Baryon Resonances and Electromagnetic Couplings

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We present a formalism for photoproduction of baryon resonances and Compton-scattering. Multichannel unitarity is used to separate baryon resonances from background. We investigate the divergence of partial wave expansions used in photoproduction. Dispersion relations are used to determine electromagnetic background.

§ 1. Introduction

Since the dispersion theoretic approach of CGLN\textsuperscript{9} supplemented by Watson's theorem\textsuperscript{9}, much experimental and theoretical works have been done in pion photoproduction processes.\textsuperscript{9} Provided that accurate data on cross sections, polarizations and asymmetries, are available in the low energy region, an energy independent multipole analysis is possible by using Watson's theorem without further dynamical assumptions.\textsuperscript{9} The extension to higher energies was carried out by Berends and Donnachie.\textsuperscript{9}

The first partial wave analysis from threshold to \( E_\gamma = 1200 \text{ MeV} \) was done by Walker,\textsuperscript{9} and it was later improved by adding more complex background.\textsuperscript{7,8}

On the other hand, British groups developed their dispersion relation analyses.\textsuperscript{9,10,11}

All analyses have some of the following deficiencies discussed by Donnachie:\textsuperscript{15}

a) Violation of unitarity at low energies,

b) The divergence of the partial wave expansion used in fixed \( t \)-dispersion relations,

c) The variation of basic resonance parameters (mass and width) to improve fits in photoproduction.

The purpose of the present paper is to discuss in detail the problems raised by Donnachie and to propose a parametrization by imposing multi-channel unitarity and analyticity.

The basic parameters are fixed in a multi-channel fit to pion-nucleon partial waves. In photoproduction only the photon coupling of the resonance is allowed to vary. In addition to the parametrization of the resonances we introduce electromagnetic background functions that dominate the partial waves in photoproduc-

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tion at low energies. These background functions ensure unitarity at low energies
but they are also the source for the divergence of the partial wave expansion in
the unphysical region. The divergent part will be treated separately and dispersion
relations are still useful for energies above $E_\gamma = 900 \text{ MeV}$.

We compare the real parts of our parametrization using multi-channel unitarity
with real parts of the partial waves calculated by dispersion relations and we are
able to determine the background functions.

New accurate data in photoproduction$^{10,11}$ together with new partial wave
analyses in pion-nucleon scattering$^{12,17}$ require more refined methods to analyze the
data.

§ 2. Unitarity and pion-nucleon scattering

In order to get the general formula satisfying the unitarity condition we start
with the couplings of a resonance to various channels and define a factorized and
symmetric $T$-matrix

$$T_{jk} = x_j x_k \sin \alpha_\ell \ell = \sqrt{\frac{\Gamma_j \Gamma_k}{\Gamma_{\text{tot}}}} \frac{M_{R \ell} \Gamma_{\text{tot}}}{M_{R \ell}^2 - s - i M_{R \ell} \Gamma_{\text{tot}}}.$$  \hspace{1cm} (2.1)

The functions $x_k^2$ are the energy-dependent ratios of the partial width $\Gamma_k$ to the
total width $\Gamma_{\text{tot}}$, where $\sum x_k^2 = 1$ by definition. The branching ratio at the reso-
nance is defined

$$a_k = x_k (W = M_R) \text{ with } \sum a_k^2 = 1.$$ \hspace{1cm} (2.2)

The matrix $X = (x_j x_k)$ has the following properties,

$$X^\dagger = X; X^2 = X; (1 - X) X = 0; (1 - X)^2 = (1 - X).$$ \hspace{1cm} (2.3)

We consider the following channels in two-body approximation,

$$1(\pi N), 2(\pi \Delta), 3(\rho N), 4(\sigma N) \text{ and } 5(\eta N).$$ \hspace{1cm} (2.4)

It is well known that this simple Breit-Wigner parametrization is not sufficient for
all partial waves in pion-nucleon scattering and resonances are usually distorted by
“background”. These examples are seen in S31 and P31 (Particle Data Group$^{10}$). To include background, we construct the whole $S$-matrix as follows:

$$S = e^{i \alpha} S(2\alpha, x) e^{i \beta},$$ \hspace{1cm} (2.5)

where

$$S(2\alpha, x) = 1 + 2i T = 1 + 2i X \sin \alpha \ e^{i \alpha} = (1 - X) + X e^{i \alpha}.$$ \hspace{1cm} (2.6)

$\sqrt{e^{i \beta}} = e^{i \beta} = (\partial_{jk} e^{i \alpha})$ represents a matrix for the elastic initial and final state inter-
action. The resulting formula for the $T$-matrix is often used in pion-nucleon scat-
tering$^{10}$
\[ T_{jk} = \delta_{jk} \sin \delta_j e^{i\beta_j} + x_j x_k \sin \alpha e^{i(\beta_j + \beta_k)}. \] (2.7)

The background-phase \( \delta_j \) is negative for repulsive background, but if it is positive it may represent the effect of another inelastic resonance in the same partial wave. We generalize formula (2.7) for two inelastic resonances which can interfere. We replace \( \sqrt{e^{i\beta}} \) by a resonance with phase \( \beta \) and branching ratios defined by a matrix \( Y \),

\[ \sqrt{S(2\beta, Y)} = (1 - Y)^2 + Y e^{i\beta} = S(\beta, Y). \] (2.8)

To take the square root we used the properties (2.3). Now we construct a unitary and symmetric \( S \)-matrix

\[ S = S(\beta, Y) S(2\alpha, X) S(\beta, Y). \] (2.9)

When the \( T \)-matrix elements are calculated using this \( S \)-matrix, an interference sum \( IS = \sum x_j \gamma_j \) between the two resonances appears. We get two simple Breit-Wigner resonances and an interference-function

\[ T_{jk} = x_j x_k \sin \alpha e^{i\alpha} + y_j y_k \sin \beta e^{i\beta} \]
\[ + [x_j y_k + x_k y_j + y_j y_k IS(e^{i\beta} - 1)] IS(e^{i\beta} - 1) \sin \alpha e^{i\alpha}. \] (2.10)

The formula for elastic scattering is simple, and it reduces to (2.7) when the resonance \( \beta \) is elastic \( \gamma_j = 1 \) and \( IS = x_j \) at low energies,

\[ T_{ii} = [x_i + y_i IS(e^{i\beta} - 1)] IS(e^{i\beta} - 1) \sin \alpha e^{i\alpha} + y_i^2 \sin \beta e^{i\beta} . \] (2.11)

In the more general case for two resonances and elastic background-phases we get as the final expression:

\[ S' = (e^{i\beta}) S(\beta, Y) S(2\alpha, X) S(\beta, Y) (e^{i\beta}) \] (2.12)

and

\[ T_{ii}' = \sin \delta_i e^{i\delta_i} + e^{i\beta_i} T_{ii} . \] (2.13)

where \( T_{ii}' \) is the same as formula (2.11).

\section*{§ 3. Unitarity and electromagnetic couplings of resonances}

Parametrizing the \( S \)-matrix for strong interactions, we left out the electromagnetic decays which we will consider now by adding another channel [compare (2.1) \( \sim \) (2.7)]

\[ 0(\gamma N) \text{ with coupling } a_0^\gamma \text{ and } \delta_0 = 0. \] (3.1)

We neglect electromagnetic initial (or final) state interactions \( \delta_0 \), because they are higher orders in the small electromagnetic coupling constant. Furthermore the
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The total width in formula (2.1) will not be changed, if the small electromagnetic width is added and, therefore, the resonance phase \(\alpha\) is not changed. Photoproduction in one of the channels (2.4) will have a structure similar to formula (2.7) with \(\delta_0 = 0\)

\[
T_{kk} = x_k x_k' \sin \alpha e^{ix_k} e^{ix_k'}.
\]

The phase of a resonance in photoproduction is essentially determined by the phase \((\alpha + \delta_k)\) of the corresponding strong channel. To include non-factorizing forces like pion-exchange in photoproduction we need a more general formalism. Unitarity and time reversal hold, if the \(T\)-matrix is written by a real and symmetric \(K\)-matrix

\[
T = K/(1 - iK) \quad \text{or} \quad T = K(1 + iT).
\]

By using this formula, the single pion photoproduction has the structure

\[
T_{kk} = K_{kk} + iK_{k0} T_{k1} + iK_{0k} T_{01} + iK_{00} T_{00} + \cdots.
\]

The \(K\)-matrix elements have poles corresponding to the resonance in formula (3.2)

\[
K_{kk} = \delta_{kk} \tan \delta_k + x_k x_k' \tan \alpha \left[ \cos \delta_j \cos \delta_k \left(1 + \tan \alpha \sum x_n^2 \tan \delta_n\right) \right]^{-1}.
\]

Unitarity still holds when we add real functions \(B_{kk}\) to the poles

\[
K_{kk} = K_{kk}^R + B_{kk}.
\]

Neglecting the background function \(B_{00}\) for the Compton scattering, we get the general formula for photoproduction

\[
T_{kk} = x_k x_k' \sin \alpha e^{ix_k} e^{ix_k'} + B_{kk} + i \sum_{n=1}^{N} B_{kn} T_{kn}.
\]

Using (2.7) for \(T_{kk}\), we rewrite this formula into

\[
T_{kk} = \left[ x_0 + i \sum_{n=1}^{N} B_{0n} x_n \right] e^{ix_k} x_k' e^{ix_k'} \sin \alpha e^{ix_n} + B_{kk} \cos \delta_k e^{ix_k}.
\]

For single pion photoproduction we define \(B_2\) (channels \(n \geq 2\)):

\[
B_2 = \sum_{n=2}^{N} B_{0n} x_n e^{ix_n}.
\]

§ 4. Dispersion relations and divergence of partial wave expansions

Here we study the conditions imposed by dispersion relations for the amplitudes \(A_t(s, t)\) introduced by CGLN\(^9\)

\[
\text{Re } A_t'(s, t) = \text{Born } A_t'(s, t) + \frac{P}{\pi} \int ds' \left[ \frac{1}{s' - s} + \frac{\eta e^{If}}{s' - u} \right] \text{Im } A_t'(s', t)
\]

\[(4.1)\]
with isospin \((0, +, -, -)\) and the crossing factors \(\gamma\) and \(\epsilon\). By projecting out partial waves from the real part and by introducing a partial wave expansion for the imaginary part, (4.1) is led to the following form:

\[
\text{Re } T_{01}^n(s) = \text{Born } T_{01}^n(s) + \frac{P}{\pi} \int \frac{ds'}{s' - s} \text{ Im } T_{01}^n(s') \\
+ \frac{1}{\pi} \int ds' \sum_n P_{mn}(s, s') \text{ Im } T_{01}^n(s').
\] (4.2)

The functions \(P_{mn}(s, s')\) are known but complicated.\(^{20}\) The sum over \(n\) is symbolically written, and it implies a sum over all imaginary parts \(\text{Im } T_{01}^n\) with the same isospin. For isovector photons there is an additional sum (coupling of isospin 3/2 to 1/2 or vice versa). In the literature\(^{20}\) a dispersion relation like (4.2) is formulated for multipoles \(E_{n\pm}, M_{n\pm}\) or \(A_{n\pm}, B_{n\pm}\).\(^{6}\) They are related to the \(T\)-matrix elements \(T_{01}^n\) by multiplying a factor \(\sqrt{kq}\) and certain normalization constants given in the Appendix. We compare the result from dispersion relations (4.2) with our result from multi-channel unitarity (3.7) for higher partial waves, where we can neglect the resonance-phase \(\alpha\) and \(\delta_k\) and the influence of the dispersion integral. At low energies below the \(\pi\piN\) threshold we have:

\[
T_{01} = B_{01} \quad \text{and} \quad \text{Re } T_{01} = \text{Born } T_{01}.
\] (4.3)

The background-function \(B_{01}\) is real (a \(K\)-matrix element) and, therefore, we have an important result. The background functions \(B_{01}\) are determined by Born terms. The Born terms in single pion photoproduction for higher partial waves are essentially dominated by gauge invariant pion exchange \(\text{Born}_\pi T_{01}\). We get from formula (3.7)

\[
T_{01} = \text{Born}_\pi T_{01} (1 + iT_{11}) \\
= \text{Born}_\pi T_{01} (1 - \text{Im } T_{11}) + i \text{ Born}_\pi T_{01} \text{ Re } T_{11}
\] (4.4)

below the \(\pi\piN\) threshold. If the real part of \(T_{11}\) is approximated by nucleon and \(\pi\pi\) exchange, we get the box graphs that determine the double spectral functions as shown in Fig. 1.

![Feynman graphs](https://example.com/feynman-graphs.png)

Fig. 1. Feynman graphs of pion exchange in photoproduction and the graphs with nucleon exchange and two-pion exchange in pion nucleon scattering determine the box graphs A~D.
The double spectral functions limit the use of partial wave expansions which were used to derive (4.2) from (4.1). In Fig. 2 we display that a fixed $t$ dispersion relation will cross the boundary of graph A (in Fig. 1) for values of $t < -1.1$ (GeV/c)$^2$. As discussed by Mandelstam, the imaginary part of a box graph with stable particles is infinite on the boundary of the double spectral function and the inclusion of spin will not change the analytical properties. The discontinuities of the box graph are calculated by using Cutkosky's rules. The imaginary part $\text{Im} A_t(s, u)$ has the structure ($u_0$ is the boundary):

$$\text{Im} A_t(s, u) = \frac{P}{\pi} \int_{u_0}^{\infty} du' a_t(s, u') \frac{1}{\sqrt{u'-u_0}} \frac{1}{u'-u}. \quad (4.5)$$

We separate the singular part

$$\text{Im} A_t(s, u) = \frac{a_t(s, u_0)}{\sqrt{u_0-u}} \delta(u_0-u)$$

$$+ \frac{P}{\pi} \int_{u_0}^{\infty} du' \frac{a_t(s, u') - a_t(s, u_0)}{\sqrt{u'-u_0}} \frac{1}{u'-u}. \quad (4.6)$$

Formula (4.6) shows that a partial wave expansion is impossible beyond $u_0 > u$. The boundary of graph A (Fig. 1) limits the region of convergence to energies $E_t$ below 900 MeV ($s < 2.57$ GeV$^2$) (Fig. 2). For higher energies formula (4.6) has to be used together with (4.1) to calculate the real part. The effects of the...
box graphs C and D (Fig. 1) are less important although the boundary in the $t$-channel is closer to the physical region (Fig. 2). The calculation of the functions $\alpha_i(s,u)$ and $\alpha_i(s,t)$ will be given in a separate paper. There we discuss the questions concerning the convergence of partial wave expansions in more detail. The isospin structure of graph A is isospin +. The divergence of the partial wave expansion due to graph A is therefore only present for isovector photons producing neutral pions.

§ 5. Multipion photoproduction, Compton-scattering and electroproduction

The processes $\gamma N \rightarrow \pi J, \rho N, \varepsilon N$ and $\eta N$ are not as well examined as single pion photoproduction. In the two-body approximation we can use multi-channel unitarity for the parametrization with formula (3.7). Only the background-functions $B_{k\alpha}$ with $k \geq 1$ are not known and all other information can be taken from pion-nucleon scattering and single pion photoproduction. In some cases (for instance, $\gamma N \rightarrow \eta N$ for S11(1550)) a multi-channel fit is much better to determine the resonance coupling because single pion photoproduction has a large background in this partial wave.

From (3.3) we derive a formula for Compton scattering

\[ T_{00} = \left[ x_0 + i \sum B_{0n} x_n e^{i\theta_n} \right] \sin \phi \ e^{i\phi} + B_{00} \]
\[ + i \sum B_{k\alpha} \cos \delta_n e^{i\theta_n}. \]  

(5.1)

The term in brackets [\ldots] is the same as for photoproduction (3.8) but the second sum is interesting because the background-functions are squared and cannot partly be canceled out due to different signs as in the first sum. In the low energy region (5.1) for higher partial waves reduces to

\[ T_{00} = B_{00} + i B_{01}. \]  

(5.2)

$B_{00}$ is given in terms of pion- and eta-exchange in the $t$-channel, while the imaginary part $iB_{01}$ is given by the product of gauge-invariant pion-exchange contributions (Fig. 1).

In electroproduction we see a dramatic change in its structure\textsuperscript{120} comparing photoproduction with electroproduction at different momentum transfers. This can be parametrized by using different form factors which are only reliable if the change of background-functions is considered, too.

§ 6. Fitting procedure

A multi-channel fit to pion-nucleon partial waves is necessary to fit the masses, widths and couplings of the resonances and to determine the background-phase $\delta_i$ in formula (2.7).
We start to analyze the data of photoproduction in the energy region of the first resonance (P33). There are two photon couplings of P33 which we have to determine (P33M and P33E). It is known from energy independent analyses that the photon coupling of P33E almost vanishes. All partial waves except P33M are therefore dominated by the background function $B_{2t}$ in formula (3.7). As long as the partial waves are elastic in pion-nucleon scattering, we have

$$T_{2t} = B_{2t} \cos (\alpha + \delta_1) e^{i(\alpha + \delta_2)} + x_0 x_1 \sin \alpha e^{i(\alpha + \delta_3)}. \quad (6.1)$$

In most cases we can neglect the resonance part at low energies and we determine $B_{2t}$. In the following step we include the data of the energy region of the second resonance. If inelastic channels are open we have to include the sum of inelastic background functions $B_x$ (formula (3.9)) which we approximate as a real function. We have to determine the photon couplings of P11M, D13E and D13M using formula (3.7) and S11E using formula (2.11). The photon couplings (isospin 1/2) are different for proton and neutron targets. If the photon couplings and background functions are adjusted to the data in the first and second resonance regions we use the dispersion relation (4.2). An energy dependent difference between the real parts from dispersion relations and unitarity will be corrected by changing the background functions. If the difference is almost energy independent, it may be caused by higher resonances and should gradually disappear if these are included in the next analysis.

§ 7. Results

In the following we will discuss the structure of baryon resonances produced in photoproduction in more detail. The results are listed in Table I.

P33 (1232) According to (3.7) the magnetic excitation P33M is proportional to the partial wave P33 if a background-phase $\delta_1$ is assumed to be zero (neglecting

<table>
<thead>
<tr>
<th>$M$ [MeV]</th>
<th>$\Gamma$ [MeV]</th>
<th>$\Gamma_s/\Gamma$</th>
<th>$\Gamma_t/\Gamma$</th>
<th>$a_0$</th>
<th>$B_{2t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1548</td>
<td>93</td>
<td>0.300</td>
<td>0.00100$^P$</td>
<td>0.0326</td>
<td>0.0300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00110$^N$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1486</td>
<td>613</td>
<td>0.627</td>
<td>0.00100$^P$</td>
<td>0.0320</td>
<td>0.0300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00260$^N$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1528</td>
<td>187</td>
<td>0.542</td>
<td>0.00500$^P$</td>
<td>0.00160</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00230$^N$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1232</td>
<td>110</td>
<td>0.994</td>
<td>0.00596$^P$</td>
<td>0.0772</td>
<td>Born$^P$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>P33M</td>
<td></td>
<td>Born$^P$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>P33E</td>
<td>-0.0005</td>
<td>Born$^P$</td>
</tr>
</tbody>
</table>
the background-function $B_{01} = \text{Born}_{\pi}, \text{P33M for the moment)$. This is the famous result from CGLN.$^{19}$}

The coupling of the electric quadrupole excitation P33E is negligibly small, and the dispersion integral contributions from other resonances are also small and cancel with each other. Therefore (4.2) reduces to an integral equation for the non-resonant background-function $B_{01}(\text{P33E})$. Using (4.4), we get

\[ \text{Born}_{\pi} \text{P33E}(1 - \text{Im P33}) = \text{Born P33E} + \frac{P}{\pi} \int ds' \left[ \frac{1}{s' - s} + P(s, s') \right] \text{Born}_{\pi} \text{P33E}(s') \text{ Re P33}(s'). \] (7.1)

At resonance the dispersion integral must cancel the (total) Born term and real and imaginary parts are zero. This relation is fulfilled to a high degree (Fig. 3) and agrees with results from energy-independent analyses.$^{4, 19}$

P11 (1480) The partial wave P11 has a noticeable background-phase $\delta_1$ which causes a double zero for $W=1.2 \text{ GeV}$ ($\alpha + \delta_1 = 0$). We compare the pion-nucleon phase P11 with the photoproduction partial wave P11M($M_{1-}$) in the elastic region ($x_1=1$ and $B_2=0$).

\[ T_{\pi}(P11) = \sin(\alpha + \delta_1) e^{i(\pi + \delta_1)}, \]
\[ T_{\pi}(P11M) = \left[ x_0 \sin \alpha + B_{01} \cos(\alpha + \delta_1) \right] e^{i(\pi + \delta_1)}. \] (7.2)

Both partial waves have the same phase (Watson's theorem) but quite a different structure. The fit results are different according as the target is proton or neutron.

proton $a_0(P11M) = 0.032$, $B_{01} = 0$,

neutron $a_0(P11M) = -0.016$, $B_{01} = -0.013$ ($W=1200 \text{ MeV}$).
In this fit we were not able to determine $B_2(\text{P11M})$ since the error of the resonance coupling constant is larger than $B_2$. The resulting Argand diagrams are compared with each other in Fig. 4.

**D13 (1520)** The background-function is again proportional to $\text{Born}_s$ but this influence is somewhat reduced at the resonance, because $B_2$ (formula (3.9)) has an opposite sign. The resonance coupling of the electric dipole D13E ($E_{2-}$) is mainly isovector, while the magnetic quadrupole radiation D13M ($M_{s-}$) is only seen for proton targets. A similar difference was observed by Devenish and Lyth, $^{29}$ and the form factor of D13E is consistent with the dipole form but that of D13M decreases more slowly.

$$
\begin{align*}
\text{isovector} & \quad B_2(\text{D13E}) = -0.005, \quad B_2(\text{D13M}) = -0.0015, \\
\text{proton} & \quad a_0(\text{D13E}) = 0.060, \quad a_0(\text{D13M}) = 0.038, \\
\text{neutron} & \quad a_0(\text{D13E}) = -0.048, \quad a_0(\text{D13M}) = -0.004.
\end{align*}
$$

The Argand diagrams are compared with each other in Fig. 5.

**S11 (1550)** The partial wave S11E is dominated by the background-function $B_{s1}$ which is almost constant apart from a threshold factor.

$$
B_{s1}(\text{S11E}) = 0.03 \, (\text{isovector photon}).
$$
The determination of the resonance coupling of S11(1550) (parametrized with $\alpha$, $x_i$) is complicated, because the resonance S11(1680) with a width of 230 MeV (parametrized with $\beta$, $y_i$) gives a strong interference. S11(1550) decays into $\pi N$ and $\eta N$, while S11(1680) decays into $\pi N$ and other channels except $\eta N$. The interference function $I_S = \sum x_n y_n$ reduces to $I_S = x_1 y_1$ in (2.11). We compare the structure of the partial wave in pion-nucleon scattering with photoproduction (isovector) (formulae (2.11) and (3.7)). The background function $B_{\text{bg}}$ was consistent with zero and we set $y_0 = 0$ [the photon coupling of S11(1700)].

\[ T_{\text{11}} = y_1^2 \sin \beta \epsilon^{1\beta} + x_1^2 \sin \alpha \epsilon^{1\alpha} (1 - y_1^2 + y_1^2 \epsilon^{1\beta})^2, \]

\[ T_{\alpha} = B_{\text{bg}} (1 + i T_{\text{11}}) + x_1 y_1 \sin \alpha \epsilon^{1\alpha} (1 - y_1^2 + y_1^2 \epsilon^{1\beta}). \]  

(7.3)

For $x_1 = 1$ and $y_1 = 1$ the phases $(\alpha + \beta)$ of these two partial waves are the same as required by Watson's theorem. The resonance coupling is mainly isovector and comparable to the background function $B_{\text{bg}}$ in their magnitudes.

\[ a_{\text{bg}}(\text{S11E}) = 0.033 \] (isovector).

The partial wave S11 shows a clear structure known as the cusp-effect. We consider a Breit-Wigner type resonance with two channels\footnote{\textsuperscript{55}}

\[ T_{\text{11}} = M_{\text{11}} \Gamma_1 / [M_{\text{11}}^2 - s - i M_{\text{11}} (\Gamma_1 + \Gamma_2)]. \]  

(7.4)

If $\Gamma_2$ has an s-wave threshold factor $\Gamma_2 \sim Q$ for $Q^2 > 0$ and $\Gamma_2 \sim i |Q|$ for $Q^2 < 0$ below threshold, the opening of the inelastic channel will introduce a cusp-effect. This cusp structure is shown in Figs. 6 and 7 where we compare the imaginary part of the resonance with the background. It depends on the magnitude of the other partial waves whether this cusp-structure is seen in the cross-section or not. It is clearly seen in $\gamma p \rightarrow \pi^+ n$ and less clear in $\gamma p \rightarrow \pi^0 p$ (Fig. 8). For neutron data the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{Imaginary parts of S11E, the resonance S11 (1550) and the photon background [formula (4.4)] which is proportional to Re S11 of pion nucleon scattering.}
\end{figure}
Fig. 7. Argand diagrams of S11 and S11E₀ (⁺⁻S11E₀).

Fig. 8. The cusp-effect (compare Fig. 7) is clearly seen in π⁺ photoproduction ¹⁰ and more difficult to detect in γp→π⁺p.

Fig. 9. Comparison of the present analysis with new data on asymmetries of polarized targets for γp→π⁺p (θ_CM=40°, 80° as example) [Nagoya, Bonn].

Fig. 10. Comparison of the present analysis with new data from Bonn ¹⁹ and other analyses, Nagoya, MW ¹⁵ and MOR ¹⁵.
effect is covered by D13E. After the completion of this analysis new data were presented at the 1977 International Symposium on Leptons and Photon Interactions at High Energies in Hamburg. Our predictions agreed very well with the new data, especially, those for proton targets. The neutron data are less precise and, therefore, predictions for polarization experiments are less reliable. The Nagoya group measured the asymmetry for $\gamma p \rightarrow \pi^+ n$ by the polarized target, which agrees well with the predictions (Fig. 9). While this measurement covers wide ranges of energy and angle, the new measurement in Bonn\textsuperscript{48} concentrates on $E_p = 850$ MeV with a high precision measurement (Fig. 10). The data reproduce our prediction for the proton target. Yoshioka et al.\textsuperscript{49} measured the differential cross-section for $\gamma p \rightarrow \pi^0 p$ and the agreement is very good especially at forward angles (Fig. 11). Fukushima et al. measured the asymmetry for $\gamma p \rightarrow \pi^0 p$ by the polarized target.

![Fig. 11. Comparison of the present analysis with new data \cite{Yoshioka et al.\textsuperscript{49}} on $d\sigma /d\Omega \gamma p \rightarrow \pi^0 p$ ($\theta_{CM}=15^\circ$, 35\(^\circ\) as example) and the analysis of Nagoya\textsuperscript{49}](image1)

![Fig. 12. Comparison of the present analysis with new data \cite{Nagoya\textsuperscript{49}} on the asymmetry for polarized target for $\gamma p \rightarrow \pi^0 p$ ($\theta_{CM}=30^\circ$ as example) and the analyses, Nagoya\textsuperscript{49}, MW\textsuperscript{77} and MOR\textsuperscript{10}](image2)

![Fig. 13. Comparison of the present analysis with new data from Nagoya\textsuperscript{49} and from Liverpool\textsuperscript{10} on the asymmetry for polarized target for $\gamma p \rightarrow \pi^0 p$ at $E_p=0.850$ GeV and the analyses, Nagoya\textsuperscript{49} and MOR\textsuperscript{10}](image3)
(Figs. 12 and 13). Again the overall agreement is very good especially at forward angles. The complete results and tables of this analysis are listed in a separate paper.\textsuperscript{310}

§ 8. Conclusions

This analysis has shown that the pion-exchange Born term in photoproduction [formula (4·4)] dominates the nonresonant background for low partial waves (for example, P33E) and determines the singularities of the amplitude in the unphysical region close to the physical one. This is an important modification of the electric Born term model proposed by Walker.\textsuperscript{9} It has been examined in detail\textsuperscript{39} that the electric Born term is too large at higher energies and has to be corrected at large angles (low partial waves). This is done by the imaginary part of $T_{II}$ in formula (4·4) and the real part of $T_{II}$ makes the pion exchange background complex. If $T_{II}$ is resonant, the background will be strongly energy dependent in contradiction to the usual assumption.

The origin of the isoscalar background may also be connected to pion exchange, since the gauge invariant Born terms necessitate s-channel proton pole term for proton target (partial waves with $J=1/2$) and u-channel proton exchange for neutron target (all partial waves). This implies relations between isoscalar and isovector background-functions which will be tested, if the data for neutron targets are improved.

A good determination of the structure of background is necessary to separate resonances with small photon couplings (S11E, D13M, P11M).

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Appendix

The isospin notation is

\[ T(\gamma p \rightarrow \pi^0 p) = \sqrt{1/3} T^0 + \sqrt{2/3} T^{v3}, \]
\[ T(\gamma p \rightarrow \pi^+ n) = -\sqrt{2/3} T^p + \sqrt{1/3} T^{v3}, \]
\[ T(\gamma n \rightarrow \pi^0 n) = -\sqrt{1/3} T^N + \sqrt{2/3} T^{v3}, \]
\[ T(\gamma n \rightarrow \pi^- p) = \sqrt{2/3} T^N + \sqrt{1/3} T^{v3}. \]
Isospin 1/2 on proton P or neutron target N, V3 isospin 3/2. Isospin 1/2 (isovector) $T^{\text{is}} = (T^P - T^N)/2$, (isoscalar) $T^{\text{ss}} = (T^P + T^N)/2$. The $T$-matrix elements are related to multipoles defined by CGLN through multiplying a factor $\sqrt{L(L+1)}/kq$, $L$ being the order of multipoles. For example,

$$S_{11}E = E_{1+}\sqrt{2}/kq, \quad P_{33}M = M_{1+}\sqrt{2}/kq,$$

$$D_{13}E = E_{2+}\sqrt{2}/kq, \quad P_{11}M = M_{1+}\sqrt{2}/kq,$$

$$P_{33}E = E_{1+}\sqrt{6}/kq, \quad D_{13}M = M_{1+}\sqrt{6}/kq.$$ 

The normalization of isospin is different from that of CGLN.\textsuperscript{3}

$$T^{\text{is}} = \sqrt{3}T^0, \quad T^{\text{ss}} = \sqrt{1/3}T^{u/z}, \quad T^{\text{rr}} = \sqrt{2/3}T^{u/z}.$$ 

The normalization factor for helicity 1/2 (A) is $\sqrt{2}$, for helicity 3/2 (B$_{n+1,-}$) it is $\sqrt{n(n+2)/2}$. For example, $P_{33}A = A_{1+}\sqrt{2}/kq, P_{33}B = B_{1+}\sqrt{3/2}/kq$.

References

7) P. Noelle, Bonn University, Pl 2-92 (1971).
18) A. Dronchak, INS Symposium on Electron and Photon Interactions at Resonance Region, University of Tokyo, 1975.
Baryon Resonances and Electromagnetic Couplings

18) Particle Data Group, Rev. Mod. Phys. 48 (1976), 211.
19) A. Barbaro-Galtieri, Advances in Particle Physics, Vol. 2 (Interscience).
   R. L. Setti and T. Lasinski, Strongly Interacting Particles (University of Chicago).
24) G. von Gehlen and P. Noelle, to be published.
26) R. H. Dalitz, Strange Particles and Strong Interactions, Oxford.
   Pak, S. Suzuki, T. Matsuda, N. Tokuda, M. Daigo and T. Ohshima, Nucl. Phys. B130
   (1977), 486.
29) M. Yoshioaka, A. Noda, M. Daigo, Y. Hemmi, R. Kikuchi, M. Minowa, K. Miyake, T.
31) P. Noelle, Preprint, Nagoya University DPNU-67-77.
33) M. Bando, S. Machida, T. Nakamura, Quoted by T. Kobayashi at INS Symposium on
   Electron and Photon Interactions at Resonance Region, University of Tokyo, 1973.