Characteristic Features of $\phi(4.16)$ and $\phi(4.4')$
in Their Decays into Charmed Mesons

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The decay properties of the resonances with their masses between 4.0 and 4.5 GeV are investigated through the analysis of the experiments on total and various types of partial cross sections. Among them the 2-prong cross section is found to play an essential role to characterize each resonance. The conclusion of our analysis is that the dominant decay channel of $\phi(4.16)$ is $F\bar{F}$ pair and that the peak at 4.4 GeV in $\sigma(\text{tot})$ is not a single resonance but is composed of at least two resonances; one of which decays mainly into $D\bar{D}^{*}$ and the other into $F\bar{F}^{*}$ or $F^*\bar{F}^*$. 

§ 1. Introduction

Experiments on $e^+e^-$ annihilation process above charm threshold show several remarkable resonance-like peaks in the total,$^{(5,6,7)}$ 2- and 4- charged prong,$^{(5)}$ $K^{0}\pi\pi$ and $D^{0,0'}$ inclusive cross sections and so on. The observed peaks in $\sigma(\text{tot})$ and $\sigma(\text{partial})$ are located around 3.77, 3.98, 4.04, 4.16 and “4.4” GeV. It is interesting to note that each peak observed in the partial cross sections shows the behaviour different from the corresponding peak in $\sigma(\text{tot})$. Some peaks (at 4.16 GeV for example) are considerably enhanced in $\sigma(\text{2pr})$ but not in $\sigma(\text{4pr})$ while for others (at 4.04 GeV for example) vice versa.

In this paper we start with the assumption that each peak corresponds to a new resonance and study the properties of those suggested resonances by comparing the structure of $\sigma(\text{partial})$ with that of $\sigma(\text{total})$, where we pay our particular attention to the 2-prong cross section. It will be found out that each resonance decays into its own characteristic charmed meson pair whose threshold energy is just below the resonance.

For the convenience of later discussion let us here summarize the situation of the experimental data. The $\phi(3.77)$ resonance decays into pure $D\bar{D}$ channel. At 3.98 GeV we observe a broad peak the detail of which is not yet known because of poor statistics. The anomalously strong coupling of $\phi(4.04)$ to $D^*\bar{D}^*$ state has induced one of the most interesting questions as to the existence of the “molecular charmonium states”. The peak observed around 4.16 GeV in PLUTO and DASP data is slightly shifted to lower energy ($\sim$4.10 GeV) in SLAC data.$^{(5,6,7)}$ Also it must be noted that the $F$ meson signal is reported at 4.16 GeV by SLAC group.$^9$ The peak around 4.4 GeV shows a complicated structure in $\sigma(\text{tot})$; sometimes it seems that
the broad peak is split into 2 or 3 narrow peaks. DASP group reports that \( \sigma(\eta_{\text{low}}X) \) amounts to a few nb around 4.4 GeV where most of the events are originated from the \( F^* \bar{F} \) or \( \bar{F}^*F^* \) productions.\(^\text{10}^\) According to the SLAC report,\(^\text{11}^\) the \( D \) meson production cross section is enhanced at 4.415 GeV, where the spectrum of the recoil mass of \( D^0 \) meson indicates that the large part of \( D \) meson production comes from a certain high mass state of \( D \) meson.

§ 2. Qualitative study of the experimental data

First of all let us define the "charm part" of the cross sections as follows:

\[
R_{\text{charm}}^{\text{(total)}} = \frac{\sigma_{\text{charm}}^{\text{(total)}}}{\sigma^{\eta_l}} = R(\text{total}) - R^{\text{hl.}}(\text{total}) - R^{\text{old}}(\text{total}), \quad (2.1a)
\]

\[
R_{\text{charm}}^{\text{(partial)}} = R(\text{partial}) - R^{\text{hl.}}(\text{partial}) - R^{\text{old}}(\text{partial}), \quad (2.1b)
\]

where \( R^{\text{hl.}} \) and \( R^{\text{old}} \) are the contributions from heavy leptons and non-charmed hadrons, respectively. Now we determine the concrete form of \( R^{\text{old}} \) and \( R^{\text{hl.}} \) in Eq. (2.1) to estimate the charm part of the cross sections. The \( R^{\text{old}}(\text{tot}) \) is the contribution from non-charmed hadrons and can be evaluated as

\[
R^{\text{old}}(\text{tot}) = R(\text{below charm threshold}) = R(\sqrt{s} = 3.6 \text{ GeV}), \quad (2.2)
\]

and heavy lepton contribution is

\[
R^{\text{hl.}} = \sqrt{1 - \frac{4M^2}{E^2} \left(1 + \frac{4M^2}{2E^2}\right)}, \quad (2.3)
\]

where recent reports indicate \( M = 1.8 \text{ GeV}.\(^\text{12}^\)

As for the old part of the \( n \)-prong cross section, we have

\[
R^{\text{old}}(n \text{ pr.}) = \left[ \frac{2}{n!} \langle n_{\text{old}} \rangle^n \exp(-\langle n_{\text{old}} \rangle) \right] \times R^{\text{old}}, \quad (2.4)
\]

in terms of \( \langle n_{\text{old}} \rangle \), the average charged multiplicity of non-charmed hadrons, where we assume the charged multiplicity distribution of the Poisson type. The \( \langle n_{\text{old}} \rangle \) can be extrapolated smoothly from the data below charm threshold:\(^\text{13}^\)

\[
\langle n_{\text{old}} \rangle = \ln (E_{\text{cm}}/1 \text{ GeV}) + 2.8. \quad (2.5)
\]

The heavy lepton contributions to the 2- and 4-prong cross sections are estimated as

\[
R^{\text{hl.}}(2 \text{ pr.}) = 0.74 \times R^{\text{hl.}}, \quad (2.6a)
\]

\[
R^{\text{hl.}}(4 \text{ pr.}) = 0.24 \times R^{\text{hl.}}, \quad (2.6b)
\]

where we use the theoretical value of \( \text{B.R.}(\tau \rightarrow 1 \text{ prong}) = 0.86.\(^\text{10}^\)
Characteristic Features of $\phi(4.16)$ and $\phi(4.4)$

Now we plot the charm part of the cross sections, $R_{\text{charm}}^{(\text{tot})}$, $R_{\text{charm}}^{(2 \text{ pr.)}}$ and $R_{\text{charm}}^{(4 \text{ pr.)}}$ using the available data measured by PLUTO collaboration. It should be noted that $R_{\text{charm}}^{(2 \text{ pr.)}}$ and $R_{\text{charm}}^{(4 \text{ pr.)}}$ show different energy dependence from each other at corresponding peaks in $\sigma_{\text{charm}}^{(\text{tot})}$. For example at 4.03 GeV we see a large enhancement in $R_{\text{charm}}^{(4 \text{ pr.)}}$ and a comparatively small enhancement in $R_{\text{charm}}^{(2 \text{ pr.)}}$, and vice versa at 4.16 GeV. Further we observe a broad peak around 4.4 GeV in $R_{\text{charm}}^{(\text{tot})}$, whereas $R_{\text{charm}}^{(4 \text{ pr.)}}$ has an enhancement at 4.44 GeV but $R_{\text{charm}}^{(2 \text{ pr.)}}$ has peaks at 4.346 GeV and at 4.385 GeV. These facts indicate that there exist two types of resonances above charm threshold, one of which decays mainly into 2 prong channel and the other decays mainly into 4 or more prong channel. Thus it is naturally recognized that the 4.4 GeV peak in $\sigma^{(\text{tot})}$ is not a single resonance but is composed of at least two resonances. Here we take the probable case with three resonances, $\phi(4.33)$ and $\phi(4.39)$ of the latter type and $\phi(4.43)$ of the former type.

In Table I we show the ratios $R_{\text{charm}}^{(2 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})}$ and $R_{\text{charm}}^{(4 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})}$ at resonance peaks. The resonances with $R_{\text{charm}}^{(2 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})} \lesssim 20\%$ and $R_{\text{charm}}^{(4 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})} \gtrsim 60\%$ can be considered to decay mainly into $D$ meson pairs such as $D\bar{D}$, $DD^*$ and $D^*\bar{D}$, since the value $R_{\text{charm}}^{(2 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})} \approx 20\%$ is consistent with the information of $B.R.(D^*\bar{D} \rightarrow 2 \text{ prong}) = 11 \pm 7\%$ and $B.R.(D^+D^- \rightarrow 2 \text{ prong}) = 17 \pm 8\%$. In fact strong signals of $D$ meson

Table I. The 2-prong and 4-prong ratio at each resonance peak evaluated from the PLUTO data.

<table>
<thead>
<tr>
<th>$E_{\text{cm.}}$ (GeV)</th>
<th>$R_{\text{charm}}^{(2 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})}$</th>
<th>$R_{\text{charm}}^{(4 \text{ pr.)}}/R_{\text{charm}}^{(\text{tot})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.028</td>
<td>0.217±0.020</td>
<td>0.612±0.024</td>
</tr>
<tr>
<td>4.156</td>
<td>0.336±0.097</td>
<td>0.510±0.098</td>
</tr>
<tr>
<td>4.336</td>
<td>0.373±0.134</td>
<td>0.537±0.132</td>
</tr>
<tr>
<td>4.385</td>
<td>0.484±0.132</td>
<td>0.410±0.109</td>
</tr>
<tr>
<td>4.444</td>
<td>0.091±0.101</td>
<td>0.802±0.149</td>
</tr>
</tbody>
</table>
pairs are observed at 4.03 and 4.415 GeV. On the other hand the high ratio of \( R_{\text{charm}}^{(2 \text{pr.})}/R_{\text{charm}}^{(\text{tot})} \) suggests that the resonances \( \psi(4.16), \psi(4.33) \) and \( \psi(4.39) \) mainly decay into another kind of charmed meson pair, i.e., \( FF*, F^*F^* \) or \( F^*F^* \) channel. Since the peak at 4.16 GeV is located below \( FF^* \) or \( F^*F^* \) threshold, \( \psi(4.16) \) decays only into \( FF \) pair. The cross section of \( \sigma (\eta'_{\text{low}}X) \) at 4.40 GeV has comparable order with \( \sigma (\text{tot}), \) which suggests that the resonance \( \psi(4.33) \) and \( \psi(4.39) \) mainly decay into \( F^*F \) or \( F^*F^* \).

All the above considerations lead us to propose that each resonance above charm threshold decays mainly into its own characteristic charmed meson pair, whose threshold energy is about 100 MeV below the resonance peak. We call this assumption “small \( Q \)-value channel dominance”. In Table II we show the channel to which each resonance couples dominantly.

<table>
<thead>
<tr>
<th>resonance</th>
<th>main decay mode</th>
<th>( Q ) value (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi(3.77) )</td>
<td>( D^0\bar{D}^+, \ D^+D^- )</td>
<td>44, 22</td>
</tr>
<tr>
<td>( \psi(3.98) )</td>
<td>( D^0\bar{D}^{<em>+}, \ D^+D^{</em>+} )</td>
<td>103, 93</td>
</tr>
<tr>
<td>( \psi(4.04) )</td>
<td>( D^{<em>+}\bar{D}^{</em>+}, \ D^{<em>+}D^{</em>+} )</td>
<td>32, 28</td>
</tr>
<tr>
<td>( \psi(4.16) )</td>
<td>( F^*F^- )</td>
<td>80</td>
</tr>
<tr>
<td>( \psi(4.33) )</td>
<td>( F^<em>F^{</em>-} )</td>
<td>130</td>
</tr>
<tr>
<td>( \psi(4.39) )</td>
<td>( F^<em>F^{</em>-} )</td>
<td>50</td>
</tr>
<tr>
<td>( \psi(4.43) )</td>
<td>( D^0\bar{D}^{<em>+}, \ D^+D^{</em>+} )</td>
<td>( \sim110, \sim110 )</td>
</tr>
</tbody>
</table>

Then the charm part of the cross section is estimated by summing up each resonance:

\[
R_{\text{charm}}^{(\text{tot})} = \sum_i R_i^{(i)} (E), \tag{2.7}
\]

using the B.W resonance formula

\[
R_i^{(i)} (E) = \frac{4m_i^2 \Gamma_{ee}^{(i)} \Gamma_{\text{tot}}^{(i)} K^{(i)} (E)}{\sigma^{ee} E^2 (E^2 - m_i^2)^2 + m_i^2 \Gamma_{\text{tot}}^{(i)} K^{(i)} (E)}, \tag{2.8}
\]

where \( m_i \), \( \Gamma_{ee}^{(i)} \) and \( \Gamma_{\text{tot}}^{(i)} \) are the mass, electronic width and total width of the \( i \)-th resonance, respectively. The effect of the form factor and threshold behaviour is represented by

\[
K^{(i)} (E) = f^{(i)} (E) / f^{(i)} (E = m_i), \tag{2.9}
\]

\[
f^{(i)} (E) = \left( \frac{P^2}{P^2 + \Lambda^2} \right)^{(2i+1)/2}, \tag{2.10}
\]

and \( f^{(i)} (E) \) behaves as

\[
f^{(i)} (E) \sim P^{2i+1} \text{ at } E \sim E_i, \tag{2.11}
\]
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where $P$ is the c.m. momentum of the produced charmed meson and $E_i$ is the threshold energy of the meson pair. The 2-prong cross section is written as follows:

$$R^\text{charm} (2 \text{ pr.}) = \sum_i \varepsilon^{(i)} (2 \text{ pr.}) R^{(i)} (E),$$

(2.12)

where $\varepsilon^{(i)} (2 \text{ pr.})$ is the branching ratio of the $i$-th resonance (i.e., of the charmed meson pair which couples dominantly to the $i$-th resonance) to 2 charged prongs.

§ 3. Numerical analysis

In this section we determine the parameters, $m^{(i)}$, $\Gamma_{\text{tot}}^{(i)}$, $\Gamma_{\text{ee}}^{(i)}$, and $\varepsilon^{(i)} (2 \text{ pr.})$ of each resonance so as to minimize the $\chi^2$ value and check quantitatively the idea obtained from the qualitative consideration.

According to the discussion in §2 we consider seven resonances in the energy region between 3.7~4.5 GeV, whose typical decay channels are listed in Table II. The charm part of $R(\text{tot})$ and $R(2 \text{ pr.})$ are represented in terms of the parameters to be determined (see Eqs. (2.7), (2.8) and (2.12)). Among the presently existing data on the cross sections, the most convenient one for our analysis turns out to be of PLUTO, which provides us the information on $R(\text{tot})$ as well as $R(2 \text{ pr.})$ above 4 GeV, and we mainly use the PLUTO data to determine the parameters of the resonances with their mass heavier than 4 GeV. The determination of the parameters is made by the following procedure.

First the parameters $m^{(i)}$, $\Gamma_{\text{ee}}^{(i)}$, and $\Gamma_{\text{tot}}^{(i)}$ of the resonances below 4 GeV are fixed to minimize the $\chi^2$ value to fit the SLAC data on $R^\text{charm}(\text{tot})$. Next the parameters $m^{(i)}$, $\Gamma_{\text{ee}}^{(i)}$, $\Gamma_{\text{tot}}^{(i)}$, and $\varepsilon^{(i)} (2 \text{ pr.})$ of the rest of five resonances are determined by $\chi^2$ fit to the PLUTO data on $R^\text{charm}(\text{tot})$ and $R^\text{charm}(2 \text{ pr.})$ for the energy range from 4.0 GeV to 4.5 GeV. Here we assume the common $\varepsilon(2 \text{ pr.})$ value for all resonances below 4.08 GeV.

All the results obtained in the above procedure are summarized in Table III. The calculated curves with these parameters are shown in Fig. 1.

It must be remarked that the set of all parameters of the resonances thus

<table>
<thead>
<tr>
<th>$M$ (GeV)</th>
<th>$\Gamma_{ee}$ (KeV)</th>
<th>$\Gamma_{tot}$ (GeV)</th>
<th>$\varepsilon_{pr}$</th>
<th>Energy Range of the fitting (GeV)</th>
<th>$\chi^2$/No. of DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PLUTO SLAC DASP</td>
<td></td>
</tr>
<tr>
<td>3.770</td>
<td>0.330</td>
<td>0.628</td>
<td>0.180</td>
<td>3.6 ~ 4.68</td>
<td>1.41 0.68 0.97</td>
</tr>
<tr>
<td>3.975</td>
<td>0.987</td>
<td>0.630</td>
<td>0.180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.044</td>
<td>0.803</td>
<td>0.687</td>
<td>0.180</td>
<td>4.08 ~ 4.20</td>
<td>0.72 3.23 1.56</td>
</tr>
<tr>
<td>4.161</td>
<td>0.807</td>
<td>0.680</td>
<td>0.260</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.330</td>
<td>0.222</td>
<td>0.031</td>
<td>0.236</td>
<td>4.20 ~ 4.50</td>
<td>1.49 1.95 1.82</td>
</tr>
<tr>
<td>4.391</td>
<td>0.172</td>
<td>0.022</td>
<td>0.513</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.430</td>
<td>0.718</td>
<td>0.071</td>
<td>0.173</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table III. The parameters for each resonance to minimize the $\chi^2$ value in fitting the PLUTO data on $R^\text{charm}(\text{tot})$ and $R^\text{charm}(2 \text{ pr.})$, where we take $X=0.3$ GeV in Eq. (2.10).
determined reproduces the behavior of \(R(\text{tot})\) measured by another groups, SLAC\(^9\) and DASP\(^9\) as well (see Table III). From our results one finds out:

(i) The total width of \(\phi(3.98)\) might be too broad to identify a stable state and the enhancement may be as a result of some kinematical effect.

(ii) The parameters of the resonances \(\phi(4.04)\) and \(\phi(4.16)\) are considerably well fixed and we notice that they belong to the two different types of resonances, the one with \(\varepsilon(2 \, \text{pr.}) = 0.18\) and the other with \(\varepsilon(2 \, \text{pr.}) = 0.26\).

(iii) The obtained values of \(\varepsilon(2 \, \text{pr.})\) of \(\psi(4.43)\) (=0.17) and of \(\psi(4.33)\) (=0.24) are compatible with what we have anticipated from their decay channels, while the one of \(\psi(4.39)\) (=0.51) is a little larger than what might be expected. We may consider an alternative case where the observed peak at 4.33 GeV does not correspond to a resonance. In this case, however, \(\chi^2\) fit requires a considerably large total width (\(\sim 100\) MeV) for \(\psi(4.39)\) and negative contribution of \(\psi(4.43)\) is needed in order to reproduce the structure of the peak. Thus we conclude that three-resonance fit is a better choice so far as we study the present experimental data.

Now we refer to the inclusive kaon production which provides an interesting information about the difference of the decay mechanism of \(F\) and \(D\). The data on \(K^0\) and/or \(K^\pm\) inclusive production are reported from SLAC\(^6\) and DESY\(^4,5\).\(^\text{a}\)\(^,\)\(^b\) We observe a strong peak at 4.04 GeV both in SLAC and DESY, whereas no enhancement is seen at 4.16 GeV. These facts imply that the branching ratio \(B(F \rightarrow K K X)\) is considerably small and the charm part of \(R(KX)\) mainly comes from \(D\) production. Thus \(R^{\text{charm}}(KX)\) is evaluated as

\[
2R^{\text{charm}}(K^0X) \sim R(DX) \quad (3.1)
\]

assuming that \(B(D \rightarrow K^\pm X) \approx B(D \rightarrow K^0 X)\) (see Fig. 2).

The rapid energy dependence of \(R^{\text{charm}}(KX)\) around 4.4 GeV is expected

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\(^{a}\) We would like to remark that as for the PLUTO's data the luminosity is located almost at 4.40 GeV for the data point between 4.40 and 4.43 GeV.
since there exist a few resonances with different decay modes in the broad peak at 4.4 GeV. At 4.415 GeV the SLAC data shows a peak corresponding to the resonance $\psi(D\bar{D}^{**})$. On the other hand no enhancement is seen in PLUTO’s data at 4.40 GeV which is well off the resonance $\psi(4.43)$.

§ 4. Discussion

So far one of the theoretical interests has been in the problem how to understand the remarkably large $D^0\bar{D}^0$ fraction in the decay of $\psi(4.04)$. There has been proposed two types of models: (1) $\psi(4.04)$ is the so called molecular resonance rather than a pure charmonium state, (2) the wave function of the radially excited state of the charmonium has its characteristic nodes, which cause the suppression of the other decay modes. The results of our analysis imply that other resonances above charm threshold have also such a characteristic property of the single channel dominance. Thus in either case of the above models the properties of those resonances should be explained consistently. In a separate paper it is shown by one of us (H.A.) that the naive charmonium model is still available for describing those resonances, where the peaks at 3.98 GeV and 4.33 GeV should be interpreted as due to just kinematical enhancement.

We would like to remark on the $F$ meson decay mode. The value of $\varepsilon(2 \text{ pr.}) = 34^{+10}_{-10}$ for $F\bar{F}$ pair indicates that the branching ratio $B(F \rightarrow 1 \text{ charged})$ is about $60^{+10}_{-10}$%, which is far larger than expected from a model calculation. Possible explanations for such a high branching ratio would be the following: (1) hadronic decay modes with low charged multiplicity such as $\eta\pi^+$, $\eta\pi^+\pi^0$, $K^+\bar{K}^0$, and $K^+\bar{K}^0\pi^0$ are enhanced, (2) pure leptonic decay mode of $F^+ \rightarrow \tau^+\nu$ is enhanced, (3) vector meson $F_{\tau}$ is lighter than pseudoscalar meson $F_{PS}$ and the mode $F_{\tau} \rightarrow l\nu$ has comparatively large branching ratio. In the case of (2), however, the decay constant of $F$ meson must be larger by a factor 3 or 4 than that of $\pi$-on so that the branching ratio $B(F^+ \rightarrow \tau^+\nu)$ becomes of the order of a few 10%. Further in the last case, a considerable amount of enhancement in anomalous $e\mu$ events and 2-prong electron events should be observed at the energy of $F\bar{F}$ resonance, whereas no enhancement is seen in the cross sections at 4.16 GeV.

We hope that experiments will confirm our version of small $Q$-value channel dominance in the near future.

Acknowledgements

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