$Cl belongs to type ($2d$, $2d$), and has spin 2. Then $^{36}$Cl-decay must be 2nd forbidden. Since matrix element $S_{ijk}$ are forbidden for the case $2\rightarrow 0$, $^{36}$Cl cannot decay due to $S_{ijk}$, although its shape of spectrum is similar to $c$-shape, and only $A_{ij}$, $T_{ij}$, $R_{ij}$ are able to contribute to this decay.

$^{18}$Be; $^{18}$Be belongs to type ($2p_{3/2}$, $2p_{1/2}$) and has spin 3. Then $^{18}$Be decay must be 2nd forbidden and rise only due to matrix element $S_{ijk}$. Indeed this is the fact.

$^{4}$Cb, $^{14}$C: ghost haunts! Although they have very large $ft$ values, the shape of energy spectrum are similar to that of allowed type, yet unsolved problem these are.

2) Case of 1st forbidden and shape ‘a’. Up to the present, 8 elements (see Fig. 9) are found to belong to this case.

Among these, $^{90}$Y, $^{90}$Sr, $^{90}$Sr, $^{90}$Y and $^{137}$Cs are solved by Feenberg and Nordheim. The situations are also adequate in Mayer's shell model. $^{38}$Cl, $^{40}$K and $^{90}$Sr are solved on this paper.

3) Case where shape are allowed but $ft$ are larger.

In this case we have not arrived to conclusion. Following possibilities are

<table>
<thead>
<tr>
<th>element</th>
<th>$W_0$ (MeV)</th>
<th>$\tau$ (sec)</th>
<th>Type of spectra</th>
<th>$ft$</th>
<th>$\bar{ft}$</th>
<th>Matrix element</th>
<th>Selection Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{90}$Y</td>
<td>5.29</td>
<td>2.2 x $10^5$</td>
<td>a</td>
<td>$1.8 \times 10^8$</td>
<td>$3 \times 10^7$</td>
<td>$B_{44}$</td>
<td>$\Delta J = \pm 2$</td>
</tr>
<tr>
<td>$^{90}$Sr</td>
<td>2.05</td>
<td>8 x $10^6$</td>
<td>a</td>
<td>$7 \times 10^7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{90}$Sr</td>
<td>3.88</td>
<td>4.7 x $10^6$</td>
<td>a</td>
<td>$3.8 \times 10^8$</td>
<td>$6.2 \times 10^7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{90}$Y</td>
<td>4.01</td>
<td>5.2 x $10^6$</td>
<td>a</td>
<td>$7 \times 10^7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{38}$Cl</td>
<td>10.43</td>
<td>2.3 x $10^6$</td>
<td>a</td>
<td>$5 \times 10^7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{42}$K</td>
<td>8.0</td>
<td>4.5 x $10^4$</td>
<td>a</td>
<td>$1.3 \times 10^6$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{39}$A</td>
<td>2.11</td>
<td>4.5 x $10^6$</td>
<td>a</td>
<td>$&lt; 4.5 \times 10^7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>2.01</td>
<td>1.2 x $10^6$</td>
<td>a</td>
<td>$4.7 \times 10^8$</td>
<td>$8.7 \times 10^7$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$ft$ shows corrected value on account of its shape a.

* RaE$^{210}$ is treated in the later paper.
The transitions of about 1st forbidden $ft$ value and of allowed shape spectra.

<table>
<thead>
<tr>
<th>element</th>
<th>$W_0$(me$^3$)</th>
<th>$r$(sec)</th>
<th>$ft$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pm$^{147}$</td>
<td>1.44</td>
<td>$1.2 \times 10^6$</td>
<td>$1.8 \times 10^6$</td>
</tr>
<tr>
<td>$^{23}$V$^{48}$</td>
<td>$t. e$</td>
<td>2.42</td>
<td>$1.4 \times 10^6$</td>
</tr>
<tr>
<td>$^{39}$P$^{123}$</td>
<td>$t. g$</td>
<td>2.83</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>$^{75}$Re$^{186}$</td>
<td>$t. e$</td>
<td>3.1</td>
<td>$3.2 \times 10^6$</td>
</tr>
<tr>
<td>$^{11}$Na$^{22}$</td>
<td>$t. e$</td>
<td>2.12</td>
<td>$8.2 \times 10^7$</td>
</tr>
<tr>
<td>$^{15}$P$^{32}$</td>
<td>$t. g$</td>
<td>4.37</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>$^{169}$Tm$^{179}$</td>
<td>$t. g$</td>
<td>2.94</td>
<td>$1.2 \times 10^7$</td>
</tr>
</tbody>
</table>

$t. e$ shows the transition to excited state,
$t. g$ " ground ".

V. Concluding remarks

Through the whole of this paper, mirror nuclei fall in the neighborhood of $ft=5 \times 10^3$. The region of $ft=10^6$ are filled by the other allowed transitions. The difference of $ft$ values may be due to the incomplete overlapping of the radial wave functions. The 1st forbidden decays fall in the region of $ft=10^7$.

As shown in Fig. 10, density-$ft$ curve has four peaks; namely,

$A$ .......... $ft \sim 5 \times 10^3$—almost of these are mirror nuclei,
$B$ .......... $ft \sim 2 \times 10^4$—the other allowed transition,
$C$ .......... $ft \sim 5 \times 10^4$,
$D$ .......... $ft \sim 5 \times 10^5$.

From this situation, almost all elements in $C, D$ have been considered to be the 1st forbidden; the elements which have 'a'-shape spectrum are all in $D$, their $ft$ values being almost the same order if one uses the corrected 1st forbidden $f$-functions.

In conclusion, we would like to express our sincere thanks to Prof. Muneo Sasaki for his valuable discussions.
Fig. 10 $\beta$-density curve of $\beta$-decay throughout the whole.

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