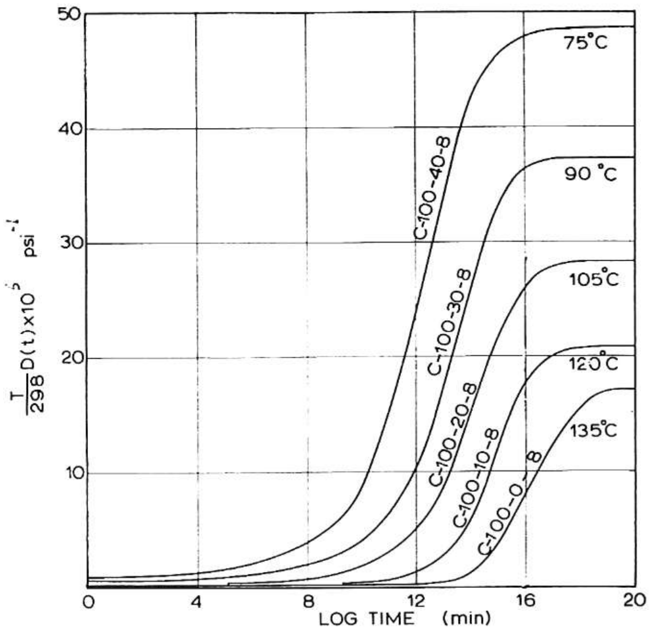

Fig. 5 The influence of the amount of plasticizer on the behavior of master creep *D(t)*-curve for a cold-setting epoxy resin C-100-0-8

Thiokol polysulfide LP-3, which proved to be an excellent material. Master creep and relaxation curves describing the mechanical and optical properties of the materials were constructed, and it was proved that the linearity of the creep and relaxation laws of these materials is valid for low, as well as for high, temperatures. This linearity is unaffected by the addition of plasticizer. Addition of plasticizer in the epoxy polymer corresponds to an increase of temperature of the pure epoxy resin. In other words, the glassy and rubbery stages are shifted along the axis of ordinates with the addition of plasticizer. Fig. 5 shows the influence of plasticizer on the creep compliance $D(t)$ of a C-100-0-8 epoxy polymer.

On a Restricted Class of Coupled Hill's Equations and Some Applications[1]

T K. CAUGHEY.[2] While the class of problems to which the paper applies is quite restricted, it is applicable to a number of problems of physical interest, which, up until now, could be analyzed only with considerable difficulty.

The writer would like to illustrate a simple extension of the theory to a restricted class of problems which include damping in addition to parametric excitation.

Consider the system of equations:

$$\mathbf{I}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{B}^{(0)} = 0 \qquad (1)$$

where $\mathbf{B}^{(0)}$ and $\mathbf{C}$ are symmetric non-negative matrices of order n. The writer[3] has shown that the system (1) will have real eigenvectors if and only if $\mathbf{C}$ is diagonalized by the same transformation which uncouples the undamped system. If $\mathbf{C}$ satisfies this condition, then $\mathbf{B}^{(0)}$ and $\mathbf{C}$ commute with one another.

[1] By C. S. Hsu, published in the December, 1961, issue of the Journal of Applied Mechanics, vol. 28, Trans. ASME, vol. 83, Series E, pp. 551–556.

[2] Associate Professor of Applied Mechanics, California Institute of Technology, Pasadena, Calif.

[3] T. K. Caughey, "Classical Normal Modes in Damped Linear Dynamic Systems," Journal of Applied Mechanics, vol. 27, Trans. ASME, vol. 82, Series E, 1960, pp. 269–271.

Note: Bold-face capitals = matrices; bold-face lower-case letters = vectors.

Consider now the system of equations:

$$\mathbf{I}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{B}^{(0)}\mathbf{x} + \sum_{i=1}^{N} f_i(t)\,\mathbf{B}^{(i)}\mathbf{x} = 0 \qquad (2)$$

where

1 $f_i(t)$ = periodic functions with period T, $i = 1, 2, \ldots N$
2 $\mathbf{B}^{(i)}$ = square matrices of order n which commute with $\mathbf{B}^{(0)}$, and hence with $\mathbf{C}$

Let

$$\mathbf{x} = \mathbf{T}\mathbf{z} \qquad (3)$$

be the transformation which uncouples the undamped system. Equation (2) may therefore be reduced to

$$\mathbf{I}\ddot{\mathbf{z}} + \boldsymbol{\Gamma}\dot{\mathbf{z}} + \boldsymbol{\lambda}^{0}\mathbf{z} + \sum_{i=1}^{N} f_i(t)\,\boldsymbol{\lambda}^{(i)}\mathbf{z} = 0 \qquad (4)$$

where

$$\left.\begin{array}{l}\boldsymbol{\Gamma} = \mathbf{T}^{-1}\mathbf{C}\mathbf{T}\\ \boldsymbol{\lambda}^{(i)} = \mathbf{T}^{-1}\mathbf{B}^{(i)}\mathbf{T}\\ i = 0, 1, 2, \ldots N\end{array}\right\}\text{ are diagonal matrices} \qquad (5)$$

Taking the jth row in (4)

$$\ddot{z}_j + \Gamma_j \dot{z}_j + \left[\lambda_j^{(0)} + \sum_{i=1}^{N} f_i(t)\lambda_j^{(i)}\right] z_j = 0 \qquad (6)$$

Let

$$z_j = u_j e^{-\frac{1}{2}\Gamma_j t} \qquad (7)$$

Hence

$$\ddot{\mu}_j + \left\{\lambda_j(0) - (\tfrac{1}{2}\Gamma_j)^2 + \sum_{i=1}^{N} \lambda_j^{(i)} f_i(t)\right\}\mu_j = 0 \qquad (8)$$

Equation (8) is a Hill's equation and can therefore be treated by Floquet's theory[4]

$$\therefore\ \mu_j = Ae^{\zeta_j t}\,\Phi(t) + Be^{-\zeta_j t}\Phi(-t) \qquad (9)$$

where $\Phi(t)$ is a periodic function, and ζ_j is either real, or pure imaginary. Combining (7) and (9)

$$z_j = Ae^{(\zeta_j - \frac{1}{2}\Gamma_j)t}\,\Phi(t) + Be^{(-\zeta_j - \frac{1}{2}\Gamma_j)t}\,\Phi(-t) \qquad (10)$$

If ζ_j is real, z_j will be stable if

$$|\zeta_j| \leq \tfrac{1}{2}\Gamma_j \qquad (11)$$

For stability of the whole system (11) must hold for all $j = 1, 2, \ldots n$.

Gas-Lubricated Cylindrical Journal Bearings of the Finite Length[1]

W. A. MICHAEL.[2] A reading of this paper raises several questions the answers for which would enhance its usefulness. In approximately increasing order of importance, they are:

1 Should not the boundary condition for p be $p = 12\pi/\Lambda$, rather than $p = 1$, as given?

2 Why is the iteration not started with $p_{i,j} = 12\pi/\Lambda$, rather than zero as stated?

[4] E. L. Ince, "Ordinary Differential Equations," Dover, 1956.

[1] By B. Sternlicht, published in the December, 1961, issue of the Journal of Applied Mechanics, vol. 28, Trans. ASME, vol. 83, Series E, pp. 535–543.

[2] Research Staff Member, International Business Machines Corporation, Research Laboratory, San Jose, Calif.

3 How is the periodicity condition imposed on the difference equations?

4 Since no bearing misalignment is assumed, the pressure is symmetric about the plane $z = 0$. Has the author taken advantage of this symmetry to reduce the number of unknowns $p_{i,j}$ and, consequently, the amount of computing time? It is meaningless otherwise to say that a 12 by 6 grid was used.

5 How are the coefficients a_0, a_1, a_2, a_3, a_4, A in equation (10) and (11) calculated? The pressure $p_{i,j}$, and, consequently, the density $\rho_{i,j}$, are available only at the grid points and not at the "in-between" points required by equation (11).

6 In connection with the tolerance factor (12) what is meant by the statement, "This tolerance was found to give sufficient accuracy"?

7 The author's difference equations (8) are complicated by the presence of fractional subscripts associated with the "in-between" points. This difficulty is easily circumvented, as was done by Michael,[3] by carrying out the differentiations of the bracketed expressions in (5) and using centered differences to approximate the resulting first derivatives. The truncation error is not increased by this method and the difference equations involve only quantities defined at nodes of the grid. Also given by Michael[3] is a simple, direct iteration that has proved to be very useful in practice. It would be interesting to know why the author has chosen the more complicated two-part iteration (10, 11). His method requires the repeated solution of a linear system interspersed with revisions of the coefficients. Since, as the author points out, the use of computers in bearing analysis represents a considerable expense, it is important to find rapidly convergent iterative schemes. It would be useful if the author would include some observations on the relative efficiency of his method.

8 Perhaps the most serious omission in the paper is a discussion of the errors that invariably arise in numerical solutions. Three kinds of errors require consideration: (*a*) Discretization error that arises from replacing derivatives by difference quotients and integrals by sums, (*b*) rounding error, and (*c*) error caused by terminating the iteration after a finite number of steps. The last two, at least in problems of this kind, can be controlled by sound numerical analysis and experimentation on the computer. Discretization error is more difficult to assess and, unfortunately, is likely to be by far the most dangerous of the three.

Scant theoretical information exists on the discretization error for nonlinear elliptic partial differential equations, so one must fall back on the linear theory, presented elsewhere,[4–6] as a guide. A typical result in the linear theory is that, as the grid is successively refined (keeping $\Delta y/\Delta x$ constant), the truncation error behaves asymptotically like $(\Delta x)^p$, where the exponent p depends upon the regularity of the coefficients and the boundary conditions. Estimates for p range anywhere from[4] $p = 2$ for Laplace's equation to a pessimistic $p = 2/5$,[6] for a more general class of equations. It is a striking fact[3] that, for the nonlinear Reynolds equation, numerical experiments involving successive grid refinements strongly suggest that the discretization error is of the order of $(\Delta x)^p$ and that such experiments can provide an entirely practical way of assessing discretization error.

Since the author has presented a considerable number of figures and tables that are of potential usefulness to designers, it is particularly important for him to discuss these questions of error, and what steps, if any, were taken to control them. In particular, how can we reconcile the five significant figures presented in Table 4 with the tolerance factor of 0.1 percent?

[3] W. A. Michael, "Numerical Solution of the Reynolds Equation for Finite Slider Bearings," *IBM Journal of Research and Development*, vol. 3, 1959, pp. 256–259.

[4] S. Gerschgorin, "Fehlerabschätzung für das Differenzenverfahren zur Lösung partieller Differentialgleichungen," *Zeitschrift für angewandte Mathematik und Physik*, vol. 10, 1930, pp. 283–325, 367, 373–382.

[5] G. E. Forsythe and W. R. Wasow, "Finite-Difference Methods for Partial Differential Equations," John Wiley & Sons, Inc., New York, N. Y. 1960.

[6] J. Nitsche and J. C. C. Nitsche, "Error Estimates for Elliptic Equations," *Archives for Rational Mechanics and Analysis*, vol. 5, 1960, pp. 293–307.

Torque and Cavitation Characteristics of Butterfly Valves[1]

J. M. ROBERTSON.[2] The author performs a commendable task in extending his previous free-streamline analysis [10, 14][3] of the plane two-dimensional flow in a butterfly valve to the considerations of cavitation and hydrodynamic torque. It would appear that he is overconfident in implying that these results completely cover the torque and cavitation characteristics of actual butterfly valves in pipelines. The use of the term "axially symmetric butterfly valve" in this regard is completely ambiguous; the author should state just what he meant by this term. This discussion contains some remarks on the free-streamline solution, its extension to axisymmetric or three-dimensional flows and the prediction of cavitation characteristics.

As was stated by M. E. McPherson et al. [10], the author's analysis of the plane two-dimensional butterfly-valve flow was an "outstanding development." Any discussion of the application of the Helmholtz free-streamline analysis to valve characteristics is incomplete without reference to the work at M.I.T. by F. F. Ehrich.[4] This work is most pertinent since it included analysis of the butterfly valve, although not in as exact a solution as the author's, but more general than that of R. E. Nece [10]. Unfortunately, Ehrich's work was not published by the Society but is summarized in a paper by W. G. Cornell[5] who compares the total pressure loss coefficient with some actual valve data by Weisbach. The free-streamline solution for the planar flow about an inclined plane lamina in a parallel-walled conduit was apparently first obtained in 1891 by A. E. H. Love[6] with the aid of J. H. Michell's logarithmic-hodograph transformation method. Love left his result in integral form owing to the considerable difficulties with the resultant elliptic integrals. The author performed a valuable service in carrying out this integration [10]. To this writer's view, the terminology applied to the nonseparated flow about a plane lamina in an infinite fluid stream is improper. U. Cisotti [20] did not develop this solution—it was already available to him in the third edition of Lamb's *Hydrodynamics* (page 80)—he merely generalized Lamb's result for 45 deg and determined the torque for any angle of attack. In fact, the author's Fig. 7 is that so given by Lamb at least as early as 1906 and indicated as being obtained from the degenerate case of his 1877 solution for the flow about elliptic cylinders. Lamb indicates earlier work in 1873 on elliptic cylinders by Beltrami, but the general nature of this nonseparated flow and the occurrence of a torque were certainly known much earlier than the separated-flow solution.

The author verifies his free-streamline solution results and their extrapolation to butterfly valves in pipes by various stratagems. It would have been better had he been content with less generalized and sweeping objectives. Free-streamline solutions

[1] By Turgut Sarpkaya, published in the December, 1961, issue of the JOURNAL OF APPLIED MECHANICS, vol. 28, TRANS. ASME, vol. 83, Series E, pp. 511–518.

[2] Professor of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill. Mem. ASME.

[3] Numbers in brackets refer to items in author's list of references.

[4] F. F. Ehrich, "Some Hydrodynamic Aspects of Valves," ASME Paper No. 55—A-114, 1955; also M.I.T. theses, 1950 and 1951.

[5] W. G. Cornell, "Some Aerodynamic Cavity Flows in Flight Propulsion Systems," Second Symposium on Naval Hydrodynamics, ONR Washington (GPO) ACR-38, 1960, pp. 391–424.

[6] A. E. H. Love, "On the Theory of Discontinuous Fluid Motion in Two Dimensions," *Proceedings of the Cambridge Philosophical Society*, vol. 7, 1891, pp. 175–201.