Note on Some Calculations in Quantum Field Theory

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June 13, 1951

In a recent paper Yukawa and Umezawa have calculated $\langle \mathbf{d}^3 x \phi^* \beta \psi \rangle$ for several coupling schemes. In this note we wish to show that, contrary to the impression given...
in their paper, their results can be derived using the methods of Pais and Epstein, i.e. using the general theorem:\(^\text{1)}\)

\[
\left\langle \frac{\partial H}{\partial \mu} \right\rangle = - \left\langle \frac{\partial H}{\partial \mu} \right\rangle,
\]

where \( H \) is the hamiltonian and \( \mu \) is a parameter which we take, for our purposes, to be the mass of the \( \phi \) field.

For the problem at hand \( H \) is, in obvious notation:\(^\text{2)}\)

\[
H = H(\phi) + H(\phi) + g \int d^4x \\
\times \left( \phi^a O \phi - \langle \phi^* O \phi \rangle_{\text{vac}} \right) \phi
\]

(we have omitted any possible tensor indices) from which it follows that

\[
\left\langle d^4x \phi^* \phi \right\rangle = \frac{\partial \langle H \rangle}{\partial \mu}
\]

\[+\frac{g}{2} \int d^4x \langle \phi \rangle \frac{\partial}{\partial \mu} \langle \phi^* O \phi \rangle_{\text{vac}}.
\]

1) J. Yukawa and H. Umezawa, Prog. Theor. Phys. 6 (1951), 112.


In applying this result we will be interested in

\[
\left\langle \frac{\partial H}{\partial \mu} \right\rangle_{\text{vac}+1} - \left\langle \frac{\partial H}{\partial \mu} \right\rangle_{\text{vac}}
\]

whence additive \( C \) numbers in \( H \) will play no role.

3) This subtraction of the vacuum current is implicitly implied in all calculations made with these theories since, in the language of Feynman diagrams, one always omits diagrams in which a point connects directly with itself (see F. J. Dyson, Phys. Rev. 75 (1949), 486.) If one would include such diagrams then, of course, most quantitative results would be changed.

4) J. Schwinger, Phys. Rev. 75 (1949), 651.