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DISCUSSION

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The authors have presented a mathematical model of a hydroelectric power plant in which important nonlinearities of the governor are taken into consideration; the turbine characteristics are globally represented by means of polynomial approximations; and a modified form of the usual hyperbolic operator is used for conduit simulation. However, more accurate methods of conduit representation, suitable for digital computer analysis, are available and this discussion is mainly concerned with this part of the study.

The momentum and continuity equations describing unsteady flow through closed conduits are quasi-linear hyperbolic partial differential equations [10, 11].[7] By either neglecting or linearizing the nonlinear terms, graphical [12, 13] and analytical [14, 15] methods have been obtained for solving these equations. These methods are approximate and cannot be used to analyze systems having a large number of pipes or systems having complex boundary conditions. Most of these methods were derived before the availability of high speed digital computers, and the method used by the authors belongs to this category. Nowadays, however, digital computers are being used extensively to solve the transients in closed conduits and various methods [16–20] suitable for computer solution have been proposed. For example, the implicit finite difference method [10] and the method of characteristics [16–20] have been used for transient analysis since the early sixties.

In the finite difference method, the partial derivatives are replaced by finite difference approximations and the resulting algebraic equations for the whole system are then solved simultaneously. Depending upon the size of the system, this involves the simultaneous solution of a large number of nonlinear equations. The analysis of transients in hydroelectric power plants by this method becomes even more complicated because the boundary condition for the turbine has to be solved by an iterative technique. Although this method has been used in reference [10] to study transients caused by large load changes in hydroelectric power plants, the method has not become popular and is not discussed any further here.

In the method of characteristics, the partial differential equations are converted into ordinary differential equations along the characteristic curves in the domain of the independent variables. These equations are then solved by a finite difference technique. Because each boundary condition and each conduit are considered separately during each time step, this method is particularly suitable for the studies reported by the authors. If this method is used, then equation (16) of the paper is replaced by a linear equation of the form

$$Q = C_1 - \frac{g}{a} H$$

where Q = discharge; H = piezometric pressure head at the turbine inlet; g = acceleration due to gravity; and a = celerity of the water hammer wave. The value of the constant C_1 is obtained from the known values of various variables for the previous time step. In addition to simplifying the analysis, this method has the following advantages:

1 Because changes in the conduit geometry, such as series and branch connections, surge tanks, etc. are included (instead of using an equivalent pipe as required in the authors' method), the water hammer wave reflections from each change in conduit section are reproduced in the analysis.
2 Friction losses are considered distributed along the pipeline and are assumed proportional to the square of the flow velocity. This method represents conduit losses more accurately than that used by the authors.
3 Transient pressures and velocities throughout the system, which are needed in the design of the penstock, are obtained.

The authors have used an iterative technique to solve the boundary condition for the turbine. Perhaps, it would be better to estimate the discharge rather than the head (Fig. 6), as the authors have done, because the discharge changes gradually while the head changes irregularly due to reflections in the system. The discharge can be estimated quite well from the known changes during the previous time steps whereas the head cannot be estimated with such accuracy. Possible errors in the estimation of the head may result in the divergence of the iterations rather than convergence to the true solution.

The writer is developing a comprehensive computer program in which the method of characteristics is used to simulate the conduit; the turbine characteristics are globally represented in a form in which the manufacturer usually supplies the model test data for the turbine; various governor nonlinearities are included; and different types of governors are analyzed. Actual field test data will be used to check the validity of the program and the results of this study will be published upon completion.

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Authors' Closure

The authors wish to express their appreciation to Dr. M. H. Chaudhry for the accompanying discussion. His point concerning the accuracy of the conduit modeling for more complex feeder systems is well taken, as the method of characteristics is generally accepted as appropriate in those cases. However, the method of the modified hyperbolic operator seems most convenient for representation of simple conduits in the work presented because of the following points:

1 The primary purpose of the modeling was that of simulation for control analysis and design. No detailed penstock information was sought.

2 The method calculates the states of the conduit-turbine interface only and is, therefore, more expedient.

3 The method may be implemented by analog computation with equal facility.

The discusser's second point deals with the choice of variable for estimation within the inner iterative loop. The authors encountered no special difficulty using the pressure parameter. Here, the iterative technique used was such that convergence was assured independent of the stability of the computations. The authors are looking forward to further publications in this developing area of hydro system simulation.