The Earth’s Distant Magnetic Field

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Summary

Low energy corpuscular radiation from the Sun interacts with the geomagnetic field and produces the magnetosphere. The magnetosphere is a region about the Earth in which the geomagnetic field is confined. When the corpuscular radiation is subsonic the magnetosphere is approximately spherical in shape. In the case of hypersonic corpuscular radiation the magnetosphere is highly asymmetrical and has the shape of a tear-drop with the tail-end pointing away from the Sun. The quantitative treatment is approximate and for hypersonic flow the Newtonian fluid theory is used. If $pU^2$ is the momentum flux of a hypersonic corpuscular radiation, it is shown that the day-side of the magnetospheric surface is at a distance proportional to $(pU^2)^{-1/6}$ and the tail-end is at a distance proportional to $(pU^2)^{1/6}$. The little that is known about the gegenschein is consistent with the idea that it is located within the magnetosphere, either in the extreme tail-end or in the “magnetic-arch”. Within the framework of an asymmetric magnetosphere there are acceleration mechanisms for accelerating protons to an energy of 100 keV and electrons to an energy of the order of 100 eV. The outstanding feature of these mechanisms is that the energetic particles are only available on the night-side of the Earth in the auroral zones. It is argued that the gegenschein and the auroral displays on the night-side of the Earth are evidence of an asymmetric magnetosphere.

1. Introduction

The study of whistlers, the discovery of the Van Allen radiation belts, and the measurements made with magnetometers on space vehicles have shown that the geomagnetic field resembles that of a dipole out to distances of several Earth radii. At greater distances the nature of the geomagnetic field has not been completely determined (Sonett & others 1960a) and we have as yet no clear idea of its spatial distribution. Many of the phenomena in the upper atmosphere and in our immediate interplanetary environment are governed in a fundamental way by the Earth’s external magnetic field, and therefore it is important that we have some idea of the nature of the distant geomagnetic field.

From the world-wide geomagnetic disturbances, which are correlated with specific features of solar activity, it is known that the Sun is an irregular emitter of low energy corpuscular radiation. In recent years Biermann (1951, 1952, 1957)
and Parker (1958, 1960) have proposed that in addition to these irregular emissions there is a permanent "solar wind" of plasma flowing radially from the Sun. The density of the solar wind is possibly $10-100$ protons cm$^{-3}$ and the velocity is typically $500$ km s$^{-1}$. In this discussion we shall adopt the neutral point of view that the distant geomagnetic field is either permanently or only occasionally affected by the low energy corpuscular radiation. Should it turn out that there is definitely a solar wind of appreciable intensity then the effects are of a permanent nature and are of considerable importance in the study of geophysical phenomena such as the aurorae. But if there is no solar wind at all, or it is completely negligible (Chamberlain 1960, 1961), then the effects are limited to the irregular occasions when the Sun is an emitter of corpuscular streams.

We shall first discuss what happens in the presence of a subsonic and a hypersonic interplanetary medium flowing round and impacting against the geomagnetic field. As shown by previous authors (Chapman & Ferraro 1931, 1932, 1933, 1940; Dungey 1954, 1958; Beard 1960; Zhigulev & Romishevskii 1960; Ferraro 1960) the geomagnetic field is compressed and confined to a region about the Earth referred to as the magnetosphere (Gold 1959). In the case of subsonic flow the magnetosphere is approximately spherical, and for a hypersonic flow the magnetosphere is highly asymmetrical and has the shape of a tear-drop with the tail-end pointing away from the Sun. It is then shown that night-side phenomena such as the aurorae and the gegenschein are apparently readily explained within the framework of an asymmetric magnetosphere.

2. The Magnetosphere

2.1. The motion relative to the Earth of a conducting interplanetary medium must mean that the geomagnetic field cannot extend indefinitely to very great distances. We shall attempt to gain some idea of the topology of the geomagnetic field in the presence of a moving conducting fluid medium.

The mean free path of a proton for large angle scattering in the interplanetary medium is large compared with the size of the Earth and in previous discussions the interaction of the interplanetary medium with the geomagnetic field has been treated primarily from the particle point of view. This view leads to a specular-type reflection from the magnetospheric surface and yields a pressure normal to the surface of $\rho U^2 \cos^2 \sigma$, where $\rho$ is the mass density of the particles, $U$ is their velocity, and $\sigma$ is the angle between $U$ and the normal to the surface.

Our first assumption is that the interplanetary medium contains a magnetic field of $h \sim 10^{-4}$ oersteds (Sonett & others 1960b); quite likely this field is highly irregular on a small scale. For a proton moving at a speed $U$ the Larmor radius is $r_L \sim 10^{-4} U/h$ cm, or $r_L \sim U/c$ cm for $h = 10^{-4}$ oersteds where $U$ is in cm s$^{-1}$. It would therefore seem that for all reasonable values of $U$ a specular-type reflection is impossible. A characteristic minimum dimension of the magnetosphere is $10^5$ km, and for $U < 10^8$ km s$^{-1}$ the corresponding Larmor radius is relatively small and is comparable with what would seem to be a reasonable boundary layer thickness. The presence of a weak magnetic field embedded in the interplanetary medium justifies therefore a treatment which uses a continuum-type theory. If $n$ is the number density of ions or electrons at temperature $T$, then the pressure in the neutral fluid is

$$p = 2nkT + \frac{h^2}{8\pi}.$$  \hspace{1cm} (1)
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We shall consider two values of $U$: 30 km s$^{-1}$ and 500 km s$^{-1}$; the first corresponds to a stationary fluid through which the Earth moves at its orbital speed, and the second is a typical speed for the corpuscular radiation. Approximately, the flow is subsonic or supersonic according to whether $p$ is larger or smaller than $\rho U^2$, where $\rho = nm$ and $m$ is the proton mass. For the characteristic values of $h = 10^{-4}$ oersteds, $n = 30$ protons cm$^{-3}$, and $T = 10^5$°K, the Mach number is unity when $U \approx 45$ km s$^{-1}$. Hence, for the Earth moving through a stationary fluid the flow is subsonic and the fluid medium can be treated as incompressible, and for the solar corpuscular radiations such as the solar wind the flow is supersonic.

2.2 Subsonic flow.—The case of subsonic flow will not be treated in detail. At the surface of the magnetosphere the normal component of the geomagnetic field vanishes. As a first approximation the magnetosphere is spherical and of radius $b$, and therefore the surface tangential component of the field is $H_t \approx 3M \sin \theta'/b^3$, where $M = H_ao^3$ is the magnetic dipole moment of the Earth's field ($H_o = 0.3$ oersted is the equatorial surface value, $a$ is the Earth’s radius, and $\theta'$ is the co-latitude measured from the magnetic axis). From the equation

$$\rho \approx \frac{H_o^2}{8\pi},$$

it follows that the radius of the magnetosphere is given by

$$\frac{b}{a} \approx \left( \frac{9}{8\pi} \frac{H_o^3}{\rho} \right)^{1/6}. \quad (2)$$

Using equations (1) and (2) and the values of $n = 30$ cm$^{-3}$, $T = 10^5$°K, $h = 10^{-4}$ oersteds, we find that $b/a \approx 18$. At subsonic speeds the interplanetary medium streams round the magnetosphere in a manner similar to a fluid flowing past a submerged solid object, and to a first approximation the magnetosphere has a radius given by equation (2).

2.3 Hypersonic flow: frontal-lobe.—A tenuous conducting medium impacting at a hypersonic velocity against the geomagnetic field presents many physical and mathematical difficulties. Adopting in the first instance a descriptive approach to the problem one might say that the geomagnetic field is compressed on the day-side by the momentum flux, and on the night-side by the hydrostatic pressure of the medium. The difference between these two forces results in a highly asymmetrical magnetosphere, shown schematically in Figure 1. Owing to the Earth's rotation about its axis the outer layers of the geomagnetic field on the day-side are peeled off and swept back to the night-side where they are permanently trapped in the form of a "magnetic arch" (Harrison 1961b). Because parts of the field rotate with the Earth and other parts are stationary, electric currents flow not only on the surface of the magnetosphere but also in the interior. One anticipates that the boundary layer is exceedingly complex and possibly exhibits several forms of instability; these instabilities and other fluctuations presumably excite a variety of modes of oscillation in the magnetosphere which have some connection with the micro pulsations of the geomagnetic field at the Earth's surface. On the surface of the frontal-lobe of the magnetosphere one form of instability might consist of prising apart of the magnetic flux lines by the fast moving relatively dense boundary layer and the injection of tongues of plasma which explode and diffuse along the
interior flux lines. A further complication of the boundary layer problem is the
tendency, particularly in the tail-end where the field is weak, for the surface
magnetic field to be combed by the boundary layer so that the flux lines lie parallel
to the flow. This suggests the possibility that beyond the tail-end of the magnetosphere
there is a tail of trailing filaments of magnetic flux. In the following
treatment we shall use an extremely simple model of the magnetosphere, and shall
assume among other things that the boundary is in a steady state and that there are
no interior currents caused by the Earth’s rotation.

At high Mach numbers the shock front tends to follow the surface of an object
and in many cases it is possible therefore to use the Newtonian flow theory.
According to this theory (Hayes & Probstein 1959) if the shock layer is thin and
inviscid, the normal component of the momentum is lost inelastically while the
tangential component is conserved. Hence the pressure on the surface is

\[ p_s = p + \rho U^2 \cos^2 \sigma, \]  

where \( p \) and \( \rho \) are the pressure and density outside the shock layer. When \( \sigma = \pi/2 \)
the shock layer ceases to be contoured to the shape of the body and a free surface
forms. It should be noted that equation (3) neglects Busemann’s (1933) correction
which takes into account the inertial force of a curved boundary layer. However, if
we equate \( p_s \) to \( H^2/8\pi \), or

\[ p + \rho U^2 \cos^2 \sigma = \frac{H^2}{8\pi}, \]  

the omission of terms taking into account the curvature of the field is to some
extent cancelled by the neglect of Busemann’s inertial term.

Let the surface of the magnetosphere be given by \( F(r, \theta, \phi) = 0 \), where \( r, \theta, \phi \)
are as shown in Figure 2, and \( U \) is parallel to the \( x \) axis. Hence, at the surface

\[ \frac{\partial F}{\partial r} dr + \frac{\partial F}{\partial \theta} d\theta + \frac{\partial F}{\partial \phi} d\phi = 0. \]  

FIG. 1.—An asymmetric magnetosphere produced by hypersonic
corpuscular radiation from the Sun. The letters \( g_1 \) and \( g_2 \) indicate the
possible locations of the gegenschein.
The magnetostatic potential is
\[ V = -\frac{M}{r^2} \cos \theta' + \sum_{n} \sum_{m} r^n P_n^m(\cos \theta')(A_n \cos m\phi' + B_n \sin m\phi'), \] (6)
where the first term on the right comes from the dipole field, the second term is due to the surface currents, and \( r, \theta', \phi' \) are with respect to the dipole axis. The normal component of the field vanishes on the surface and therefore \( \nabla F \cdot \nabla V = 0 \), or
\[ \frac{\partial F}{\partial r} \frac{\partial V}{\partial r} + \frac{1}{r^2} \frac{\partial F}{\partial \theta} \frac{\partial V}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial F}{\partial \phi} \frac{\partial V}{\partial \phi} = 0. \] (7)

On converting \( \sigma \) into \( r, \theta, \phi \) co-ordinates, equation (4) becomes
\[ \rho + \rho U^2 \left( \frac{\sin \theta \cos \theta}{\partial r} \frac{\partial F}{\partial r} + \cos \theta \cos \phi \frac{1}{r} \frac{\partial F}{\partial \theta} - \frac{\sin \phi}{r \sin \theta} \frac{\partial F}{\partial \phi} \right)^2 = \frac{H_r^2}{8\pi}. \] (8)

We assume that the magnetic axis is inclined at an angle \( \epsilon \) in the \( xy \) plane; any inclination in the \( yz \) plane is eliminated by rotating the coordinate system about the \( x \) axis. This has the advantage of making the magnetosphere symmetrical about the \( xy \) plane and therefore \( B_n = 0 \) in equation (6). The following simplifying assumptions are now made: (i) \( \partial V / \partial \phi = 0 \) and hence \( H_r^{-1} \partial r = H_{\theta}^{-1} \partial \theta \). For a small inclination \( \epsilon \) this is reasonable for the frontal-lobe, and for the tail-end the shape of the magnetosphere is almost independent of the geomagnetic field. (ii) To evaluate the surface field strength only the first term \( n = 1 \) is used. Therefore,
\[ V \approx \left( A_1 r - \frac{M}{r^2} \right) \cos \theta' = \left( A_1 r - \frac{M}{r^2} \right)(\cos \theta \cos \epsilon + \sin \theta \cos \phi \sin \epsilon) \]
\[ \approx \left( A_1 r - \frac{M}{r^2} \right) (\cos \theta + \epsilon \sin \theta \cos \phi), \] (9)
for small $\epsilon$. (iii) The previous assumption is equivalent to regarding the frontal-lobe as approximately hemispherical and therefore

$$\left(\frac{dr}{rd\theta}\right)^2 \ll 1, \quad \left(\frac{dr}{rd\phi}\right)^2 \ll 1.$$  

Using (i) and (ii), equation (7) now becomes

$$\frac{\partial F}{\partial r} \left(A_1 + \frac{2M}{r^3}\right) \left(\cos \theta + \epsilon \sin \theta \cos \phi\right) - \frac{\partial F}{\partial \theta} \left(A_1 - \frac{M}{r^3}\right) \left(\sin \theta - \epsilon \cos \theta \cos \phi\right) = 0.$$  

Consider first the $xx$ plane. For small $\epsilon$, $\partial F/\partial \theta = 0$ and therefore

$$H_t = \frac{\partial V}{r \partial \theta} \left(1 + \left(\frac{dr}{r d\theta}\right)^{2/3}\right) \approx 3 \frac{M}{r^3},$$  

from (iii). Also, $\partial F/\partial \phi = -\left(dr/d\theta\right)\partial F/\partial r$, and hence equation (8) is

$$\cos \phi + \sin \phi \frac{dr}{r d\phi} = \pm \frac{1}{r^3} \left(\frac{9M^2}{8\pi \rho U^2}\right)^{1/2},$$  

for hypersonic flow in which $\rho$ is negligibly small. Using $z = -r \sin \phi$, and choosing the positive sign (because $dz/dx < 0$ for $0 < \phi < \pi$), equation (11) is

$$\frac{dz}{d\phi} = -\left(\frac{9M^2}{8\pi \rho U^2}\right)^{1/2} \frac{\sin^2 \phi}{z^2}.$$  

Hence it follows

$$-z^3 = r^3 \sin^3 \phi = \frac{3}{4} r_0^3 \left(\phi - \sin \phi \cos \phi\right),$$  

$$r_0 = \left(\frac{8\pi}{16\rho U^2}\right)^{1/6}.$$  

This equation is plotted in Figure 3. For $\phi = 0$ we find $r = r_0$; also

$$r = \left(\frac{3\pi}{4}\right)^{1/3} r_0, \quad \text{for } \phi = \frac{\pi}{2},$$  

$$z = \pm \left(\frac{3\pi}{2}\right)^{1/3} r_0, \quad \text{for } \phi = \mp \pi.$$  

Consider now the $xy$ plane. We have

$$H_t \approx \frac{\partial V}{r \partial \theta} \approx 3 \frac{M}{r^3} \sin(\theta - \epsilon),$$  

and since $\partial F/\partial \phi = 0$, it follows that equation (8) is now

$$\sin \theta - \cos \theta \frac{dr}{r d\theta} = \pm \left(\frac{r_0}{r}\right)^3 \sin(\theta - \epsilon).$$  

Using $y = r \cos \theta$ and integrating as before,

$$y^3 = \pm r_0^3 \left(\cos^3 \theta - \epsilon (\sin^3 \theta - 3 \sin \theta)\right) + \text{constant}.  $$
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The shape of the magnetosphere in the $xz$ plane for $pU^2 = 100\rho$, and $r_0 = 8$ Earth radii. The corpuscular radiation is from the right and is parallel with the $x$ axis.

On choosing the positive sign for $\epsilon \leq \theta \leq \pi + \epsilon$:

$$y^3 = r^3 \cos^3 \theta = r_0^3 \cos^3 \theta - \epsilon(\sin^3 \theta - 3 \sin \theta + 2),$$

and hence

$$r = r_0(1 - \epsilon)^{2/3}, \quad \text{for} \quad \theta = \epsilon,$$

$$r = r_0(1 + \epsilon)^{2/3}, \quad \text{for} \quad \theta = \pi + \epsilon.$$  \hfill (17a, b)

Equation (15) is plotted in Figure 4 for $\epsilon = 23^\circ.5$, which is the maximum inclination of the Earth's axis of rotation in the $xy$ plane. Choosing the negative sign for

the night-side ($dy/dz$ is still positive for $\pi + \epsilon < \theta < 3\pi/2$) we find from equations (15) and (17a, b)

$$y^3 = -r_0^3(2 + \cos^3 \theta - \epsilon(\sin^3 \theta - 3 \sin \theta - 2)), \quad \text{for} \quad \pi + \epsilon \leq \theta \leq \frac{3\pi}{2},$$

$$y^3 = r_0^3(2 - \cos^3 \theta + \epsilon(\sin^3 \theta - 3 \sin \theta - 2)), \quad \text{for} \quad \frac{3\pi}{2} \leq \theta \leq \epsilon.$$  \hfill (18a, b)
as shown in Figure 4. The asymptotic values of \( y \) for \( \theta = 3\pi/2 \) are \( y = \pm r_0 z^{1/3} \) and are independent of the angle \( \epsilon \).

2.4 Hypersonic flow: tail end. On the night-side of the Earth the geomagnetic field strength falls off rapidly and the boundary layer is virtually a free surface which is affected only by the pressure \( p \). The neglect of the pressure \( p \) in the above equations means that we must now seek alternative solutions for the surface of the magnetosphere on the night-side. Let the boundary layer contain a mass \( \gamma \) per unit length per unit angle about the \( x \) axis (using \( \tilde{\omega}, \psi, x \) cylindrical coordinates). The total mass of the interplanetary medium swept up per second is

\[
2\pi \gamma u = \pi (3\pi)^{1/3} r_0^2 \rho U
\]

where \( u \) is the mean speed of the free shock layer and it is assumed that \( \gamma \) is independent of \( \psi \): it is also assumed that the cross-section of the magnetosphere in the \( \omega, \psi \) plane is elliptical with the major and minor axes having maximum values of \((3\pi/2)^{1/3} r_0 \) and \((2)^{1/3} r_0 \), respectively. To a first approximation \( u \) is the same as that for a hemisphere:

\[
\frac{\pi^2}{4} r_0^2 \int_0^{\psi} \cos^2 \theta \sin \theta \, d\theta = \frac{5}{6} U. \tag{20}
\]

Since the cross-section in the \( \tilde{\omega}, \psi \) plane (that is, the \( yz \) plane) is nearly circular the equation of motion of the free shock layer is approximately

\[
\gamma d\psi \frac{d^2\tilde{\omega}}{dt^2} = -p\tilde{\omega} \, d\psi. \tag{21}
\]

If now \( t = -(x-x_1)/u \), where \( x \) is measured from the point \( x_1 \) at which \( d\tilde{\omega}/dt = 0 \), \( \tilde{\omega} = \tilde{\omega}_0 \), it follows

\[
\tilde{\omega} = \tilde{\omega}_0 \cos \left( \frac{x-x_1}{u} \left( \frac{r}{\gamma} \right)^{1/2} \right). \tag{22}
\]

The difficulty, of course, is to match continuously the frontal and tail-end solutions for the magnetospheric surface. In view of our approximations a sophisticated treatment is scarcely justified. We shall therefore put \( x_1 = 0 \), and we find from equations (19), (20) and (22) that the extreme tail-end of the magnetosphere is at a distance of

\[
x_0 \simeq -\frac{\pi}{2} r_0 (3\pi)^{1/6} \left( \frac{\rho U^2}{3 \rho} \right)^{1/2}, \tag{23}
\]

as shown in Figures 3 and 4.

Using the values \( n = 30 \) protons \( \text{cm}^{-3} \), \( T = 10^5 \, \text{K} \), \( h = 10^{-4} \) oersteds, \( U = 5 \times 10^7 \, \text{cm} \, \text{s}^{-1} \), it follows that \( \rho U^2 \simeq 100 \rho \), \( r_0 = 8 \) Earth radii, and \( x_0 = 1.3 r_0 \) or approximately 100 Earth radii. It can be seen that \( x_0 \propto (\rho U^2)^{1/3} \), but \( r_0 \propto (\rho U^2)^{-1/6} \) and therefore as the momentum flux \( \rho U^2 \) is increased the frontal-lobe of the magnetosphere contracts while the tail-end extends to greater distances.

In the above treatment we have avoided any enquiry into the nature of the boundary layer itself; one unfortunate consequence of this omission is that the
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The tail-end solution of the magnetosphere may be in considerable error. When the relatively dense shock layer forms into a free surface it must, because of its own internal pressure, begin to expand. Therefore the solutions given above are only approximate and can be regarded as an estimate of the mean position of a diffuse magnetospheric surface.

3. The gegenschein

The gegenschein is a faint luminosity in the night sky close to the countergon setsolar point. Very little is known about the gegenschein and the recorded observations are not all in agreement. Certain features however are of significant importance in the theory of the magnetosphere. The apparent size of the gegenschein is variable (Barnard 1888, 1892, 1893, 1897, 1899, Astapovitch 1950, and Fesenkov 1950) but generally it is elliptical in shape with the major axis roughly parallel with the ecliptic plane; often the major axis subtends an angle of about 10° and the minor axis an angle of about 6°. According to the above observers, and Roach & Rees (1956), the gegenschein occasionally presents a spotty appearance. It has been stated frequently that the gegenschein is displaced approximately 3° west of the midnight meridian, and that also there is a small displacement from the ecliptic plane. Furthermore, the Russian observers (Hope 1957) have claimed that there is a parallactic shift during the night and from this have deduced that the gegenschein is at a distance of approximately 20 Earth radii.

Houzeau (1881) and Evershed (1899, 1902) independently proposed the theory that the Earth possesses a luminous gaseous tail directed away from the Sun, very much like the tail of a comet. Although the theory encounters a number of serious difficulties it has in recent years been revived by Astapovitch and Fesenkov (see Hope 1957). More recently Piddington (1960) has elaborated on this idea and has suggested that the gegenschein is located in the Earth's magnetic tail. He suggests that excited atoms or ions, possibly of atmospheric origin, are enmeshed in a magnetic tail which is an occasional feature that may persist as a remnant of a magnetic storm. However, a recent examination of the emission spectrum seems to show that there are very few atoms of atmospheric origin in the gegenschein (Pariiskii & Gindilis 1959).

With the present model the natural place for the gegenschein is either in the magnetic-arch or in the tail-end of the magnetosphere, as indicated in Figure 1 at positions $g_1$ and $g_2$. In a plane perpendicular to the $x$ axis the magnetosphere is elliptical in shape, with the minor and major axes having a ratio of approximately $(4/3\pi)^{1/3} = 0.75$, as compared with the observed ratio of 0.6. In view of the approximate nature of the calculations, and of the difficulty of making precise observations, the difference is not so very large. The spotty appearance is consistent with the idea of strands of magnetic flux embedded in a conducting medium.

If the interplanetary medium has a velocity $U$ directed radially away from the Sun, the $x$ axis of the magnetosphere is directed at an angle $\alpha$ west of the countergon setsolar point, where

$$\tan \alpha = \frac{U_B}{U}$$

and $U_B$ is the orbital velocity of the Earth (Harrison 1961b). Since $\alpha$ is typically 3° and $U_B = 30\,\text{km}\,\text{s}^{-1}$, it follows that $U = 600\,\text{km}\,\text{s}^{-1}$, which is significantly close to the values usually proposed for the velocity of the solar wind. The
displacement in latitude however is not so easily explained. Roach and Rees suggest that the gegenschein is situated not in the ecliptic plane but in the invariable plane of the solar system. Gingilis (1960) points out that this possibility is consistent with Astapovitch's (1950) observations which reveal an annual latitude variation. With the present model it is impossible to explain this effect unless the interplanetary medium at a distance of 1 AU from the Sun has a tendency to move parallel to the invariable plane. In this case the magnetosphere will have an annual latitude variation of

$$\beta \simeq 1° 34' \sin(\lambda - 107°)$$  \hspace{1cm} (25)$$

where $\lambda - 107°$ is the longitude measured from the ascending node of the invariable plane which has an inclination of $1° 34'$. The phase and amplitude of this variation are in reasonable agreement with Astapovitch's observations.

4. The aurorae

4.1 As yet there is no generally accepted theory for the aurorae. In this discussion we shall advance the point of view that the aurorae are explained by the topology and general nature of the magnetosphere. The occurrence of auroral displays predominately on the night-side of the Earth (as shown by radio studies) is evidence in support of this point of view. Given a magnetosphere distributed in an asymmetric way about the Earth, we have the following reasons for thinking that energetic particles will penetrate to the auroral zones on the night-side of the Earth.

4.2 Fermi interactions.—Parker (1958b) has suggested that magnetic inhomogeneities embedded in a turbulent plasma at the magnetospheric boundary will accelerate ions to high energies by means of Fermi interactions. If $\beta = nkT/(H^2/8\pi)$, this acceleration mechanism requires that $\beta \ll 1$, and consequently only a small fraction of the ions are in the process of being accelerated (Harrison 1960a); when $\beta \sim 1$, no acceleration occurs and the Fermi interactions merely heat the plasma.

The production of energetic ions in the boundary layer cannot by itself account for the night-side occurrence of the auroral displays in the auroral zones. However, an essential feature of an elongated magnetosphere is that it consists of two distinct regions: an inner region, which rotates with the Earth and extends to a distance of approximately $r_0 = (gM^2/8\pi\rho U^2)^{1/6}$; and an outer region, which is stationary and consists of the magnetic-arch and the tail-end of the magnetosphere. The co-rotating region contains therefore all the magnetic flux lines which intersect with the Earth's surface at latitudes below the auroral zones, and the stationary outer region contains all the flux lines which intersect with the Earth's surface in the polar regions. Separating the two is an intermediate region in the form of a shell that envelopes the night-side of the Earth and extends down into the auroral zones (Figure 1). Considerable shear occurs in this shell and presumably it contains highly agitated plasma and magnetic field fluctuations within which Fermi interactions are an efficient mechanism for accelerating protons. One might therefore apply Parker's theory to this region, and the accelerated protons are then guided along the magnetic field and contribute to the auroral phenomena on the night-side of the Earth.

Consider an element of volume $dS_0 \, ds_0$, where $dS_0$ is the cross-sectional area of a magnetic tube (or shell) of flux and $ds_0$ is an element of length in the direction of the field. If the accelerated ions are collision-free and have at a point $s_0$ an isotropic
distribution function \( f(v, s_0) \), then at the point \( s_1 \),

\[
f(v, s_1)dS, \ ds_1 = \frac{H_0}{H_1}f(v, s_0)\ dS_0ds_0,
\]

from the adiabatic theory (Spitzer 1956). But \( H_1 \ dS_1 = H_0 \ dS_0 \), and therefore \( f(v) \) is constant along the flux lines and remains isotropic. This means that if the shell contains energetic ions which are collision-free and have an isotropic distribution function (and therefore the end losses into the auroral zones are relatively small) then the density of the ions is uniform throughout the shell. For simplicity let us suppose \( f(v) \) is Maxwellian and the number density of the energetic ions is \( n' \) and their temperature is \( T' \). The maximum pressure possible for these suprathermal or energetic ions is

\[n'kT' = \frac{H_0^3}{8\pi}\]

(26)

where \( H_0 \) is the minimum field strength and is therefore in the equatorial plane. Or, since the shell occurs at a distance of approximately \( r_0 \) where \( H_0/8\pi \approx \rho U^2 \)
we have

\[n'kT' \approx \rho U^2\]

(27)

as a very approximate upper limit for the pressure of the energetic ions.

Close to the Earth \( f(v) \) is no longer isotropic, and for a one-sided Maxwellian distribution the mean energy flux into the auroral zone is

\[\frac{1}{2}n'm\langle v^2O_l \rangle = n'kT'\left(\frac{2kT'}{\pi m}\right)^{1/2} \approx \rho U^2\left(\frac{2m}{\pi n'}\right)^{1/2}.
\]

(28)

Assuming \( n' \approx 0.1 \) and \( n = 10 \) protons cm\(^{-3} \) and \( U = 500 \) km s\(^{-1} \), the total energy flux is 16 erg s\(^{-1} \) cm\(^{-2} \), or the mean energy per proton is \( 2.5 \times 10^5 \) eV and the flux is approximately \( 2 \times 10^7 \) protons s\(^{-1} \) cm\(^{-2} \). These figures are obtained by an extremely crude method, yet nevertheless they are in reasonable agreement with the deductions from observations (Chamberlain 1954, 1957).

It should be pointed out that this method of acceleration depends on the velocity of the particle and the corresponding energy for electrons is therefore only about 100 eV. However, the energy required by the electrons is not large (Fan 1954; Chamberlain & Meinel 1954) and is of the order of several hundred electron volts.

4.3 Adiabatic acceleration.—It has been pointed out by Johnson (1960) that as the Earth rotates about its axis the magnetic flux lines in the magnetic-arch must also rotate about each other. This is best illustrated by referring to Figure 1. The flux line marked \( aa \) becomes the flux line \( a'a' \) when the Earth has revolved through 180° (allowance must be made for the change in the tilt of the magnetic axis). Each flux line moves round a closed orbit in a clockwise direction (viewed from above the north pole) once in every 24 hours. Consider a tube of flux \( aa \) which contains protons and electrons that have leaked in from the interplanetary medium. Now because the field is continuously varying as the tilt of the magnetic axis changes, and because of possible local inhomogeneities in the field itself, the plasma is virtually confined or frozen to a given flux tube. As the flux lines move
round and their length varies, the contained particles are accelerated and de-
celerated in a cyclic manner. It is seen that the particles with maximum energy are in that region of the magnetic field where the flux lines intersect with the Earth’s surface in the auroral zones on the night-side.

If the particle velocities are randomized by collisions during each revolution of the Earth then their energy is \( W \propto L^{3/2} \), where \( L \) is the length of a flux line. Thus if the electrons have an initial mean energy of \( 10 \) eV, which is largely thermal, and if \( L \) changes by as much as a factor of \( 10 \), then their peak mean energy is about \( 50 \) eV. Similarly, if the protons have an initial energy of \( 10^3 \) eV, corresponding to \( U = 500 \text{ km s}^{-1} \), their peak energy is \( 5 \times 10^3 \) eV. However, an examination of the mean collision time for large angle scattering (Spitzer 1956) shows that the protons are virtually collision-free during a complete acceleration-deceleration cycle. It follows that if \( W \) is their translation energy along the field lines, then \( W \propto L^2 \).

For an injection energy of \( 10^3 \) eV at \( a a \), the peak energy at \( a'a' \) can be as high as \( 10^5 \) eV.

### 4.4 Field acceleration of electrons

When an electric field is applied to a plasma there are always some electrons in the high energy tail of the distribution which are freely accelerated and form a runaway current (Harrison 1960a). If the electric field exceeds a critical value of

\[
E_c \simeq 10^{-8} n / T \text{ V cm}^{-1},
\]

then all the electrons tend to run away. Using a density of \( n \sim 10^3 \text{ cm}^{-3} \) and a temperature of \( T \sim 10^5 \text{ K} \) for the magnetosphere at altitudes greater than \( 1000 \) km, this equation gives \( E_c \sim 10^{-8} \text{ V cm}^{-1} \). If now we consider a circuit whose linear dimensions are about ten times the Earth’s diameter, the potential difference required to cause complete runaway along the magnetic flux lines is of the order of \( 100 \) volts.

There is one obvious cause for potential differences of this magnitude. Because of the Earth’s rotation about its axis the entire magnetospheric medium is in motion in the magnetic-arch and is churned round as the flux lines move from \( a a \) to \( a'a' \) and back again. The drag of this medium on the magnetic field causes the polar magnetic flux lines to be twisted about each other in a helical fashion. It is readily seen that the associated electric field is directed radially outward from the magnetic poles and upward at the auroral zones. Thus electrons are dragged down into the outer zones of the polar caps and may well contribute a significant part to the electron streams which appear to be an essential feature of auroral displays. The maximum possible potential difference developed in the magnetosphere due to this effect is

\[
\phi_{\text{max}} = 2a^2 \omega \frac{H_a}{c} \int_{\theta_0}^{\pi/2} \sin \theta \cos \theta \, d\theta \simeq 10^4 \text{ V},
\]

(\( \theta_0 \) is the colatitude of the auroral zone and \( \omega \) is the Earth’s angular velocity of rotation) and is the potential difference in a stationary external circuit connecting the magnetic pole and the auroral zone.

From what has been said it is clear that for any potential difference larger than \( 100 \) eV, the current that flows is limited not by resistance but by the inductance of the circuit. If \( v \) is the velocity of the electrons, and \( j = -nev/c \) is the current
The Earth's distant magnetic field

density, then from the equation of motion and Maxwell's equations:

\[ \mathbf{E} \approx -\frac{m_e}{e} \frac{\partial \mathbf{v}}{\partial t} \approx \frac{m_e c}{n e^2} \frac{\partial j}{\partial t} = \frac{m_e c}{4\pi n e^2} \text{curl} \frac{\partial \mathbf{H}}{\partial t}. \]

Hence, if \( \lambda_e = (m_e c / 4\pi n e^2)^{1/2} \) is the "collision-free penetration depth", it follows

\[ \nabla^2 \mathbf{E} - \lambda_e^{-2} \mathbf{E} \approx 0 \]

and the electric field is confined to a region of thickness \( \lambda_e \). This is approximately the thickness of auroral arcs and \( \lambda_e \) is a significant parameter in the study of the stability of auroral structures (Harrison 1960b). Hence it seems that quite small electric fields in the magnetosphere can cause relatively thin current sheets to form parallel to the magnetic field and to intersect with the Earth's surface (or rather the ionosphere). An essential feature of these currents is that they are induced by the time-varying magnetic fields of the magnetosphere.

Finally, under this heading one might also mention the possibility that collective electrostatic interactions between the proton streams and the atmospheric electrons might cause the electrons to be dragged down with the protons (Harrison 1960b). Until more is known about the distribution function of the protons it is impossible at this stage to determine the relative importance of this process.

In conclusion one might say it is evident that within the general framework of an asymmetric magnetosphere the possibilities are rich for explaining many of the properties of auroral displays. In particular, problems such as the origin of the streams of charged particles which excite the aurorae, the nature of the associated acceleration mechanisms, and the location of the displays predominantly on the night-side within the auroral zones, apparently can be solved with such a theory.

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