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Seeking optimal groundwater pumping strategies at Pinggu District in Beijing, China

A. L. Yang, G. H. Huang, X. S. Qin, L. Li and W. Li

ABSTRACT

A simulation-based fuzzy optimization method (SFOM) was proposed for regional groundwater pumping management in considering uncertainties. SFOM enhanced the traditional groundwater management models by incorporating a response matrix model (RMM) into a fuzzy chance-constrained programming (FCCP) framework. RMM was used to approximate the input–output relationship between pumping actions and subsurface hydrologic responses. Due to its explicit expression, RMM could be easily embedded into an optimization model to help seek cost-effective pumping solutions. A groundwater management case in Pinggu District of Beijing, China, was used to demonstrate the method's applicability. The study results showed that the obtained system cost and pumping rates would vary significantly under different confidence levels of constraints satisfaction. The decision-makers could identify the best groundwater pumping strategy through analyzing the tradeoff between the risk of violating the related water resources conservation target and the economic benefit. Compared with traditional approaches, SFOM was particularly advantageous in linking simulation and optimization models together, and tackling uncertainties using fuzzy chance constraints.

Key words | chance-constrained programming, fuzzy sets, groundwater, response matrix

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INTRODUCTION

Groundwater withdrawal plays a vital role in supporting sustainable urban water management in many countries (Bayer *et al.* 2009). Previously, many simulation and optimization approaches have been developed for supporting groundwater management (Das & Datta 2001; Psilovikos & Tzimopoulos 2004; Theodossiou 2004; Tung & Chou 2004; Psilovikos 2006; Saffi & Cheddadi 2007; Ayvaz & Karahan 2008; Ayvaz 2009). In most cases, simulation models were applied to provide predictions of groundwater responses under various pumping actions, and optimization models were used to seek best management strategies (Ayvaz & Karahan 2008). Over the past years, there has been an increasing awareness that the groundwater management system is intrinsically complicated with uncertainties that may affect the relevant modeling analysis (He *et al.* 2008). These uncertainties may be derived from aquifer heterogeneity, modeling mechanism, measurement/monitoring errors and human judgment, and result in difficulties in generating effective solutions (Isukapalli 1999; Baalousha & Köngeter 2006). Thus, it is desirable that effective inexact modeling methodologies be developed to explicitly incorporate uncertainties into the regional groundwater management.

Over the past decades, various groundwater management approaches have been proposed, with a majority of them focused on stochastic theories (Tiedeman & Gorelick 1993; Domenico & Alex 2006; Mantoglou & Kourakos 2007; Singh & Minsker 2008; Li *et al.* 2009; Yan & Minsker 2011). However, a potential problem associated with the stochastic method is that it may suffer from strict data requirement and may encounter difficulties when data sources are highly insufficient. Fuzzy techniques are capable of describing possibilistic type of uncertainties and the related fuzzy information is relatively flexible to define; they are viable alternatives for tackling uncertainties in groundwater

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management. Dou *et al.* (1997) developed a fuzzy groundwater flow model by linking the finite-difference method with fuzzy number representations, which could handle imprecise information directly without generating many realizations. Guan & Aral (2004) computed the membership functions of contaminant concentrations at two control points through a steady-state flow and a time-dependent contaminant transport model. Baudrit *et al.* (2007) applied possibility distributions to address the uncertain parameters (e.g. dispersivity) in the model of Galya (1987). Kentel & Aral (2007) proposed a fuzzy multiobjective decision-making approach to optimize additional groundwater withdrawal at multiple demand locations. Prasad & Mathur (2010) treated the longitudinal dispersivity and transverse dispersivity as fuzzy sets, and used fuzzy simulation coupled with BIOPLUME III to obtain the numerical results. Among various fuzzy approaches, the fuzzy chance-constrained programming (FCCP) method, proposed by Liu & Iwamura (1998), has been extensively applied in various research fields (He *et al.* 2008; Rong & Lahdelma 2008; Cao *et al.* 2009). FCCP adopts a possibilistic type of chance constraints into optimization framework and allows involvement of fuzzy variables in models where the related membership functions could be straightforwardly defined. However, in the field of groundwater management, the related studies were very limited.

Another challenge in applying FCCP in groundwater management lies in the difficulties of coupling a simulation model into an optimization framework. The response matrix method (RMM) has been proved effective in describing linear, nonlinear, or mixed-binary linear relationships between pumping actions and groundwater responses (Doughterty & Marryott 1991). There are several early studies (Deininger 1970; Peralta *et al.* 1991; Takahashi & Peralta 1995; Kwanyuen & Fontane 1998). More recently, Theodossiou (2004) used RMM to integrate simulation and optimization models for groundwater management in a large-scale site. Psilovikos (2006) used RMM in groundwater management based on the space-superposition method or the fusing of space and time superposition. Generally, RMM has gained in popularity in groundwater management for many years, and exhibited great potential to help link a simulation model to an optimization platform. However, RMM has never been applied in a FCCP framework.

Thus, the objective of this study is to develop a simulation-based fuzzy optimization method (SFOM) for supporting the optimal design of regional groundwater management which has uncertainties. The groundwater flow under pumping operations will be simulated by MODFLOW (e.g. Visual MODFLOW 4.0, Waterloo Hydrogeologic Inc.) based on different initial and boundary conditions (McDonald & Harbaugh 1988), and the related input and output datasets will be approximated by RMM. A FCCP model will then be formulated to seek the least costly pumping strategy subject to slightly violable hydraulic constraints, where RMM is embedded to help build the linkage between decision variables and groundwater drawdown levels. The proposed method will be validated by a real case located in Pinggu District, Beijing, China.

GROUNDWATER MANAGEMENT UNDER UNCERTAINTY

The groundwater management system involves processes of groundwater flow modeling and optimization of operation strategies, which are more or less found to be associated with many types of uncertainties (Huang *et al.* 1999). Several studies have shown that groundwater geologic environments are highly heterogeneous and could easily lead to imprecise description under the condition of limited borehole data (Dou *et al.* 1997). In the optimization process, the constraints of environmental standards and water supply capacity are somewhat subjective due to human-biased judgment and/or scarcity of knowledge and research. From the point of view of system optimization, the management problem under consideration is how to pump groundwater to achieve maximum economic benefit and groundwater resource conservation, given an allowable risk of violating the specific regulations.

Generally, the accuracy of the groundwater model is affected by the model inputs and parameters. To reduce the influence of uncertainty from such sources, we can try to improve the accuracy of input parameters. For example, the study area can be divided into several subareas within which each one can have a specific set of parameters through on-site measurement; this is better than having a single parameter for the whole area with assumed distributions. These parameters can be further calibrated and verified through a trial-and-error procedure or use of an automatic calibration algorithm (i.e. a calibration tool of GMS). In a word, the uncertainty sources from a simulation model could be somewhat mitigated through additional measurement efforts. However, the uncertainty associated with environmental standards, policies and risks are subjective information due to the preferences of local authorities. It is more desirable that such information be defined as fuzzy variables. The proposed SFOM can be used for tackling such a case.

METHODOLOGY

Framework of the general method

SFOM integrates RMM and FCCP into a general framework (see Figure 1), and can be used to handle uncertainties expressed as fuzzy sets. The first step is site characterization, which attempts to identify aquifer property, system domain, and modeling parameters. The second step is to construct the response matrices, and then validate their accuracy. Simulation (see Section S1 of the Supplementary document, available online at http://www.iwaponline.com/jh/015/006.pdf) and analysis of the response matrix (Section S2 of the Supplementary document) are performed to predict groundwater behaviors and establish the response matrices. Then, FCCP are integrated with the obtained response matrices to optimize pumping strategies and generate a trade-off curve between risk and system cost under different uncertainty levels. The information related to standard, policies and risks are tackled as fuzzy variables based on expert consultations or surveys.

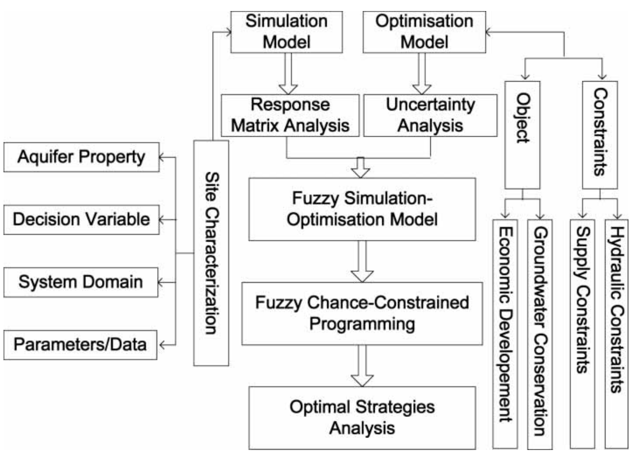

Figure 1 | General framework of SFOM.

Fuzzy chance-constrained programming (FCCP) model

A general FCCP model can be formulated as follows (Liu & Iwamura 1998):

$$\text{Max} \quad f = CX \tag{1a}$$

Subject to:

$$\text{Pos}\{AX \leq \underset{\sim}{B}\} \geq \alpha \tag{1b}$$

$$GX \geq E \tag{1c}$$

$$X \geq 0 \tag{1d}$$

where X is a vector of decision variables; $\underset{\sim}{B}$ is a vector containing the right-hand side items of constraints; $\mu(\underset{\sim}{B})$ is the membership function of $\underset{\sim}{B}$; C, A, G and E are the coefficient vectors; α is a predefined confidence level; $\text{Pos}\{\cdot\}$ is the event possibility. For model structure, Equation (1a) is the objective function; Equation (1b) represents the group of chance constraints with fuzzy variables; Equation (1c) represents the group of deterministic constraints; Equation (1d) is the group of technical constraints (i.e. non-negativity requirement).

The fuzzy chance constraints (1b) can be converted into their respective crisp equivalents (Liu & Iwamura 1998). Figure 2 shows that, for any given confidence levels of αm

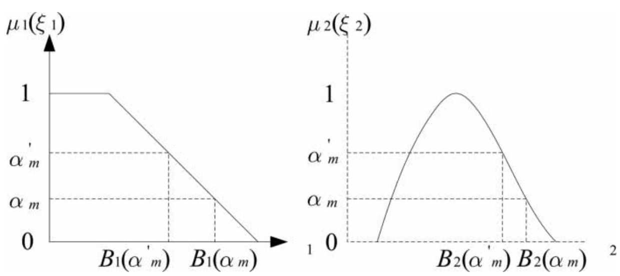

Figure 2 | Fuzzy membership functions.

($0 \le \alpha_m \le 1$, $m = 1, 2, \ldots, p$) in the fuzzy numbers ξ_m with membership functions $\mu_m(\xi_m)$, there exist the maximum values of all potential values $B(\alpha m)$ such that (Liu & Iwamura 1998):

$$\mathrm{Pos}\{\xi_m | B(\alpha_m) \le \xi_m\} = \alpha_m \tag{2a}$$

$$\mathrm{Pos}\{\xi_m | B(\alpha_m) \le \xi_m\} = \mu_m\{B(\alpha_m)\} \tag{2b}$$

$$B(\alpha_m) = \mu_m^{-1}(\alpha_m) \tag{2c}$$

$$B(\alpha_m) = \sup\{B \big| B = \mu_m^{-1}(\alpha_m)\} \tag{2d}$$

where μ_m^{-1} is the inverse of μ_m, $m = 1, 2, \ldots, p$. As $B(\alpha'_m)$ (as indicated in Figure 2) are smaller than $B(\alpha_m)$, there exists

$$\mathrm{Pos}\{\xi_m | B(\alpha_m) \le \xi_m\} = \sup\{\mu_m(\xi_m) | \xi_m \ge B(\alpha_m)\} \le \sup\{\mu_m(\xi_m) | \xi_m \ge B'(\alpha_m)\} = \mathrm{Pos}\{\xi_m | B'(\alpha_m) \le \xi_m\} \tag{3}$$

Hence, the crisp equivalents of Equation (1b) can be represented as $AX \le \mu^{-1}(\alpha)$ (Liu & Iwamura 1998).

Simulation-based fuzzy optimization model (SFOM) for groundwater management

For a groundwater management system, the decision-makers are responsible for satisfying the water demand and controlling the expansion of the drawdown funnel to control the declining trend of the groundwater level. The establishment of the groundwater drawdown standard is normally decided by the local government in light of hydrogeological, socio-economic, and eco-environmental conditions; it is best described by fuzzy sets due to the lack of information or subjectivity of human judgment. Meanwhile, the evaluation of the allowable extraction amount may also suffer from uncertainty because of the aquifer complexities. Therefore, to tackle the uncertainties associated with groundwater management, a SFOM can be formulated as:

$$\mathrm{Max} \quad f = \sum_{j=1}^{\mathrm{NW}} \sum_{k=1}^{\mathrm{NV}} Q(j, k) \tag{4a}$$

subject to

$$\mathrm{Pos}\left\{D(i, l) \le \tilde{D}_{\max}(i)\right\} \ge \alpha_m \tag{4b}$$

$$\mathrm{Pos}\left\{\sum_{j=1}^{\mathrm{NW}} Q(j, k) \le \tilde{G}_{\max}(l)\right\} \ge \alpha_m \quad \forall k \tag{4c}$$

$$\sum_{j=1}^{\mathrm{NW}} Q(j, k) \ge G(l) \quad \forall k \tag{4d}$$

$$Q(j, k) \le Q_{\max}(j, k) \quad \forall k \tag{4e}$$

$$0 \le \alpha_m \le 1 \tag{4f}$$

$$Q(j, k) \ge 0 \quad \forall k \tag{4g}$$

where $Q(j, k)$ is the pumping rate of well j at time period k, [L^3/T]; α_m is the confidence level; NW is the total number of pumping wells; NV is the total number of planning period; $D(i, l)$ is the drawdown of well i in period l, [L]; $\tilde{D}_{\max}(i)$ is the fuzzy maximum drawdown of well i, [L]; $\tilde{G}_{\max}(l)$ is the fuzzy maximum allowable extraction amount in the management period l, [L^3/T]; $\tilde{G}(l)$ is the water amount for meeting the demand of economic development at different management period, [L^3/T]; $Q_{\max}(j, k)$ is the maximum pumping capacity of well j, [L^3/T].

Objective (4a) is represented as the total pumping rate of all potential pumping wells. Environmental constraint (4b) requires that the drawdown values at the pumping wells should be less than the environmental standard. Taking into account the aquifer's water capacity, constraint (4c) requires that the total pumping rate cannot exceed the limit of the allowable extraction amount. Constraint (4d) ensures an adequate amount of water for maintaining economic development. Constraint (4e) requires that the pumping rate at each well cannot exceed its maximum capacity. Technical constraints (4f) and (4g) are provided to ensure the effectiveness of pumping wells.

To solve the above model, RMM (Section S2 of the Supplementary document, http://www.iwaponline.com/jh/015/006.pdf) will be used to combine simulation and optimization models (Psilovikos 2006):

$$D(i,l) = H(i,0) - H_0(i,l) + \sum_{j=1}^{m}\sum_{k=1}^{l}\beta(i,j,l-k+1)Q(j,k) \quad (5)$$

where $H(i, 0)$ is the initial water head of node i, [L]; H_0 (i, l) is the natural hydraulic head of node i at time period l, [L]; $\beta(i, j, l–k+1)$ is the unit response coefficient that describes the effect on the hydraulic head at node i in period l caused by a unit pumping volume at node j in the current or previous period k. According to the predetermined confidence levels, the SFOM for groundwater management can be formulated as follows:

$$\text{Max} \quad f = \sum_{j=1}^{NW}\sum_{k=1}^{NV} Q(j,k) \quad (6a)$$

Subject to:

$$H(i,0) - H_0(i,k) + \sum_{j=1}^{NW}\sum_{k=1}^{NV}\beta(i,j,l-k+1)Q(j,k) \le D(\alpha_m) \quad (6b)$$

$$\sum_{j=1}^{NW} Q(j,k) \le G(\alpha_m) \quad \forall k \quad (6c)$$

$$\sum_{j=1}^{NW} Q(j,k) \ge G(l) \quad \forall k \quad (6d)$$

$$Q(j,k) \le Q_{\max}(j,k) \quad \forall k \quad (6e)$$

$$0 \le \alpha_m \le 1 \quad (6f)$$

$$Q(j,k) \ge 0 \quad \forall k \quad (6g)$$

$$D(\alpha_m) = \sup\{D \mid D = \mu_1^{-1}(\alpha_m)\} \quad (6h)$$

$$G(\alpha_m) = \sup\{K \mid K = \mu_2^{-1}(\alpha_m)\} \quad (6i)$$

where $D(\alpha_m)$, $G(\alpha_m)$ are the maximum values of all potential values of drawdown and extraction amount under the confidence level of α_m.

CASE STUDY

Background of the study site

From 1999, Beijing experienced 9 years of consecutive drought, with an average annual precipitation of 505.8 mm in 1999–2007, which caused a sharp decline in surface water, and groundwater level was also dropping year by year. In August 2004, the Pinggu emergency water project was launched and this, to some extent, expanded the working area of groundwater extraction, and resulted in an increased drop in the groundwater level. Table 1 illustrates the statistics of the extraction volume of groundwater from Xiagezhuang during 2004–2007. Over the past years, it is found that issues related to seasonal funnels, secondary salinization, and water shortage (for dry land, woodland and grassland) are seriously hindering local agricultural development and eco-environmental conservation. Therefore, it is desired that a regional groundwater management model be developed to help analyze the effect of pumping on local economic development, eco-environmental quality and water resources availability.

The study area, Pinggu plain, is located in Pinggu district of Beijing and has an area of 362.38 km^2. The town of Xiagezhuang, the main study area containing the pumping wells, is located in the southeast part of Pinggu plain (Figure 3). The aquifer system is semi-confined, heterogeneous, and anisotropic, consisting of surface clay

Table 1 | Statistics of the groundwater extraction amounts (10×10^3 m^3) in Xiagezhuang during 2004–2007

Year	Total water utilization amount	Industry	Residential water consumption	Public service	Agriculture
2004	680.514	22.297	110.707	16.40	531.11
2005	685.35	34.95	92.57	16.20	541.63
2006	620.56	15.91	73.06	8.54	523.05
2007	603.94	13.47	71.15	11.53	507.79

Data source: Li (2009).

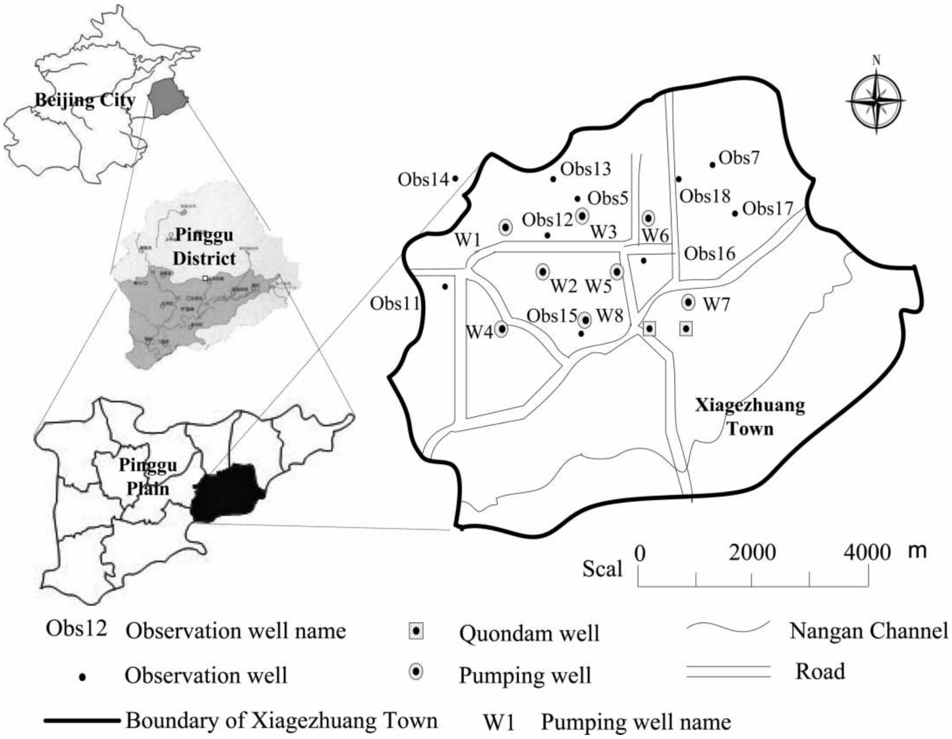

Figure 3 | The location of wells.

underlain by scree-gravel aquifer subsidiary interbeds, with the thickness decreasing from north to south. The primary groundwater source of Xiagezhuang contains a huge scree-gravel aquifer with a thickness of 150 m. According to the site investigations, the horizontal hydraulic conductivity of the subsurface decreases from 153 to 4 m/d from north to south, and the vertical hydraulic conductivity decreases from 4 to 0.0012 m/d. The studied groundwater system consists of two aquifers: (i) the superficial aquifer is unconfined and accepts recharge from rainfall with an infiltration coefficient around 0.11 per year; it releases discharge to the external environment through evapotranspiration and artificial drainage; and (ii) the confined aquifer contains the groundwater with preferable quality for exploitation. From the hydrogeological condition of the site (see Figure A in Supplementary document, available online at http://www.iwaponline.com/jh/015/006.pdf), it is found that the elevations at the top of the superficial aquifer range from 20 to 40 m, those at the bottom of the superficial aquifer range from −68 to −50 m, and those at the bottom of the confined aquifer range from −115 to −243 m. The confined aquifer contains a large amount of fresh water and is used as the extraction zone.

A conceptual numerical model based on the field condition is constructed. The study area is a three-dimensional domain, where the aquifer is assumed horizontal and of semi-finite extent. The pumping wells fully penetrate the second layer. Groundwater flow is horizontal and radial towards the pumping wells. The numerical simulation is applied to the entire domain of Pinggu plain, which is discretized uniformly into 14,918 effective square grids with a spacing of 200 by 200 m. In terms of boundary conditions, the southern boundary will be treated as a general head boundary (GHB), while the northern boundary will be treated as a fixed head boundary (see Figure 4). The recharge items include precipitation, lateral recharge, and irrigation recharge, and the discharge items include evapotranspiration, abstraction, outflow, and others.

Model calibration and verification

Due to the highly variable geologic environments, the study area is divided into 32 subareas. The initial condition of each subarea is obtained according to the result of pumping tests and references (Li 2009). The model parameters are

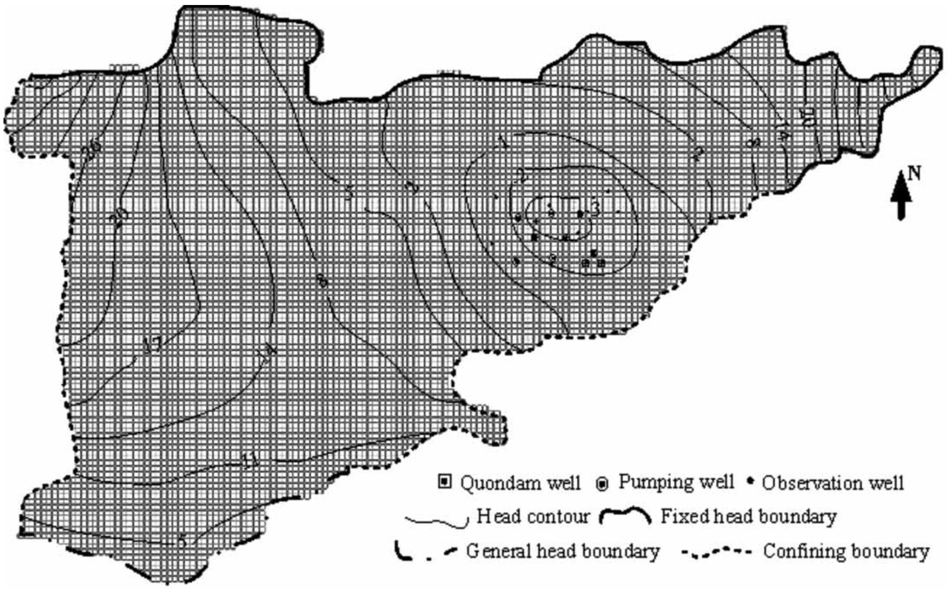

Figure 4 | The simulation domain.

calibrated through a trial-and-error procedure. In this study, the simulator is calibrated by using the monitoring data in 2008, and verified by using data in 2009. The analysis of groundwater budget during the simulation periods shows that the aquifer is under the condition of over-exploitation, with the volume of recharge at 107.53 million m^3 and discharge item at 168.53 million m^3. These values are in relatively good agreement with the work of Li (2009), where the volume of recharge is about 92.84 million m^3 and discharge item 165.58 million m^3. The verification results indicate that the absolute errors between the simulated and observed water heads range from 0.0088 to 0.2953 m, and the mean absolute error is around 0.1589 m; the root mean square error (RMSE) is about 0.1795 m and the correlation coefficient is about 0.952 (see Figure B in the Supplementary document, available online at http://www.iwaponline.com/jh/015/006.pdf). These results imply that the numerical model has satisfactory prediction accuracy, and is acceptable for supporting further simulation and optimization studies. Table 2 lists the major input parameters for the area in which the well is located.

Table 2 | Major input parameters (for zone of wells located)

Parameter	Value	Unit
Longitudinal hydraulic conductivity in the unconfined aquifer	30	m/d
Horizontal hydraulic conductivity in the unconfined aquifer	30	m/d
Vertical hydraulic conductivity in the unconfined aquifer	0.005	m/d
Storage coefficient in the unconfined aquifer	0.08	–
Effective porosity in the unconfined aquifer	0.4	–
Longitudinal hydraulic conductivity in the confined aquifer	32	m/d
Horizontal hydraulic conductivity in the confined aquifer	32	m/d
Vertical hydraulic conductivity in the confined aquifer	0.006	m/d
Storage coefficient in the confined aquifer	0.13	–
Effective porosity in the confined aquifer	0.38	–
Precipitation infiltration coefficient	0.25	–
Osmotic coefficient of irrigation return	0.16	–

Based on the numerical simulation outputs, the response technique is then used to generate the response matrix between pumping rates and drawdown. The unit impulse response function depends on the characteristics of groundwater system, including the aquifer type, boundary shape, hydro-geological parameters and the distance between observation points. The selection of unit pumping volume is generally based on the fact that it should cause a notable response on water level of each controlled node but incur negligible influences on system boundary. According to such a principle, the unit pulse pumping rate of the study district is identified as 50,000 m^3/d after repeated runs of calculation. In the Supplementary document (http://www.iwaponline.com/jh/015/006.pdf), Table A shows the response value and Figures C and D present the detailed spatial distributions of the drawdown values. The drawdown ranges from 11.68 m at the center of the depression cone to 0.05 m in the edges near the location of the GHB. This shows that the unit impulse pumping rate could cause a notable response on water level of each controlled node but incur negligible influences on system boundary.

The accuracy of the response coefficients is an indication of the RMM's ability to predict the response of the calculated system state to changes in stress, and plays an important role in the solution of a ground-water management model. The response matrix technique depends on the validity of the linear correlation between the pumping rate and the hydraulic head. In spite of the fact that the aquifer is semi-confined, the assumption of linear superposition between water extraction and drawdown is considered valid as the abstraction aquifer is confined. Also, the linearity assumption is tested and verified by using various scenarios of pumping wells. According to the drawdown information that is obtained by pumping each well continuously for 1 year at a rate of 1,000 m^3/d, it is revealed that the mean absolute error of drawdown outputs between RMM and direct numerical simulation is 0.0015 m, the RMSE is 0.0387 m, and the correlation coefficient is 0.976 (Figure E in the Supplementary document, http://www.iwaponline.com/jh/015/006.pdf). The results demonstrate that the established RMM has a high prediction accuracy and could reasonably help transform the numerical models to explicit linear expressions.

Management model settings

The developed methodology is applied to the planning of groundwater extraction rates for a number of pumping wells, with a target to offer the needed water as much as possible while, at the same time, ensure the hydraulic balance and groundwater conservation are within standards. The planning horizon is 1 year. At the beginning of the planning horizon, there exist two wells (with a total water supply capacity at 2,270 m^3/d) in the planning area of Xiagezhuang town. There is a need to exploit 8 additional pumping wells for emergency during the planning period (see Figure 3). In this study, the water demand value is set to 8,603 m^3/d according to the statistics of water demand during 2004–2007 (Li 2009). However, the standard of the maximum allowable drawdown and the maximum allowable extraction amount of groundwater are subjective information and hardly verifiable due to the preference of local authorities. Thus, they are defined as fuzzy variables. Constructions of the membership functions for fuzzy sets are based on the efforts of questionnaire surveys and/or expert consultations. To make sure the membership functions are reasonably defined, the experts and/or stakeholders should understand well the related concept and offer their inputs for uncertainty quantification. The core of the maximum allowable drawdown in this study is defined as 0.8 m, and the maximum degree of violation is based on expert consultations (in this study the support is set as from 0.4 to 1.2 m). The allowable extraction amount may vary from 9,293 to 13,933 m^3/d according to Li (2009). Its possibility distribution is defined as a fuzzy set with a core at 11,613 m^3/d and a support from 9,293 to 13,933 m^3/d. The related parameters for the optimization problem are listed in Table 3. Then, a simulation-based fuzzy model can be formulated and solved by converting the inexact model into its equivalent crisp form.

Table 3 | Parameters for the optimization model

Item description	Value	Unit
Water demand for Xiagezhuang[(a)]	8,603	m^3/d
Maximum allowable extraction amount[(a)]	Support = (9,293, 13,933), Core = 11,613	m^3/d
Maximum allowable drawdown[(a)]	Support = (0.4, 1.2), Core = 0.8	m
Maximum pumping capacity	2,000	m^3/d
Maximum number of pumping well for Xiagezhuang	10	none

Note: (a) Information referred to Li (2009).

Result analysis

Figure 5 and Table 4 present the obtained solutions through SFOM under different α-cut levels (from 0.0 to 1.0) for the management period of 2010. Figure 5 shows the pumping rate distribution over the eight wells. The significant differences are attributed to the different confidence levels and the uneven productivity of the aquifer. When the α-cut level decreases from 1.0 to 0.0, the objective function value (i.e. the total pumping yield) would vary from 8,684.85 to 13,898.52 m^3/d. It indicates that the higher the confidence level, the lower the objective function value. This implies that a higher level of concern for the groundwater drawdown would compromise the extraction target. Taking the α-cut level of 1.0 for example, the allowable drawdown value is 0.8 m, and the corresponding optimal pumping well number is 6 with the total pumping yield of 8,684.85 m^3/d; while at the α-cut level of 0.4, the allowable drawdown value is 1.1 m, and the corresponding optimal well number changes to 7 with the total pumping yield of 12,053.40 m^3/d.

Table 4 shows the optimal pumping rates for different wells under various α-cut levels. Generally, as the confidence level increases, the pumping rate of each pumping well would decrease. In detail, wells W1, W3 and W6 are desired to operate at their full capacities (i.e. 2,000.00 m^3/d), because these wells are in the Wangduzhuang water resources field, which has huge water capacities. As mentioned above, the thickness and hydraulic conductivity of the aquifer decrease from north to south. Therefore, the wells in the south have less water capacity. Wells W2, W4, W5 and W7 tend to have lower pumping rates in order not to cause serious hydraulic problems. For W8, the pumping rate turns out to be zero under most confidence levels. This is because W8 is close to the confining boundary and the two quondam wells. The drawdown level of W8 could easily violate the regulatory standard if it is used for groundwater extraction. Thus, its exploitation potential is limited.

Table 5 presents the drawdown levels obtained from the solutions under different α-cut levels. Generally, as the α-cut level decreases, the allowable drawdown values of each well would increase, leading to higher rates of water extraction. For example, under the α-cut level of 0.8, the maximum total pumping yield is 9,808.43 m^3/d and the maximum drawdown is 0.8692 m; while, when the α-cut level decreases to 0.2, their values would increase to 13,100.00 m^3/d and 1.1918 m, respectively. The solution leading to a lower total pumping yield corresponds to a more conservative consideration where the allowable drawdown is relatively strict. Conversely, the pumping strategy under a lower confidence level corresponds to a more optimistic condition where the allowable drawdown requirement is relatively loose.

Figure 6 shows the predicted drawdown distributions under different α-cut levels. Generally, the cone of depression would be formed around each pumping well. The drawdown extent would increase as the α-cut levels decreases. This is due to the fact that the confidence level controls the strictness of satisfying the hydraulic constraints, which, in turn, influence the pumping yields;

Table 4 | Optimal pumping rates under various α-cut levels

α-cut levels	1.0	0.8	0.6	0.4	0.2	0.0
W1 optimal pumping yield (m^3/d)	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
W2 optimal pumping yield (m^3/d)	822.45	1,402.31	1,519.61	1,636.92	1,815.54	2,000.00
W3 optimal pumping yield (m^3/d)	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
W4 optimal pumping yield (m^3/d)	1,165.18	1,289.58	1,493.37	1,697.17	1,924.52	2,000.00
W5 optimal pumping yield (m^3/d)	0.00	245.43	923.86	1,602.30	2,000.00	2,000.00
W6 optimal pumping yield (m^3/d)	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
W7 optimal pumping yield (m^3/d)	697.22	871.12	994.07	1,117.01	1,294.93	1,578.98
W8 optimal pumping yield (m^3/d)	0.00	0.00	0.00	0.00	65.01	319.54

Table 5 | Drawdown distributions at different wells using the optimal pumping rates

α-cut levels	1.0	0.8	0.6	0.4	0.2	0.0
W1 drawdown (m)	0.7040	0.7817	0.8613	0.9360	1.0071	1.0618
W2 drawdown (m)	0.6727	0.8192	0.9209	1.0233	1.1252	1.2081
W3 drawdown (m)	0.6673	0.7452	0.8215	0.9005	0.9709	1.0232
W4 drawdown (m)	0.6954	0.7945	0.8927	0.9919	1.0907	1.1589
W5 drawdown (m)	0.5697	0.6852	0.8427	1.0002	1.1240	1.1935
W6 drawdown (m)	0.6916	0.7642	0.8435	0.9221	0.9929	1.0470
W7 drawdown (m)	0.7613	0.8692	0.9769	1.0845	1.1918	1.2980
W8 drawdown (m)	0.7190	0.8218	0.9240	1.0269	1.1291	1.2319

a more stringent drawdown requirement (i.e. a high α-cut level) would need a lower groundwater extraction rate, whereas less strict requirement would allow a more extensive pumping action, resulting in more serious drawdown problems.

Generally, the model solutions with information related to violation risks would be useful for decision makers to look into the balance between system reliability and economy. For the study case, a lower pumping rate may guarantee that hydraulic drawdown is within the required limit (i.e. higher system reliability) but this would lead to a lower economic benefit. Conversely, when the planners desire a higher pumping yield, the economic reward would be more attractive but this may correspond to a higher system risk. Therefore, a trade-off between economic development and the risk of overusing groundwater resources could be examined to help recognize the most cost-effective groundwater management strategy.

Discussion

This study was novel in a number of aspects. From the methodology point of view, SFOM could effectively link the simulation and optimization models together through the application of RMM, where uncertainty could be effectively addressed in a FCCP framework. No such study has been reported in the field of groundwater management. From the practical application point of view, many northern cities in China have been overusing groundwater resources for many years due to shortage of effective surface water resources. Many areas are facing the risk of long-term groundwater drawdown and surface subsidence problems. How to effectively use groundwater resources in a sustainable manner, considering the complexities of the management system (like uncertainty issues), is an important topic for the local water managers. The proposed management model could help generate a full spectrum of solutions at different confidence levels, and assist the related

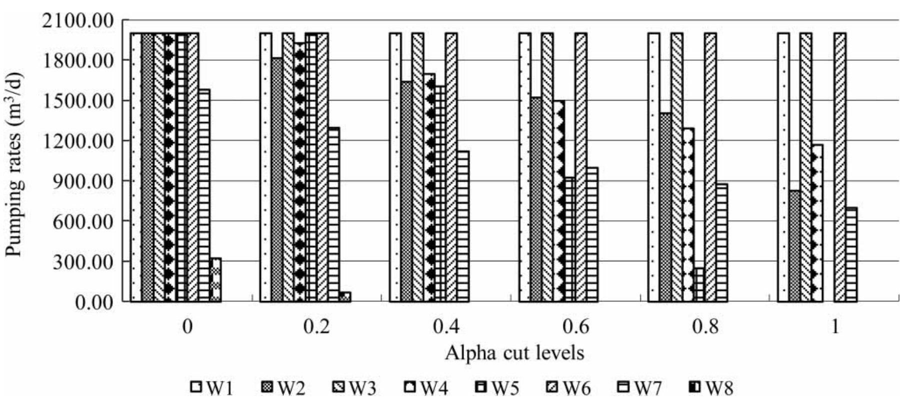

Figure 5 | Optimal pumping rates under various α-cut levels.

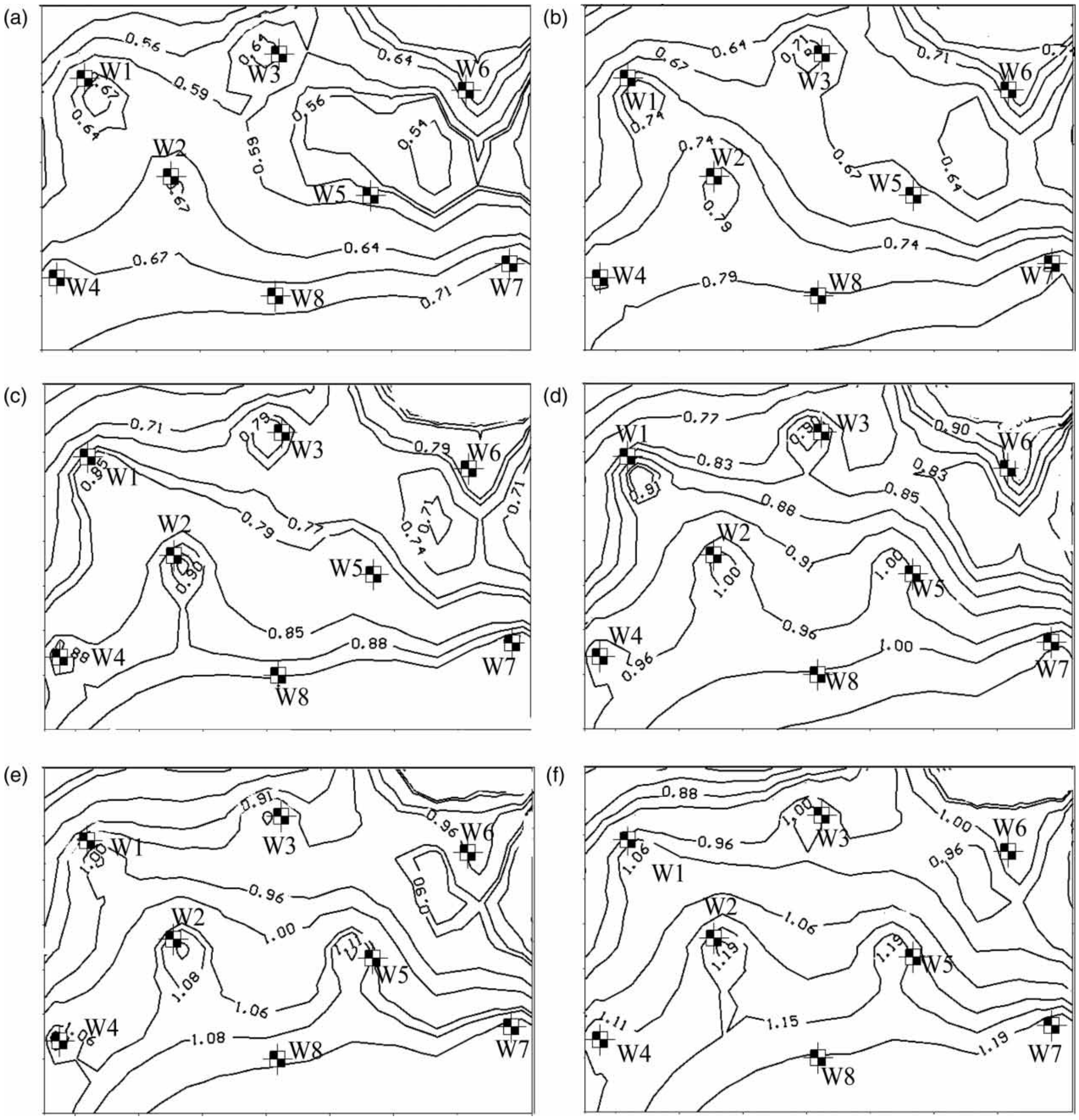

Figure 6 | Predicted drawdown distributions (m) under different confidence levels.

decision makers in identifying the most-efficient operation actions. Previously, very few related studies in this region have been reported.

This study also has a number of limitations that need to be tackled in the future. Firstly, SFOM allows representation of uncertainty by fuzzy sets where the corresponding membership functions could be more flexibly defined than the probability distribution functions (PDFs). However, in groundwater management models, uncertainty may also be associated with model inputs and parameters. To simplify our analysis and highlight the advantages of SFOM, we have ignored them. In addition, the fuzzy chance-constraint programming used in this study could only deal with uncertain parameters in the right-hand side of the model constraints, more sophisticated fuzzy programming methods are needed to handle more complicated cases when both sides are associated with uncertainties. Secondly, we only made a 1-year planning of groundwater extraction, given constant allowable drawdown levels, water demand/capacity, and pumping rates.

As many parameters (like the water demand and supply capacity) may be dynamic over time, the management model may have to be formulated as dynamic over different periods. Considering the response matrixes could be developed for multiple managing periods, the problem could be solved successively using a sequential linearization approach (Ahlfeld *et al*. 2005). Finally, the ecological and economic constraints were not accounted for during the model development. This is because the focus of this study was to demonstrate the methodology and the optimization model has been somewhat simplified. If the decision makers intend to address ecological safety, the related constraints could be added to ensure the water table drawdown is less than a critical buried depth. The budget limitation of well installation and operation costs could also be added, if required by the decision makers.

CONCLUSIONS

A SFOM was proposed for regional groundwater pumping management in consideration of uncertainties. A groundwater management case in Pinggu District of Beijing, China, was used to demonstrate the method's applicability. It was found that SFOM improved upon the traditional groundwater management models through embedding response matrix model (RMM) into a FCCP framework. RMM built an approximation of the relationships between aquifer's responses and pumping actions. The dimensions of the response matrixes relied on the number of observation points. The uncertainty originated from the optimization system could be effectively handled by FCCP. A spectrum of the objective-function values and decision variables were available under several α-cut levels; the decision-makers could choose the best solution from multiple alternatives, considering the degree of imprecision derived from uncertain information. Moreover, the proposed method did not oblige all the constraints be strictly met; thus a trade-off between economic development and the risk of overusing groundwater resources could be examined. This study is useful in helping decision makers identify cost-effective groundwater pumping strategies, and setting up a good example of groundwater management under uncertainty for the related areas.

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