



Fig. 5 The influence of the amount of plasticizer on the behavior of master creep $D(t)$ -curve for a cold-setting epoxy resin C-100-0-8

Thiokol polysulfide LP-3, which proved to be an excellent material. Master creep and relaxation curves describing the mechanical and optical properties of the materials were constructed, and it was proved that the linearity of the creep and relaxation laws of these materials is valid for low, as well as for high, temperatures. This linearity is unaffected by the addition of plasticizer. Addition of plasticizer in the epoxy polymer corresponds to an increase of temperature of the pure epoxy resin. In other words, the glassy and rubbery stages are shifted along the axis of ordinates with the addition of plasticizer. Fig. 5 shows the influence of plasticizer on the creep compliance $D(t)$ of a C-100-0-8 epoxy polymer.

On a Restricted Class of Coupled Hill's Equations and Some Applications¹

T. K. CAUGHEY.² While the class of problems to which the paper applies is quite restricted, it is applicable to a number of problems of physical interest, which, up until now, could be analyzed only with considerable difficulty.

The writer would like to illustrate a simple extension of the theory to a restricted class of problems which include damping in addition to parametric excitation.

Consider the system of equations:

$$I\ddot{x} + C\dot{x} + B^{(0)}x = 0 \quad (1)$$

where $B^{(0)}$ and C are symmetric non-negative matrices of order n . The writer³ has shown that the system (1) will have real eigenvectors if and only if C is diagonalized by the same transformation which uncouples the undamped system. If C satisfies this condition, then $B^{(0)}$ and C commute with one another.

¹ By C. S. Hsu, published in the December, 1961, issue of the JOURNAL OF APPLIED MECHANICS, vol. 28, TRANS. ASME, vol. 83, Series E, pp. 551-556.

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³ T. K. Caughey, "Classical Normal Modes in Damped Linear Dynamic Systems," JOURNAL OF APPLIED MECHANICS, vol. 27, TRANS. ASME, vol. 82, Series E, 1960, pp. 269-271.

NOTE: Bold-face capitals = matrices; bold-face lower-case letters = vectors.

Consider now the system of equations:

$$I\ddot{x} + C\dot{x} + B^{(0)}x + \sum_{i=1}^N f_i(t) B^{(i)}x = 0 \quad (2)$$

where

- 1 $f_i(t)$ = periodic functions with period T , $i = 1, 2, \dots, N$
- 2 $B^{(i)}$ = square matrices of order n which commute with $B^{(0)}$, and hence with C

Let

$$x = Tz \quad (3)$$

be the transformation which uncouples the undamped system. Equation (2) may therefore be reduced to

$$I\ddot{z} + \Gamma\dot{z} + \lambda^0 z + \sum_{i=1}^N f_i(t) \lambda^{(i)} z = 0 \quad (4)$$

where

$$\left. \begin{aligned} \Gamma &= T^{-1}CT \\ \lambda^{(i)} &= T^{-1}B^{(i)}T \\ i &= 0, 1, 2, \dots, N \end{aligned} \right\} \text{are diagonal matrices} \quad (5)$$

Taking the j th row in (4)

$$\ddot{z}_j + \Gamma_j \dot{z}_j + \left[\lambda_j^{(0)} + \sum_{i=1}^N f_i(t) \lambda_j^{(i)} \right] z_j = 0 \quad (6)$$

Let

$$z_j = u_j e^{-\frac{1}{2}\Gamma_j t} \quad (7)$$

Hence

$$\ddot{\mu}_j + \left\{ \lambda_j^{(0)} - \left(\frac{1}{2}\Gamma_j\right)^2 + \sum_{i=1}^N \lambda_j^{(i)} f_i(t) \right\} \mu_j = 0 \quad (8)$$

Equation (8) is a Hill's equation and can therefore be treated by Floquet's theory⁴

$$\therefore \mu_j = A e^{\zeta_j t} \Phi(t) + B e^{-\zeta_j t} \Phi(-t) \quad (9)$$

where $\Phi(t)$ is a periodic function, and ζ_j is either real, or pure imaginary. Combining (7) and (9)

$$z_j = A e^{(\zeta_j - \frac{1}{2}\Gamma_j)t} \Phi(t) + B e^{(-\zeta_j - \frac{1}{2}\Gamma_j)t} \Phi(-t) \quad (10)$$

If ζ_j is real, z_j will be stable if

$$|\zeta_j| \leq \frac{1}{2}\Gamma_j \quad (11)$$

For stability of the whole system (11) must hold for all $j = 1, 2, \dots, n$.

Gas-Lubricated Cylindrical Journal Bearings of the Finite Length¹

W. A. MICHAEL.² A reading of this paper raises several questions the answers for which would enhance its usefulness. In approximately increasing order of importance, they are:

- 1 Should not the boundary condition for p be $p = 12\pi/\Lambda$, rather than $p = 1$, as given?
- 2 Why is the iteration not started with $p_{i,j} = 12\pi/\Lambda$, rather than zero as stated?

⁴ E. L. Ince, "Ordinary Differential Equations," Dover, 1956.

¹ By B. Sternlicht, published in the December, 1961, issue of the JOURNAL OF APPLIED MECHANICS, vol. 28, TRANS. ASME, vol. 83, Series E, pp. 535-543.

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