Direction of Approach of Microseisms

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Summary

Three methods for determination of direction of arrival of microseisms (Jensen's method, the amplitude method and Teisseyre–Siemek's method), which can conveniently be applied to standard seismograph records in three matched components, have been investigated in this paper. Theories are given for all methods with special emphasis on limitations in the use of the methods and the influence of different wave types. The methods are applied to the Uppsala long-period records in two cases, each with a single source, representing two typical cases of Scandinavian microseisms. The results agree well within error limits, and the errors are comparable in the three methods. Jensen's method and the amplitude method are preferred because of less evaluation work and fewer theoretical restrictions in the application, compared to the third method. Those methods are just as reliable as any other methods, including the tripartite station method, as the spread in the observations are in our cases mainly caused by arrival of Rayleigh waves from several directions. Cases with two or more sources can also be analysed by Jensen's method or the amplitude method, provided the angular separation between the sources is sufficiently large and that the sources are of comparable strength.

1. Introduction

The author's earlier investigations of microseisms in the years 1947–1953 were based on older type seismographs, which were practically the only ones existing in the Scandinavian area at that time. As modern and much more efficient instrumentation has since then been set up, I consider it to be of interest to continue the research of microseisms by these new means. Especially the long-period Columbia seismographs (Press–Ewing, E & N, Sprengnether, Z) with a pendulum period of 15 s and a galvanometer period of 85–90 s, in operation at Uppsala since the IGY, have shown to be very useful for microseism studies in the period range of 3–10 s.

Quite a number of different techniques for determination of the direction of approach of microseisms have been developed in recent years. Some of these methods permit a simple and reliable direction determination from the records of three matched components, excluding the use of special seismographs or elaborate technique for the evaluation of the measurements. As such simple methods can be applied to many stations, they have an undisputable value. In this paper I
shall test and compare three such methods, i.e. Jensen's method (1958, 1959, 1961), the amplitude method and Teisseyre-Siemek's method (1960). The amplitude method has been given by Blaik & Donn (1954) and by Hollinderbäumer (1959), but these authors do not use the vertical component. The amplitude method as applied here includes all three components. The vertical component is used in the selection of the times for measuring the horizontal amplitudes and for unambiguous determination of the direction of approach.

2. Theory for the direction methods used

Jensen (1958, 1959, 1961) has not given any theory for his method, but in his texts he has clearly stated the conditions under which his method is valid. We start by giving a theory for his method and introduce the following notation:

\[ x, y, z = \text{rectangular coordinates directed eastwards, northwards and up respectively with the station at the origin}, \]
\[ H = \text{horizontal (occurs only as index to denote horizontal component)}, \]
\[ \alpha = \text{direction of approach of microseisms, counted from north over east}, \]
\[ R, Q = \text{Rayleigh and Love wave amplitudes respectively}, \]
\[ t = \text{time}, \]
\[ \omega = \text{angular frequency (assumed to be the same for } R \text{ and } Q), \]
\[ U = \text{displacement, with components } U_x, U_y, U_z, \]
\[ \gamma = \text{phase displacement between } R \text{ and } Q \text{ waves}, \]
\[ n = \text{number of observations included in one direction determination}. \]

The method assumes the microseisms to be a composite wave motion, consisting of Rayleigh and Love waves. The \( x \) and \( y \) components of \( R \) and \( Q \) can then be written as follows:

\[
\begin{align*}
R_x &= RH \sin \alpha; \quad Q_x = Q \cos \alpha \\
R_y &= RH \cos \alpha; \quad Q_y = Q \sin \alpha.
\end{align*}
\]

(1)

The displacements at time \( t \) along the coordinate axes are

\[
\begin{align*}
U_x &= R_x \sin \omega t + Q_x \sin(\omega t + \gamma) \\
U_y &= R_y \sin \omega t + Q_y \sin(\omega t + \gamma) \\
U_z &= R_z \sin(\omega t - \frac{\pi}{2}).
\end{align*}
\]

(2)

Derivation with regard to time gives

\[
\begin{align*}
\frac{dU_x}{dt} &= \frac{dR_x}{dt} \sin \omega t + \omega R_x \cos \omega t + \frac{dQ_x}{dt} \sin(\omega t + \gamma) + \\
&\quad + \left( \omega + \frac{d\gamma}{dt} \right) Q_x \cos(\omega t + \gamma)
\end{align*}
\]

(3)

\[
\begin{align*}
\frac{dU_y}{dt} &= \frac{dR_y}{dt} \sin \omega t + \omega R_y \cos \omega t + \frac{dQ_y}{dt} \sin(\omega t + \gamma) + \\
&\quad + \left( \omega + \frac{d\gamma}{dt} \right) Q_y \cos(\omega t + \gamma).
\end{align*}
\]
As moments for measuring the slopes on the horizontal records, Jensen chooses $|U_x| = \max$, i.e. $t = 0$. Then equations (3) become

$$\begin{align*}
\frac{dU_x}{dt} &= \omega R_x + \frac{dQ_x}{dt} \sin \gamma + \left(\omega + \frac{dy}{dt}\right) Q_x \cos \gamma \\
\frac{dU_y}{dt} &= \omega R_y + \frac{dQ_y}{dt} \sin \gamma + \left(\omega + \frac{dy}{dt}\right) Q_y \cos \gamma.
\end{align*}$$

(4)

As we have good reason to assume the $R$ and $Q$ waves to be uncorrelated with each other, the phase angle $\gamma$ assumes all values between $-\pi$ and $+\pi$. For a sufficiently large number of observations, we then have

$$\begin{align*}
\sum_n \sin \gamma &= \sum_n \cos \gamma = 0; \\
\sum_n \frac{dU_x}{dt} &= \omega R_x; \quad \sum_n \frac{dU_y}{dt} = \omega R_y
\end{align*}$$

(5)

and finally

$$\sum_n \frac{dU_x}{dt} / \sum_n \frac{dU_y}{dt} = R_x/R_y = \tan \alpha.$$  

(6)

It is immediately obvious that this development is valid also for different angular frequencies of $R$ and $Q$. The method is valid if the microseisms consist of uncorrelated $R$ and $Q$ waves. We shall investigate this a little further in the light of other findings.

(a) Monakhov (1959) found from an azimuthal seismograph installation at Yalta that the microseisms consist almost exclusively of Rayleigh waves, mainly with inclined planes of vibration. The theoretical possibility of such waves has been demonstrated by Caloi (1951), and they have also been found in explosion records. In this case the transverse component can still be denoted $Q$, but this $Q$ component is then correlated to the $R$ waves, i.e. equations (5) and (6) are not valid.

(b) In case the microseisms are continental channel waves, $Lg$ and $Rg$ (Gutenberg 1958a, 1958b; Båth 1956), it is possible to develop an analogous theory, considering that a $Q$ motion dominates in $Lg$ but that also longitudinal and vertical components exist. Jensen’s method is then still valid.

(c) Some microseisms may be higher-mode surface waves. A clear example of this is an observation by Gutenberg (1958b) of two-second microseisms in California with mainly a vertical component. According to our experience such cases are rare, and Jensen’s method is not affected.

(d) If the microseisms consist of $R$ waves simultaneously arriving from different directions, it is also possible to develop an analogous theory. As the directions are usually not perpendicular to each other, equations (1) and (2) and the following are changed. It is possible to determine the individual directions by Jensen’s method, provided that the angular separation between the sources considerably exceeds the errors in both determinations, and provided that the two sources produce microseisms of comparable strength, as microseisms from a comparatively weak source will be masked.
In the amplitude method the measuring moments correspond to $U_z = 0$ in well developed wave groups on the vertical component, and the horizontal amplitudes are measured. If we put $U_z = 0$ in equations (2) we find that

$$
\begin{align*}
U_x &= R_x + Q_x \cos \gamma \\
U_y &= R_y + Q_y \cos \gamma.
\end{align*}
$$

(7)

Again if $\gamma$ is arbitrarily distributed, the mean ratio $U_x/U_y$ of a sufficiently large number of observations will give the direction to the source.

Teisseyre & Siemek (1960) have given a theory for their method in their paper. Like Jensen, they use $R$ waves, but as measuring moments they select $U_z = 0$ in well developed wave groups on the vertical component. They arrive at the following vector equation, equivalent to equation (7):

$$
\frac{U_H}{R_z} = \frac{R_H}{R_z} + \frac{Q}{R_z} \cos \gamma
$$

(8)

which corresponds to a straight line in a coordinate system with $U_x/R_z$ and $U_y/R_z$ as axes, under the conditions that $R_H/R_z$ is constant and the $Q$ waves are polarized perpendicularly to $R_H$. The arrival direction is perpendicular to this straight line.

It is to be noted that the existence of $Q$ waves is necessary for a successful application of Teisseyre–Siemek’s method, whereas they are not required in the other two methods.

The three methods are summarized in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>Method</th>
<th>Measuring moment</th>
<th>Quantities measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jensen’s method</td>
<td>$U_s = \text{maximum}$</td>
<td>$dU_x/dt, dU_y/dt$</td>
</tr>
<tr>
<td>Amplitude method</td>
<td>$U_z = 0$ in well developed wave group</td>
<td>$U_x, U_y$</td>
</tr>
<tr>
<td>Teisseyre–Siemek’s method</td>
<td>$U_z = 0$ in well developed wave group</td>
<td>$U_z, U_y, R_z$</td>
</tr>
</tbody>
</table>

**3. Application to individual cases**

(a) A single source.

The three methods of direction determination have been applied to two cases, both with one dominating source of microseisms. The two cases represent two totally different situations: one of them (1961 March 27) with a typical coast effect from the Norwegian coast and a cyclone centre situated in Northern Europe, the other (1960 December 30) with an Atlantic origin and a cyclone centre over the Atlantic between the British Isles and Iceland. The results from the study of the long-period Columbia records at Uppsala of these two cases are presented in Figures 1 and 2 and in Table 2.

All errors given in Table 2 are standard errors of the mean. The results by the Teisseyre–Siemek’s method (T.–S.) have been computed by the method of least squares.

It is evident from Table 2 that all three methods yield results which agree within their limits of error. The errors are comparable in the cases studied, but...
in the case of Norwegian coast microseisms the amplitude method gives a range of $\alpha$ which is considerably larger than from Jensen's method. This is probably due to increased precision in the slope measurements with larger amplitudes. In the case of Atlantic microseisms, the range is almost exactly the same in both methods. Still, the range is in both these cases with one dominating source considerably smaller than 180° ('the empty half-plane' according to Jensen).

FIG. 1.—Direction determinations for 1961 March 27, 06.09-07.07 GMT, at Uppsala:

I. Jensen's method (dots and dashed lines, indicating mean and extreme directions) and the amplitude method (circles and full lines).

II. Teisseyre–Siemek's method with its least-squares solution.

III. Portion of the long-period vertical record at Uppsala.

IV. Weather situation on 1961 March 27, at 06h GMT (Swedish weather map) with the directions from I. included. Isobars are drawn for every fifth millibar.

The unit for $U_x, U_y$ is mm trace amplitude, and for $dU_x/dt, dU_y/dt$ it is cm/min.
Table 2

Results from measurements of two cases, each with a single source

<table>
<thead>
<tr>
<th>Date</th>
<th>Time interval measured GMT</th>
<th>Number of observations</th>
<th>Wave period</th>
<th>$U_H/R_z$</th>
<th>Method</th>
<th>Direction determinations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0609–0707</td>
<td>55</td>
<td>~ 6.4</td>
<td>0.98 ± 0.02</td>
<td>Jensen</td>
<td>$-42.5 \pm 1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ampl.</td>
<td>$-44.5 \pm 1.8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>T.–S.</td>
<td>$-44.1 \pm 1.1$</td>
</tr>
<tr>
<td>1960 December 30</td>
<td>2002–2309</td>
<td>50</td>
<td>~ 9.6</td>
<td>0.69 ± 0.02</td>
<td>Jensen</td>
<td>$-91.2 \pm 3.7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ampl.</td>
<td>$-91.7 \pm 3.7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>T.–S.</td>
<td>$-95.9 \pm 2.9$</td>
</tr>
</tbody>
</table>
From the practical point of view, Jensen's method and the amplitude method are equivalent, requiring about the same amount of work. In this respect the Teisseyre–Siemek's method is inferior to the other two, partly because it requires one more quantity to be measured, partly because the points are so scattered in the diagrams that a reliable solution can only be found by a least-squares calculation, which is more time-consuming than the calculations of mean directions in the other methods. A theoretical drawback of Teisseyre–Siemek's method is that the existence of $Q$ waves polarized perpendicularly to $R_H$ is required. If the $Q$ waves are unimportant or polarized in all directions (as has been demonstrated by Iyer

FIG. 2.—Direction determinations for 1960 December 30, 20.02–23.09 GMT, at Uppsala:

I. Jensen's method (dots and dashed lines) and the amplitude method (circles and full lines).
II. Teisseyre–Siemek's method with its least-squares solution.
III. Portion of the long-period vertical record at Uppsala.
IV. Weather situation on 1960 December 31, at 06$^h$ GMT with the directions from I. included.
Direction of approach of microseisms

1958), this method breaks down. The other two methods are not subject to these limitations in their use.

The range $\Delta \alpha$ is explained to some extent by an admixture of $Q$ waves, but is mainly due to arrival of $R$ waves from different directions. This is explained by the aerial extent of the source and incoherent wave propagation, i.e. propagation in form of wave packets. On direct inspection of the records it is obvious that pure $Q$ waves are very rare; for the case 1961 March 27, only two well developed $Q$ wave groups were found in the interval investigated; they give the directions $\alpha = -42^\circ.8$ and $-42^\circ.6$; likewise, in the case 1960 December 30, two $Q$ wave groups were found, which give an arrival from due W. Comparison of the values of $U_{H/R}$ for the two cases (Table 2) is also of interest. For the Atlantic microseisms (1960 December 30) this ratio agrees almost exactly with the theoretical value of $R_{H/R}$ on a homogeneous medium, which may indicate only a small admixture of $Q$ waves.

The directions $\alpha$ have a normal frequency distribution in the case of 1961 March 27, but for the case of 1960 December 30, both Jensen’s method and the amplitude method give directions with a double-humped frequency curve. The two maxima correspond roughly to $\alpha = -60^\circ$ and $-110^\circ$ to $-115^\circ$. The material is not statistically sufficient to demonstrate that two simultaneous sources are active. However, from the weather situation this does not seem to be excluded. The sources would then correspond respectively to an open ocean area south of Iceland, where rapidly changing wind directions would be able to produce standing ocean waves, and to the coastal areas of the western part of the British Isles. The microseisms from these areas have longer periods than those from the Norwegian coastal area (Table 2). The difference in period is so obvious that it permits an immediate distinction between the two cases upon inspection of the records. In the case of the Atlantic microseisms, there is an admixture of shorter period microseisms of much less amplitude, probably of much nearer origin. This may contribute to the greater range in $\alpha$ for this case. Similar clear cases of Atlantic microseisms have recently been recorded at Uppsala on 1960 November 11 and 24, and 1961 February 2–3.

The microseisms from the Norwegian coastal area produce the largest amplitudes in Scandinavia. It is clear from Figure 2 that the range in the directions determined comprises just that portion of the Norwegian coast, where we have the strongest winds nearly perpendicular to the coast, but that only a small fraction of the cold air sector over Northern Atlantic is included.

In both cases there is a time-lag of at least 12 hours between the development of the weather situation and the increase of the microseisms. A third type of microseisms, probably from near-by sources, is recorded from time to time at Uppsala. They are of short period, usually around 3 s. A typical case occurred in the night between 1961 July 14 and 15.

(b) Two or more simultaneous sources.

Only Jensen’s method has been applied to this case, but the amplitude method can be just as easily used. The requirements for successful direction determinations in such cases were described in Section 2, and only cases which fulfil these requirements have been investigated.

The first case (1958 January 10, 04.01-04.02 GMT, 12 observations, Figure 3) was given already by Jensen (1958, p. 16). The three distinct point groups as shown in Jensen’s graph were interpreted by him as $R$ waves from a northerly direction accompanied by $Q$ waves, which give rise to the eastern and
western point groups. Considering the fact that $Q$ waves are relatively unimportant in comparison with $R$ waves, it is more likely that all three point groups represent $R$ waves from three simultaneous sources of comparable strength. A comparison with the weather situation confirms this view. The three directions from Copenhagen ($\alpha = +75^\circ$, $-10^\circ$ and $-80^\circ$ respectively) all point towards cyclones.

![Figure 3. Weather map for 1958 January 10, at 06 GMT with the directions to Copenhagen as obtained from Jensen's (1958) measurements.](https://academic.oup.com/gji/article-abstract/6/4/450/663732)

The second (case 1961 February 8, 05.30-06.30 GMT, 55 observations, Figure 4) was investigated on the long-period Columbia records at Uppsala. The situation was selected from the weather maps so that it seemed likely that two different sources of comparable strength and sufficient angular separation were active. The results by Jensen's method (Figure 4) clearly demonstrate that two different directions prevail, well separated from each other, and each pointing into the cold air sectors of two different cyclones. By direct inspection of the records we find that the waves are definitely $R$ waves from both directions, and not $R$ waves from one direction accompanied by $Q$ waves. In this case there is no essential difference in periods between the microseisms from the two sources. In other cases with clear period differences, this may be helpful in separating the directions.

4. Conclusions

Our conclusions may be summarized in the following points:

(a) 'The three methods of direction determination studied in this paper (Jensen's method, the amplitude method, Teisseyre-Siemek's method) can all be applied to standard seismograph records in three matched components, and do not require any elaborate technique for the evaluation. The two first-mentioned methods are superior to the third, partly because they require less work, partly because they are applicable also if $Q$ waves do not exist or are not polarized perpendicularly to $R_H$. 

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FIG. 4.—Direction determinations for 1961 February 8, 05.30–06.30 GMT, at Uppsala by Jensen’s method (upper figure) and the weather map for 1961 Feb 8, 06th GMT, with the mean directions to Uppsala included.
(b) Applied to the same cases, all three methods yield results which agree well within the error limits. This is not surprising, as the methods are related to each other. The errors are comparable in the three methods. A time interval of one to three hours on the records is usually sufficient for the determination of a reliable direction, comprising about 50 observations. Considering the fact that in our cases the spread in the direction is mainly explained by arrival of $R$ waves from different directions, due to aerial extent of the source and to incoherent wave propagation, we may state that these methods are just as reliable as any other method for direction determinations, including the tripartite station method.

(c) Jensen's method and the amplitude method can conveniently be used to separate two or more simultaneous sources, provided that they have an angular separation well exceeding the spreads in each direction determination and that the sources are of comparable strength.

(d) As these methods, especially Jensen's and the amplitude method, have proved to be both simple and reliable, it is my belief that much useful information on microseisms can be obtained by studies of simultaneous records at a large number of stations, in specially selected cases as well as statistically, with the use of one of these methods, combined with studies of time variations of amplitudes, periods, and weather situation. A sufficiently dense net of stations would permit studies of the propagation in more detail, including lateral refractions and reflexions of the waves.

5. Acknowledgments

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