the tube radius must be extremely small and the viscosity very high. Greenberg's [3] report includes a study of the effect of \( \theta \) and \( \eta \) on the maximum displacement and time of maximum displacement. This information is useful when only the first cycle or first half cycle is to be used. For \( \frac{\epsilon}{2\beta} = 10^{-1} \), no damping effects are evident during the first cycle so that the results given in [3] hold. For \( \frac{\epsilon}{2\beta} = 10^{-1} \), some damping appears but not an appreciable amount and the general conclusions of [3] are nearly the same.

It is of interest to point out that for these low values of \( \frac{\epsilon}{2\beta} \), as for Greenberg's calculation with \( \epsilon/\beta = 0 \), the maximum displacement is nearly constant for a large range in \( \eta \). The lower limit of this range depends on \( \theta \). For \( \theta = \frac{1}{4} \), this behavior begins for \( \eta \) less than one and remains nearly constant as \( \eta \) increases. For smaller values of \( \theta \), there is a sharper rise up to the constant value (the asymptote of these curves). A similar observation is made for the curves showing time of maximum displacement as a function of \( \eta \).

The two solutions of Section 3 and those calculated by Greenberg for \( \epsilon/\beta = 0 \) are based on the approximation that the displacement of the mass is very small. For \( \theta \) very small, this need not be true. In this case, the initial motion of the mass due to the first wave arriving from the near end may be large. With this in mind, Miranker [4] calculated the nonviscous, plane, acoustic case (\( \epsilon/\beta = 0 \)) for the situation in which the mass motion may be large. This entails applying the boundary conditions of Equations (25), (26), and (27) on the moving mass at its local position at any time and not at its original position \( X = \theta \pi \). To do this, it is necessary to follow the path of the wave fronts and to effectively plot out the mass motion. The behavior of the mass during the intervals between the times when it is struck by incident or reflected waves is governed by ordinary differential equations. The solution of each of these in its time interval not only gives the reflected waves is governed by ordinary differential equations. This kind of calculation has been carried out so as to include viscous effects but since these effects have been shown to have such slight influence in the linear case it may be inferred that this will be nearly the same in the more exact treatment. A comparison of the exact treatment and the nonviscous, acoustic results may be seen in [4].

It seems quite clear that damping effects due to viscosity and the presence of boundaries will only be appreciable when the tube radius is extremely small or the viscosity very high. This means that the acoustic solution of Greenberg [3] will probably be correct for most applications. The results of the nonlinear boundary condition calculation of [4] also bear out this conclusion. Thus, in studying time-dependent pressure inputs, for instance, it is certainly reasonable to neglect viscous effects.

References

Discussion

F. T. Brown

The authors have treated in elegant style a specific problem involving a superposition of the two fundamental modes of fluid motion in a tube: The longitudinal primary mode and the "toroidal" secondary mode, involving a series of counterrotating toroidal-like eddies. In general, the primary mode strongly predominates in a long tube, except in some cases near the ends. For a short tube the secondary mode becomes significant if the end conditions are those which require such a mode. For example, the authors' piston requires a toroidal flow to be superposed on the longitudinal flow, but this mode is insignificant far from the piston face. A pure pressure end condition does not of itself generate the secondary mode, but will of course reflect it.

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Nichols\(^5\) has mathematically demonstrated the superposition of the linearized modes, and made great progress toward the frequency analysis of the secondary mode (as well as a third or rotary mode). Iberall,\(^6\) Nichols,\(^7\) and the writer\(^8\) have given the frequency response of the linearized primary mode, and the writer has also given the transient response. In none of this work is it necessary to distinguish boundary layer and core flows.

In the authors' problem and others like it the dispersion of the primary mode alone is a good indication of the effect of viscosity on the response of the system. The writer has shown\(^8\) that impulses and steps of pressure at one end of a liquid-filled tube produce primary-mode pressure waves of the following shapes at position \(L\) downstream, assuming laminar flow, neglecting any reflected wave, and assuming the tube is not too long. Modifications are made for long tubes. The writer will add the long "tails" to these responses in a future paper.

\(^6\) N. B. Nichols, of Taylor Instrument Company, is urged to publish this considerable advance at an early date!


It may be directly seen, for example, that the value \((\epsilon/2\beta) = 10^{-4}\) produces negligible dispersion, as indicated by the authors. It is hoped that, after all of the foregoing developments are completely published, a wide scope of practical problems can be solved easily, without resorting to the sophisticated techniques of the authors.