Quark-Diquark Model and Nonleptonic Decays of Hyperons and Charmed Baryons

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We obtain a consistent explanation of both S- and P-wave amplitude of nonleptonic hyperon decays assuming the quark-\(0^+\)diquark picture of \(1/2^+\)-baryons, and we also derive the interesting selection rules for charm-changing decays.

§ 1. Introduction

Recently, the diquark has been considered in relation to some exotic resonances in the \(NN\) channel with masses ranging from below thresholds to about 3 GeV.\(^9\) The quark-diquark picture of the proton also seems favourable for the absence of the second dip up to \(|t|\sim10\text{ GeV}^2\) in the \(p-p\) elastic differential cross section in FNAL and CERN-ISR regions.\(^9\)

The diquark is assumed to be an S-wave bound state of two quarks and belongs to a \(21\{36\}\)-dimensional representation of the group \(SU(6)\) [\(SU(8)\)]. Thus there are two kinds of diquarks; (a) an \(SU(3)\) sextet [\(SU(4)\) decuplet] of spin one and (b) an \(SU(3)\) triplet [\(SU(4)\) sextet] of spin zero. Though the masses of these diquarks degenerate in the limit of the \(SU(6)\) [\(SU(8)\)] symmetry, they will be split by a spin-spin force between two quarks which may, for example, come from gluon exchanges. The \(1/2^+\)-baryon is the mixed state of the quark-(1\(^+)\)diquark and the quark-\(0^+\)diquark one.

The quark-(1\(^+)\)diquark model has been discussed in connection with strong and electromagnetic properties of baryons.\(^{9,8}\) In this paper we want to stress that the quark-(0\(^+)\)diquark component dominates in the nonleptonic weak processes of baryons.

The first attempt to adopt the quark-(0\(^+)\)diquark picture in the discussion of nonleptonic hyperon decays was seen in the unitary symmetrical \([\chi\bar{\theta}]\) composite model\(^9\) of baryons, where \(\chi\) and \(\bar{\theta}\) were assumed to be triplet spinor and scalar (or pseudo-scalar) fields, respectively, and considered to correspond either to the urbaryon triplet \(\chi\) and \(\chi\bar{\theta}\) of the quartet model\(^9\) (\(\chi\bar{\theta}\) being the singlet), or to the quark \(q\) and the diquark \(qq\). This model succeeded in a neat explanation of \(S(\Sigma^+) = P(\Sigma^-) = 0\), where, e.g., \(S(\Sigma^+)\) denotes the S-wave amplitude of \(\Sigma^+\)

\(^{8})\) As far as we calculate the mass differences of the \(SU(3)\)-octet baryons,\(^9\) there survives the possibility that the octet baryon consists of a quark and a 0\(^+\)diquark.
→nπ⁺, but the too simple mechanism assumed there for the decays of constituents led to the wrong relation among P-wave amplitudes. As to this, we will discuss later.

§ 2. Selection rules

Assumptions Our basic assumptions are:
(i) The lowest 1/2⁺-baryon consists of a 0⁺-diquark and a 1/2⁺-quark. Those quark and diquark are the 4- and the 6-plet of SU(4).
(ii) The quark and the diquark behave just as free particles throughout the decays. They also have very small overlapping of their wave functions at the origin; they are sufficiently far apart inside the hadron compared with the size of diquark.

Selection rules Since the angular momentum conservation requires
\[ d(0^+ \rightarrow d(0^+) + M_i : l\text{-wave decay}, \]
where \( d \) is \( J=0 \) diquark and \( M_i \) is \( J=l \) meson, assumptions (i) and (ii), if we take the GIM-current, immediately lead to the interesting selection rules for the charmed 1/2⁺-baryon decays;
\[
\begin{align*}
B_{s}^{++} ([c \rho] \rho) &\rightarrow BM_i) \quad \text{\( l\)-wave decay, for \( \cos^2 \theta_c \) terms,} \\
B_{s}^{+} ([c \lambda] \lambda) &\rightarrow B'M_i \quad \text{\( l\)-wave decay, for \( \cos^2 \theta_c \) terms,}
\end{align*}
\]
where \( B \) and \( B' \) are uncharmed 1/2⁺-baryons. For ordinary processes, we obtain
\[ \Sigma^- \rightarrow n\pi^- : S\text{-wave decay.} \]
Note that all these processes must undergo meson emission from only diquark (see Fig. 1). The rules predict the vanishing of the asymmetry parameters for the Cabibbo favoured decays of \( B_{s}^{++} \) and \( B_{s}^{+} \) into 1/2⁺-baryon and meson. This will offer a good test of our model.

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extension of $SU(3)$ to $SU(4)$, our model is essentially the same as the $[\chi\bar{\theta}]$-model. The major difference lies in the structure of weak interaction. In the $[\chi\bar{\theta}]$-model it is assumed that the decays occur through the following two mechanisms: (A) $\bar{\theta}\to\bar{\theta}+\pi$, $\chi\to\chi$ and (B) $\bar{\theta}\to\bar{\theta}$, $\chi\to\chi+\pi$, where $\bar{\theta}\to\bar{\theta}+\pi$ is considered to occur only through $S$-wave $\pi$ emission. $P(S^-) = 0$ is then naturally explained in this model. By further assuming an octet ($\lambda_8$) effective Hamiltonian, the model derives $S(S^+) = 0$ and the Lee-Sugawara relation for $S$-wave amplitudes, but wrong relation among $P$-wave ones: $2P(S^-) + P(A^+) = (1/\sqrt{3}) P(S^+)$. However, we note that this difficulty is not inherent to the quark-diquark picture but only due to the assumed simple mechanisms (A) and (B).

We show the way of dealing it successfully as follows. As to the ordinary nonleptonic decays, we make a further assumption:

(iii) The two-body interaction $\bar{\pi}\lambda + h.c.$ is dominant for the $\Delta S=1$ and $\Delta C=0$ processes ($S$ and $C$ being the strangeness and the charm quantum numbers, respectively). Then, noting that the weak two-body transition of $0^-$-diquark is parity conserving one (recall assumption (ii)) and thus contributes to $P$-wave amplitude only, and that the single-quark-pole diagram also contributes to $P$-wave amplitude only, the set of all possible diagrams of nonleptonic hyperon decays is limited to Figs. 2 and 3. In these figures, the $\bar{\pi}\lambda$ transition of constituent quark in meson-meson transition is not explicitly shown as it is unambiguously specified.

Now we get the amplitudes;

Fig. 2. Diagrams for the $S$-wave decays of hyperons. $\times$ denotes the weak $\bar{\pi}\lambda$ transition.

In the meson-meson transition, $\bar{\pi}\lambda$ interaction is not explicitly shown. Note that diagram (a) has no pole-contribution.

Fig. 3. Diagrams for the $P$-wave decays of hyperons.

\* The assumption of dominant contribution of $\bar{\pi}\lambda$ interaction may have some reasoning such as the recent argument based on the introduction of the right-hand current.\*

We can show that so far as $S$-wave amplitudes are concerned, the results of this section are obtained for the case of usual four-Fermi interaction instead of assumption (iii).
Quark-Diquark Model and Nonleptonic Decays

\[ S(\Lambda^0) = -\sqrt{2} S(\Lambda^0_8) = \left( a - b - 2c \right) / \sqrt{6} , \tag{1a} \]
\[ S(\Sigma^+ - 0) = 0 , \tag{1b} \]
\[ S(\Sigma^- - 0) = -\sqrt{2} S(\Sigma_0^+) = a - b , \tag{1c} \]
\[ S(\Xi^- - 0) = -\sqrt{2} S(\Xi_0^+) = \left( -2a + 2b + c \right) / \sqrt{6} , \tag{1d} \]

and

\[ P(\Lambda^0_8) = -\sqrt{2} P(\Lambda^0_8) = -\left( a' + 2c' \right) / \sqrt{6} - 2b' \Lambda / \sqrt{6} , \tag{2a} \]
\[ P(\Sigma^+_8) = \sqrt{2} P(\Sigma_0^+) = -a' , \tag{2b} \]
\[ P(\Xi^+_8) = 0 , \tag{2c} \]
\[ P(\Xi^-_8) = -\sqrt{2} P(\Xi_0^+) = c' / \sqrt{6} + b' \Xi / \sqrt{6} . \tag{2d} \]

In the above equations, \( a, b \text{ and } c \) denote the contributions of Figs. 2(a), 2(b) and 2(c), respectively, \(^*\) \( a' \) that of non-pole term of Fig. 3(a'), \(^*^*\) and \( b' \text{ and } c' \) the overall contributions of Figs. 3(b') and 3(c'), respectively.

**S-wave amplitudes**

The S-wave amplitudes satisfy the Lee-Sugawara relation as well as the \( \Delta I = 1/2 \) relations and \( S(\Sigma^+ - 0) = 0 \). The correspondence between our amplitudes and those of \([\chi\bar{\theta}]\) model is obvious, as seen from Fig. 2; \( a - b \text{ and } c \) of our amplitudes correspond to the two amplitudes \( -\lambda_1 \text{ and } -\lambda_4 \text{ terms, respectively} \) of the latter model. Our model gives more concrete and informative picture through discriminating (a) and (b) of Fig. 2 which in the case of \([\chi\bar{\theta}]\) model, are combined into a single mechanism (A). It seems worthwhile observing the necessity of the contribution from (c)-type diagram in both models, since if we discard its contribution we are led to the \( 20^\circ \text{-relation}^{10} \) of \( SU(4) \); \( S(\Lambda^0_8), S(\Sigma^+_8), S(\Xi^-_8) = 1; -\sqrt{3}; -2 \), which considerably disagree with experimental values.

**P-wave amplitudes**

The P-wave amplitudes satisfy the \( \Delta I = 1/2 \) relations as well as \( P(\Sigma^-_8) = 0 \), but not the Lee-Sugawara relation. Now we assume the dominance of the \( 1/2^*\)-baryon and \( 0^-\)-meson pole contributions as to Figs. 3(b') and 3(c'), respectively. Then we readily see

\[ b'(\Xi) / b'(\Lambda) = (m_\Xi - m_\Lambda) / (m_\Xi - m_\Lambda) , \tag{3} \]

and the Lee-Sugawara relation, however, is replaced by a new relation;

\[ 2P(\Xi^-) + P(\Lambda^-) = P(\Sigma^+_8) / \sqrt{3} + 2b'(\Xi) - b'(\Lambda) / \sqrt{6} \]
\[ = P(\Sigma^+_8) / \sqrt{3} + 2b'(\Xi) \left[ 1 - \frac{m_\Xi - m_\Lambda}{m_\Lambda - m_\Sigma} \right] / \sqrt{6} , \tag{4a} \]

\(^*\) The non-pole contribution of Fig. 3(b')-type diagram can be included in the parameter \( c \).

\(^*^*\) The contribution of pole term of Fig. 3(a') should vanish due to the rediagonalization of diquark-diquark transition. Note that one gluon emission does not occur for \( d(0^-) \to d'(0^-) \) transition, whereas it occurs for \( \lambda \to \eta \) transition which then cannot be absorbed by rediagonalization into quark mass terms.
Eq. (3) being used in the last equality.

Now we define

$$R = c'/b' (\Xi),$$

then by noticing $P(\Xi^-) = (1 + R) b' (\Xi) / \sqrt{3}$, we get

$$2P(\Xi^-) + P(A_s^e) = P(\Sigma_0^+) / \sqrt{3} + 2P(\Xi^-) \left(1 - \frac{m_Z - m_{\pi}}{m_{\pi} - m_N}\right) / (1 + R). \quad (4b)$$

If we put the experimental values* of the relevant amplitudes into the above equation, we find

$$R_{\exp} \approx -0.76. \quad (5)$$

The problem is now if we can find out a reasonable theoretical account of this value of the ratio $R$. An affirmative answer is given by assuming the following relation to hold:

$$\frac{d''}{f' + d'} = \frac{\alpha_p}{\alpha_f + \alpha_p} = \frac{2(m_K - m_{\pi})}{(m_Z - m_N)(1 + \alpha_p / \alpha_f)}, \quad (6)$$

where $d''$ and $(f', d')$ are the two-body ($\lambda_e$-type) meson- and baryon-coupling constant, respectively, and $\alpha_p$ and $(\alpha_f, \alpha_f)$ the mass-breaking ($\lambda_e$-type) parameters of mesons and of baryons respectively, $(f, F)$ and $(d, D)$ denoting the $F$-type and the $D$-type couplings. The above relation is no more than the assumption of similarity of the ratios of two-body octet couplings of mesons to those of baryons (at least, those of $\lambda_e$ and $\lambda_e$-type interactions). We think this is possible and worthy to be tested in the other processes, but here we just assume it. Then we get, with $\alpha_p / \alpha_f \approx -0.31$,

$$R_{tb} = c'/b' (\Xi) = \frac{m_Z - m_{\pi}}{m_{\pi} - m_K} \frac{d''}{f' + d'}$$

$$= \frac{2(m_Z - m_{\pi})}{(m_Z - m_N)(1 + \alpha_p / \alpha_f)} \approx -0.95. \quad (7)$$

This value seems to give a justification of $R \approx R_{\exp} \approx -0.76$ and we can reconcile our new relation of Eq. (4) with experiments.

The difference between our model and $[\chi \tilde{\phi}]$ model in treatment of $P$-wave processes lies also in the discrimination of the contributions of the diagrams $(b')$ and $(c')$ in Fig. 3 in our case and the indiscrimination of them by putting them into single mechanism (B) in $[\chi \tilde{\phi}]$ model.

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* The data are taken from Review of Particle Properties, 1978.
§ 4. Concluding remarks and comments

We have obtained the successful explanation of nonleptonic hyperon decays of both $S$- and $P$-wave, especially $S(\Sigma^{+}) = 0$ and $P(\Sigma^{-}) = 0$, assuming (i) the quark-(0\textsuperscript{+}) diquark picture of 1/2\textsuperscript{+}-baryons, (ii) the free picture of quarks and diquarks and (iii) the dominance of $\bar{q}q$ interaction, as well as the interesting selection rules for charm-changing decays. Even if we take the four-body interaction instead of the two-body one, we can obtain $S(\Sigma^{+}) = 0$. The vanishing of $S(\Sigma^{+})$ for either of two-body and four-body interactions is the characteristic feature of our model. We also make such comment that the present model also predicts the $P$-wave properties of $A_{1}^{+}(c\bar{p}n) \rightarrow K^{+}\Xi^{0}$, $\pi^{0}\Sigma^{+}$ and $\rho^{+}\Sigma^{0}$, which shall be tested by future experiments.

Among various analyses, recently Igarashi and Shin-Mura have examined the hyperon decays on the basis of constituent-rearrangement diagrams with the $SU(4)$ 15- and 20\textsuperscript{−} weak spurion using the universality of $d/f$ ratios and the spectator assumption. Their results are the following: 1) The 15-contribution dominates over the 20\textsuperscript{−} one for $S$-wave decays. 2) The 20\textsuperscript{−} contribution is important for $P$-wave decays as well as the 15 one. Our model is, consequently, the same as their one as to $S$-wave part since we take only the $n\bar{q}$ transition. For $P$-wave decays, however, our situation is different from theirs. They need the four-body (20\textsuperscript{−}) interaction since they express the contributions of ($b^{\prime}$) and ($c^{\prime}$) in Fig. 3 by single amplitude, while we do not need it since we express them by distinct ones; the baryon- and the meson-pole one, although we assume a weaker condition, Eq. (6), than the universal $d/f$ ratios.

In this paper we have discussed the quark-(0\textsuperscript{+}) diquark model of 1/2\textsuperscript{+}-baryons in connection with their nonleptonic decays. It should, however, be noted that we have to introduce the spin-one diquark component in addition to the spin-zero diquark in calculating, e.g., the asymmetry parameter of the $\beta$-decay process $\Sigma^{-} \rightarrow ne^{-}\nu$, the amplitudes of $\Omega^{-}$ decays and some properties of the strong and electromagnetic interactions of baryons.\textsuperscript{81} Although we find no convincing explanation, at the present stage, as to why the quark-(0\textsuperscript{+}) diquark picture dominates in the nonleptonic decay processes, we think it may be related to the dynamics of quarks and especially to the origin of the $\Delta I = 1/2$ enhancement.\textsuperscript{82}

Acknowledgement

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\textsuperscript{81} As we readily see, we obtain the vanishing of the asymmetry parameter of the radiative transition, $\Sigma^{-} \rightarrow p\bar{\nu}$, even if we use the Fritsch-Minkowski current.\textsuperscript{82}
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