VARIATION OF BASAL METABOLIC RATE PER UNIT SURFACE AREA WITH AGE

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Basal metabolism empirically referred to surface area is theoretically related indirectly to its hypothetical determining factor. Definition of the basal metabolic rate per unit surface area as a function of age then constitutes proximate definition of the basal metabolic gradient.

The age gradient of calories per square meter per hour outlined by Du Bois is the foundation of the Du Bois standards. Base lines for males in the prime adult years for surface area computed from the Meeh formula (1) were located by averaging selected series of reported normals with normal controls (2, 3). A scatter diagram for males and females to the age of 24 years was plotted for similar series and smoothed curves drawn. A second scatter diagram for a series revised for surface area computed from the Lissauer formula (4) for infants and from the linear (5) or height-weight formula (6) for children and adults was plotted for the life span of males (7). New base lines were located for boys in later childhood, and for several age groups of men and women, and an average percentage decrement for females calculated (8). Readings taken from a gradient similar to that plotted for males and diminished a uniform percentage for females were tabulated as provisional age group standards (9). In the section of the gradient corresponding to the standards (Chart 1) a hyperbolic curve runs into a horizontal line for 20 to 30 years which runs into a sloped line to the upper age limit. Readings from a gradient in which the plateau of constant rate reaches to 35 years are assigned to age groups extending symmetrically to either side of them. The standards for females are in general calculated approximately 7 per cent below those for males of the same age groups (Table I).

The gradient and the standards are not consistent. The graph of the gradient is in succession a hyperbolic curve, a horizontal line, and a sloped line. The graphs of the standards are series of horizontal lines (Chart 1). Within its range the gradient has two discontinuities,

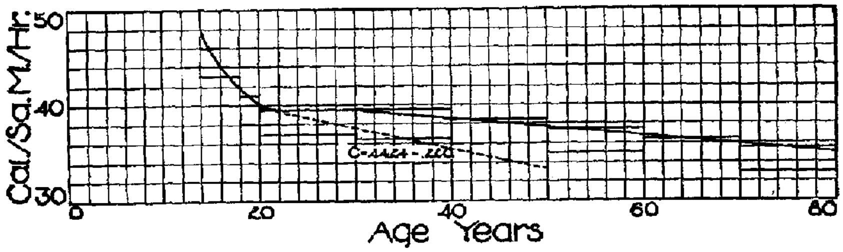

CHART 1. Basal metabolic rate (Aub-Du Bois).

TABLE I

Calories per Square Meter of Body Surface per Hour (Aub-Du Bois)

Sex	Age	Age group	Rate	Linear equation*	Exponential equation*
	yrs.	*yrs.*	*cal.*	$y = A + Bx$	$y = Ae^{Bx}$
Male	15	14–16	46.0	$\log C = 1.8488 - 0.01247\, a$	$C = 70.60\, e^{-0.0288\, a}$
	17	16–18	43.0		
	19	18–20	41.0		
	30	20–40	39.5	$\log C = 1.6301 - 0.00104\, a$	$C = 42.67\, e^{-0.0024\, a}$
	45	40–50	38.5		
	55	50–60	37.5		
	65	60–70	36.5		
	75	70–80	35.5		
Female	15	14–16	43.0	$\log C = 1.8329 - 0.01342\, a$	$C = 68.06\, e^{-0.0301\, a}$
	17	16–18	40.0		
	19	18–20	38.0		
	25	20–30	37.0	$\log C = 1.5972 - 0.00101\, a$	$C = 39.55\, e^{-0.0023\, a}$
	35	30–40	36.5		
	45	40–50	36.0		
	55	50–60	35.0		
	65	60–70	34.0		
	75	70–80	33.0		

* C = calories per square meter per hour, a = age in years, e = natural logarithmic base.

while the standards have seven and eight discontinuities for males and females respectively. While the successive segments of the gradient are expressible as a hyperbolic equation, a Fourier series, and a linear equation, the partial intervals of the standards are expressible as Fourier series.

Linear regression equations calculated by Harris and Benedict (10) represent the adult gradient as

$$\text{For men} \ldots\ldots\ldots\ldots \quad h_D = 1022.17 - 3.60\,a$$
$$\text{For women} \ldots\ldots\ldots\ldots \quad h_D = 924.25 - 2.96\,a$$

in which h_D = heat production in calories per square meter per day, and a = age in years. The equation for the major part of the data (3) on which the middle stretch of the Du Bois gradient rests

$$h_D = 1061.81 - 5.25\,a$$

is drawn on a comparable hourly scale in Chart 1. Such equations do not necessarily define the variation of the basal metabolic rate with age. A linear regression equation represents the most probable value of the dependent variable only on the assumption and providing that it be a linear function of the independent variable. If this relationship is not functional but only empirical these equations do not define metabolic gradients but substitutive straight lines.

The basal metabolic rate per unit surface area may be treated as an exponential function of age. The logarithm of the rate is then a linear function of age. Thus the midpoints of the Du Bois standards for each sex plotted against age on an arithlog grid (Chart 2) lie approximately in two straight lines. The most probable constants for the first degree polynomial equations of their logarithms are calculated by the method of least squares. The linear equations of the logarithms are transformed into exponential equations of the rates (Table I). The fitted straight lines are plotted (Chart 2).

In a definitive revision of the Du Bois standards Boothby and Sandiford (11) give for each sex self-consistent series of means for halved age groups (Table II). Between the ages of 5 and 19, and 22 and 77, for males, and 5 and 17, and 18 and 77, for females, the yearly first order numerical differences as well as the first order logarithmic differences are approximately equal. But in order to obtain equations congruent with a rational mechanism of variation the logarithm of the basal metabolic rate per unit surface area is expressed as a linear function of age and the basal metabolic rate per unit surface

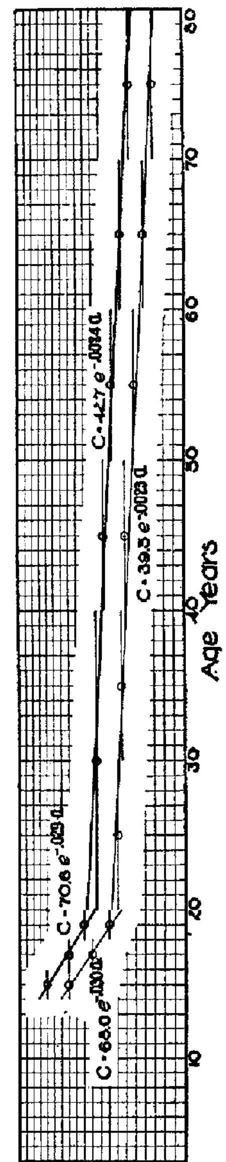

CHART 2. Basal metabolic rate (Aub-Du Bois).

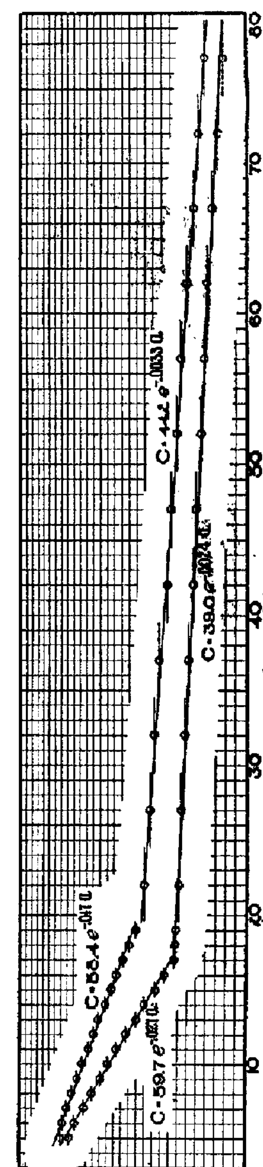

CHART 3. Basal metabolic rate (Boothby and Sandiford).

TABLE II

Calories per Square Meter of Body Surface per Hour (*Boothby and Sandiford*)

Sex	Age group*	Mid-value	Rate	Linear equation	Exponential equation
	yrs.	*yrs.*	*cal.*	$y = A + Bx$	$y = Ae^{Bx}$
Male	5	5	(53.0)	$\log C = 1.7667 - 0.00736\,a$	$C = 58.44\,e^{-0.0170\,a}$
	6	6	52.7		
	7	7	52.0		
	8	8	51.2		
	9	9	50.4		
	10	10	49.5		
	11	11	48.6		
	12	12	47.8		
	13	13	47.1		
	14	14	46.2		
	15	15	45.3		
	16	16	44.7		
	17	17	43.7		
	18	18	42.9		
	19	19	42.1		
	20–24	22	41.0	$\log C = 1.6450 - 0.00144\,a$	$C = 44.16\,e^{-0.0033\,a}$
	25–29	27	40.3		
	30–34	32	39.8		
	35–39	37	39.2		
	40–44	42	38.3		
	45–49	47	37.8		
	50–54	52	37.2		
	55–59	57	36.6		
	60–64	62	36.0		
	65–69	67	35.3		
	70–74	72	(34.8)		
	75–79	77	(34.2)		
Female	5	5	(51.6)	$\log C = 1.7759 - 0.01178\,a$	$C = 59.68\,e^{-0.0271\,a}$
	6	6	50.7		
	7	7	49.3		
	8	8	48.1		
	9	9	46.9		
	10	10	45.8		
	11	11	44.6		
	12	12	43.4		
	13	13	42.0		
	14	14	41.0		
	15	15	39.6		
	16	16	38.5		
	17	17	37.4		

TABLE II—*Concluded*

Sex	Age group*	Mid-value	Rate	Linear equation	Exponential equation
	yrs.	*yrs.*	*cal.*	$y = A + Bx$	$y = Ae^{Bx}$
Female—*Continued*	18	18	37.3	$\log C = 1.5909 - 0.00103\ a$	$C = 38.98\ e^{-0.0024\ a}$
	19	19	37.2		
	20–24	22	36.9		
	25–29	27	36.6		
	30–34	32	36.2		
	35–39	37	35.8		
	40–44	42	35.3		
	45–49	47	35.0		
	50–54	52	34.5		
	55–59	57	34.1		
	60–64	62	33.8		
	65–69	67	33.4		
	70–74	72	(32.8)		
	75–79	77	(32.3)		

* Class limits 7th month of preceding and 6th month of concluding year inclusive.

area as an exponential function of age. The formulas are calculated (Table II) and their graphs plotted (Chart 3). For children of each sex the residuals sensibly vary with age as though a subordinate factor which arose and subsided during this period were neglected. The residuals for adults are negligible.

The fitted straight lines for the provisional Du Bois standards may be compared with those for the Boothby and Sandiford revision. The discontinuities for males and females respectively occur at 19.1 and 19.0 years in the Du Bois standards and at 20.6 and 17.2 years in the Boothby and Sandiford revision. The fitted straight line for the Du Bois standards for younger males intersects that for the Boothby and Sandiford revision at 16.1 years and diverges from it from 2.5 per cent to −3.4 per cent between their common lower and upper extremes. The line for the Du Bois standards for adult males intersects that for the Boothby and Sandiford revision at 37.3 years and diverges −1.6 per cent to 4.5 per cent. The line for the Du Bois standards is for younger females from 8.2 per cent to 6.6 per cent, and for adult females 1.5 per cent to 1.8 per cent above that for the Boothby and Sandiford revision. The differences are varied and considerable.

The exponential formulas (Table II) define the gradients plotted by the Boothby and Sandiford standards. The rates graduated for continuous variation may be read off their graphs (Chart 4). Four-place logarithms for them may be computed from the linear formulas (Table II). Regular computations involving them can be made on alignments charts (12) or four variable straight line diagrams with logarithmic rectangular coordinates (13) so modified as to convert the serial variable into a continuous variable (14).

The not comparable, but self-consistent, determinations of minimum heat production per square meter of body surface per 24 hours for the basal conditions of infancy and childhood, of Benedict and Talbot (15), plot an age gradient for each sex from the end of the first week to

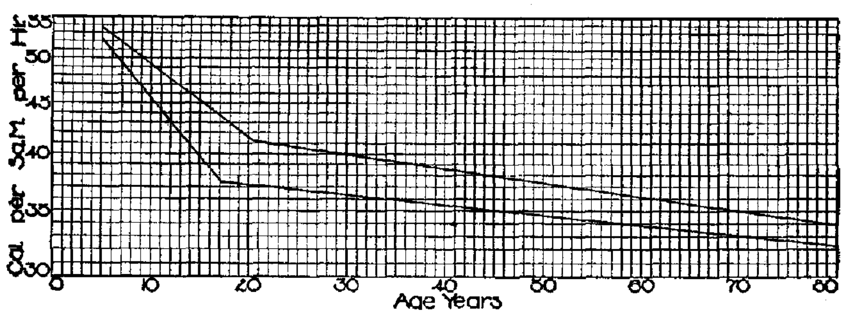

CHART 4. Basal metabolic rate.

the onset of the Boothby and Sandiford standards. The function represented by the smoothed curves of the medians (Charts 5, 6) is expressible as the difference of two negative exponential terms. The rising and falling age gradient of a physiological activity is the resultant of decrease and diminishing increase simultaneously proceeding exponentially (16, 17). On this ground Brody (17) deduced the equation

$$y = 56.7\, e^{-0.024t} - 32.0\, e^{-1.224\, t}$$

in which y = calories per square meter per hour, e = the natural logarithmic base, and t = years birth age, for the tentative Meeh formula gradient for males (7). Exponential equations for the decrease of the basal metabolic rate with age are derived for the parts of the curves where increase has become negligible and their graphs projected to their origins. The residuals for decrease counteracted by increase are plotted with their signs changed and exponential equations for the negative of the increase of the basal metabolic rate

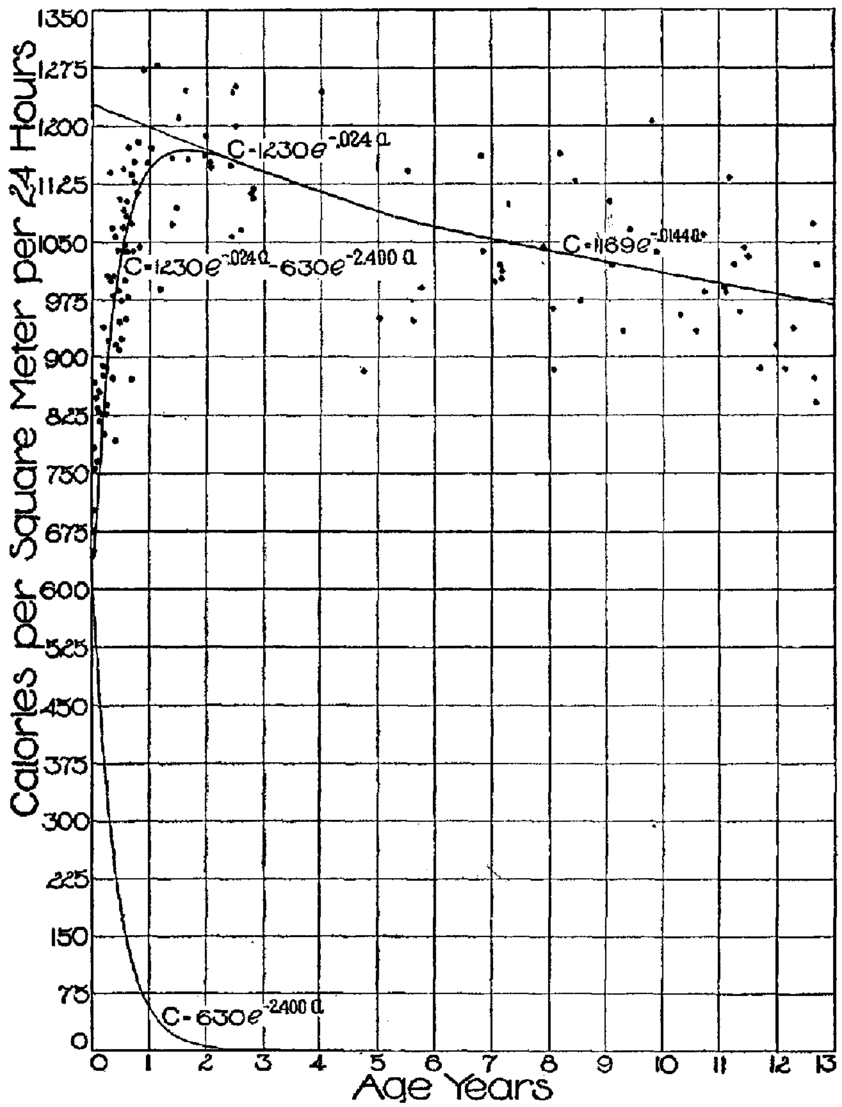

CHART 5. Basal metabolic rate of boys (scatter diagram after Benedict and Talbot).

with age derived for them. The gradient is defined by the term for decrease minus the term for negative increase (Charts 5, 6).

Thus the basal metabolic gradient is proximately defined throughout

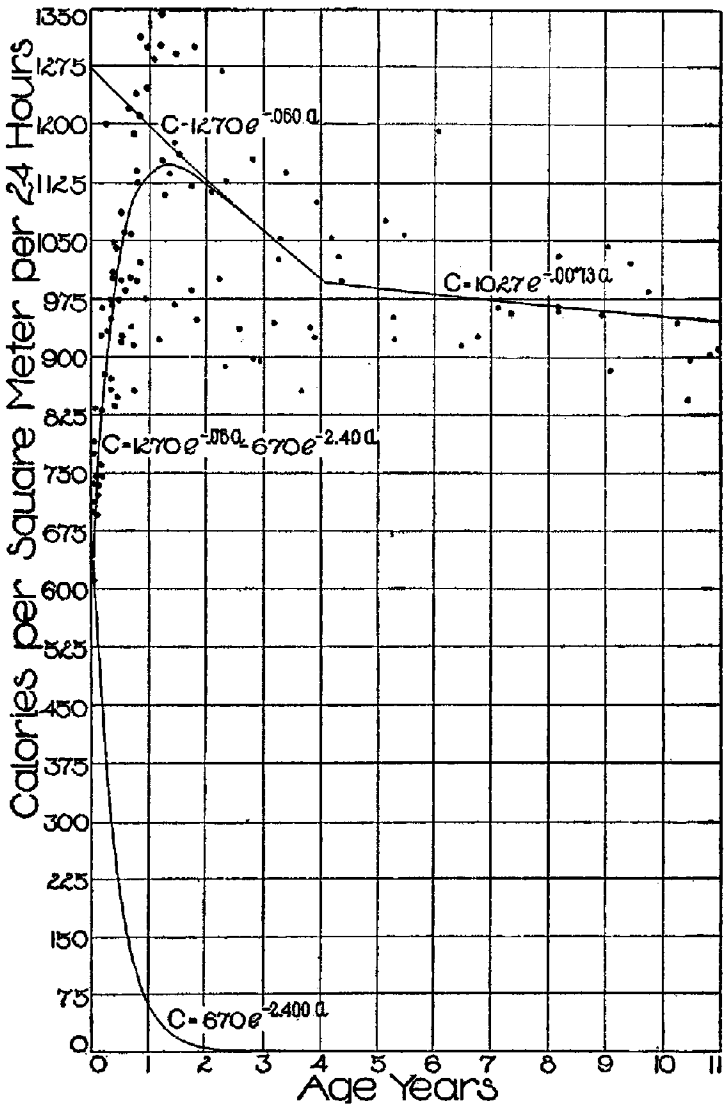

CHART 6. Basal metabolic rate of girls (scatter diagram after Benedict and Talbot).

post-natal life. The first day of extra-uterine readjustment and stabilization establishes basal metabolism at the level it maintains to the end of the first week (18). From this point an acceleration which is the same for both sexes begins, culminating in an inflection which is earlier, lower, and more acute for females than for males, followed by three successive retardations the first two of which are shorter and more rapid for females than for males. The rise and fall in the basal metabolic gradient is the resultant of concomitant decrease and increase. Basal metabolism decreases in each sex at three successive constant percentage rates which diminish in lessening degrees with abrupt transitions at the terminations of lengthening intervals of time. This decline in basal metabolism is opposed by an increase the negative of which diminishes at a constant percentage rate which is the same for both sexes.

BIBLIOGRAPHY

1. Meeh, K., *Ztschr. f. Biol.*, 1879, **15**, 425.
2. Coleman, W., and Du Bois, E. F., *Arch. Int. Med.*, 1914, **14**, 168.
3. Gephart, F. C., and Du Bois, E. F., *Arch. Int. Med.*, 1915, **15**, 835.
4. Lissauer, W., *Jahrb. f. Kinderh.*, 1902, **58**, 392.
5. Du Bois, D., and Du Bois, E. F., *Arch. Int. Med.*, 1915, **15**, 868.
6. Du Bois, D., and Du Bois, E. F., *Arch. Int. Med.*, 1916, **17**, 863.
7. Du Bois, E. F., *Arch. Int. Med.*, 1916, **17**, 887.
8. Gephart, F. C., and Du Bois, E. F., *Arch. Int. Med.*, 1916, **17**, 902.
9. Aub, J. C., and Du Bois, E. F., *Arch. Int. Med.*, 1917, **19**, 823.
10. Harris, J. A., and Benedict, F. G., *Carnegie Inst. Washington, Pub. 279*, 1919, 114–116.
11. Boothby, W. M., and Sandiford, I., *Abs. Com. xiiith Internat. Physiol. Cong., Amer. J. Physiol.*, 1929, **90**, 290.
12. Boothby, W. M., and Sandiford, R. B., *Boston Med. and Surg. J.*, 1921, **185**, 337.
13. Bruen, C., *J. Biol. Chem.*, 1930, **85**, 607.
14. Bruen, C., *New England J. Med.*, 1930, **202**, 531.
15. Benedict, F. G., and Talbot, F. B., *Carnegie Inst. Washington, Pub. 302*, 1921, 173–176.
16. Brody, S., Ragsdale, A. C., and Turner, C., *J. General Physiol.*, 1923, **6**, 31.
17. Brody, S., *J. General Physiol.*, 1923–24, **6**, 245.
18. Benedict, F. G., and Talbot, F. B., *Carnegie Inst. Washington, Pub. 233*, 1915, 102–106.