Heavy Fermions in the Standard Sequential Scheme

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Some of the qualitatively new features associated with the possible existence of heavier Fermions in the standard sequential scheme are discussed. As for the heavy Fermions with masses less than the W-boson mass $M_W$, the weak decay of constituent quarks in the lowest $^1S_0$ state of the heavy quarkonium is expected to become appreciable for $M_q \lesssim 20$ GeV and its implications are studied. We then discuss the stability of the Higgs mechanism in the presence of superheavy Fermions with masses greater than $M_W$. The weak interaction mediated by the W-boson in general becomes strong for such heavy objects, but some of the models such as the vector-like scheme could lead to a dynamically stable limit for superheavy Fermions.

§ 1. Introduction

The discovery of the heavy lepton $^0 \tau$ and the heavy quark $^0$ associated with $\Upsilon (9.5)$ suggests the possible existence of further heavier Fermions. The success of the standard Weinberg-Salam model of weak and electromagnetic interactions with regard to the neutral current phenomena and $V-A$ structure of the $\tau$-current may indicate that the standard sequential scheme is still applicable to the weak phenomena of these heavier Fermions.

On the basis of a simple scaling argument, one expects several qualitatively new features for these heavy objects, if they exist, and we discuss some of these novel aspects in the present paper. Although the discussion of these heavy Fermions is necessarily speculative, those heavy objects nicely illustrate the possible new domains as well as limitations of the gauge theory of weak interactions.

§ 2. Heavy Fermions with masses less than the W-boson mass

We first illustrate the effects of the W-boson pole on the heavy Fermion by considering the weak decay of the doublet

$$\begin{pmatrix} L \\ \nu \end{pmatrix}$$

into

$$L \rightarrow \nu + \mu + \bar{\nu}_\mu.$$  \hspace{1cm} (2.2)

In Fig. 1, we show the decay rate for (2.2) with $m_\nu = 0$ and 5 GeV in the standard sequential scheme. The mass of the W-boson is taken as
$M_w = 75$ GeV, \hspace{1cm} (2.3)

corresponding to $\sin^2 \theta_w = 1/4$; this value of $\sin^2 \theta_w$ is adopted throughout the present paper.

The effects of the $W$-boson pole become appreciable for $M_L \gtrsim 30$ GeV and start to modify the decay rate significantly for $M_L \gtrsim 50$ GeV (e.g., $\sim 35\%$ increase in the decay rate for $M_L = 50$ GeV). The energy spectrum of $\mu$ in (2.2) is also affected by the $W$-pole, and the energy spectrum is shifted toward the high energy side. But the effects are, in general, not drastic except for an extremely heavy lepton with $M_L \gtrsim 60$ GeV. We illustrate this by showing the normalized energy spectrum in Fig. 2. Some of the relevant formulas for these calculations are relegated to the Appendix.

**Decay of the Heavy Quarkonium**

The "strong" weak interaction could appear in a more dramatic way for the
decay of the lowest lying $^3S_1$ state of the heavy quarkonium.\textsuperscript{6} We illustrate it by assuming that the mass of the $t$-quark in the doublet

\[
\begin{pmatrix}
t \\ b
\end{pmatrix}
\]  \hspace{1cm} (2.4)

is sufficiently heavy.

The strong and electromagnetic decays of the $^3S_1$ state of a heavy quarkonium $Q\bar{Q}$ have been discussed by various authors\textsuperscript{7,8} in the past. The electromagnetic one-photon decay of the $^3S_1$ state is assumed to be described by the Van Royen-Weisskopf formula\textsuperscript{9}:

\[
\Gamma(Q\bar{Q} \to \gamma \to l\bar{l}) = 16\pi\alpha_e^2 c_q^2 |\psi(0)|^2/M(v)^2
\]  \hspace{1cm} (2.5)

with $c_q$ being the charge of the heavy quark $Q$ in units of the electron charge, $M(v)$ the mass of the $^3S_1$ state, and $\psi(0)$ the value of the $Q\bar{Q}$ wave function at the origin. The strong decay in the quarkonium picture\textsuperscript{6} is given by

\[
\Gamma(Q\bar{Q} \to 3g \to \text{hadrons}) \approx \frac{160}{81} (\pi^2 - 9) \alpha_s^3 |\psi(0)|^2/M(v)^2.
\]  \hspace{1cm} (2.6)

Although the parameter $\alpha_s$ in (2.6) has a definite meaning as the running coupling constant of QCD in the simplest quarkonium scheme, we take it as a convenient parametrization in the present paper.

The mass dependence of (2.5) is controlled by a scaling law of the non-relativistic Schrödinger equation\textsuperscript{8}. For the popular QCD plus confinement potential\textsuperscript{6,7}

\[
V(r) = \frac{r}{a^2} - \frac{4}{3} \alpha_s \frac{1}{r},
\]  \hspace{1cm} (2.7)

(2.5) is expected to behave as

\[
\Gamma(Q\bar{Q} \to \gamma \to l\bar{l}) \sim 1/m_q
\]  \hspace{1cm} (2.8)

up to some value of $m_q$, and then it would gradually turn into the asymptotic form

\[
\Gamma(Q\bar{Q} \to \gamma \to l\bar{l}) \sim m_q
\]  \hspace{1cm} (2.9)

for the sufficiently heavy quarkonium (presumably for $m_q > 50$ GeV). For the logarithmic potential

\[
V(r) = c \ln r
\]  \hspace{1cm} (2.10)

suggested by Quigg and Rosner\textsuperscript{8}, the width (2.5) follows

\[
\Gamma(Q\bar{Q} \to \gamma \to l\bar{l}) \sim 1/m_q^{1/2}.
\]  \hspace{1cm} (2.11)

The logarithmic potential may interpolate the two extreme cases (2.8) and (2.9), and it seems to be more favorable for the description of $\Gamma$, which we as-
sume to be the lowest $^3S_1$ state of $b\bar{b}$. We here adopt the scaling law (2.11) to estimate $\Gamma(Q\bar{Q} \rightarrow \gamma \rightarrow ll)$. Consequently

$$\Gamma(Q\bar{Q} \rightarrow \gamma \rightarrow ll) \approx 4.8(e_q/(2/3))^3 \left(\frac{m_c}{m_q}\right)^{1/2}. \quad (2.12)$$

This estimate gives (by taking $m_c\approx 1.5$ GeV and $e_q=2/3$)

$$\Gamma(Q\bar{Q} \rightarrow \gamma \rightarrow ll) \approx 1.3 \text{ keV}, \quad m_q=20 \text{ GeV}, \quad 1.1 \text{ keV}, \quad 30 \text{ GeV}, \quad (2.13)$$

$$0.83 \text{ keV}, \quad 50 \text{ GeV}.$$

The strong decay in (2.6) is estimated:

$$\frac{\Gamma(Q\bar{Q} \rightarrow Z \rightarrow \text{hadrons})}{\Gamma(Q\bar{Q} \rightarrow \gamma \rightarrow ll)} \approx 10\left(\frac{\pi^2-9}{81}\right)^3 \frac{M(v)^4}{81\pi a^2 e_q^{1/2}} = 10 \text{ for } J/\psi(31) \quad (2.14)$$

and this ratio for $e_q=2/3$ is expected not to vary drastically from the value of $J/\psi(31)$ when $M_q$ becomes larger.

As for the decay through $Z$-boson annihilation, we have

$$\frac{\Gamma(Q\bar{Q} \rightarrow Z \rightarrow e\bar{e} + \nu\bar{\nu})}{\Gamma(Q\bar{Q} \rightarrow \gamma \rightarrow ll)} \approx 3\left(\frac{1}{3e_q}\right)^2 \frac{M(v)^4}{(M(v)^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}, \quad (2.15)$$

$$\frac{\Gamma(Q\bar{Q} \rightarrow Z \rightarrow u\bar{u} + d\bar{d})}{\Gamma(Q\bar{Q} \rightarrow \gamma \rightarrow ll)} \approx \left(\frac{23}{3}\right)\left(\frac{1}{3e_q}\right)^2 \frac{M(v)^4}{(M(v)^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

in analogy with (2.5) for $\sin^2\theta_W = 1/4$; for $e_q=2/3$, (2.15) give a small fraction for almost all values of $M(v)$, except for the $Q\bar{Q}$ state very close to the $Z$-boson pole.

Consequently, the major "conventional" decay modes of the $^3S_1$ state of the heavy quarkonium may be estimated by (2.13) and (2.14).

On the other hand, Fig. 1 gives for the doublet (2.4) (with the replacement $L \rightarrow t$ and $\nu \rightarrow b$ with $m_b=5$ GeV),

$$\Gamma(t \rightarrow b\bar{t}\nu_b) \approx 0.047 \text{ keV \quad for } m_t=20 \text{ GeV},$$

$$0.48 \text{ keV}, \quad 30 \text{ GeV}, \quad (2.16)$$

$$8.8 \text{ keV}, \quad 50 \text{ GeV},$$

by assuming that the possible mixing of $t$ and $b$ with light quarks is small. This gives, if $m_t=30$ GeV, the weak decay rate of the constituent quark in the quarkonium $t\bar{t}$.

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* In addition to the estimate in (2.17), we also have the contribution corresponding to the process $t\bar{t} \rightarrow b\bar{b}$ via $t$-channel $W$ exchange. This is estimated at $\Gamma(t\bar{t} \rightarrow b\bar{b})/\Gamma(t\bar{t} \rightarrow \gamma ll) \sim 2[1/2e_q]^4 \times [M(v)^4/[M_{W^\pm} + M(v)/4]^3]$ which is of the order of unity for $m_t=30$ GeV.
\[ \Gamma (t \bar{t} \rightarrow \text{weak decay}) \approx 2 \times 9 \Gamma (t \rightarrow b \bar{\nu} \nu) \]
\[ \approx 8.6 \text{ keV} \quad (2.17) \]

which is comparable with \( \sim 18 \text{ keV} \), the sum of the conventional decay modes (2.13) and (2.14). Here we have counted 3-lepton and 4-light quark degrees of freedom in the final state. We see that the weak decay of the constituent quark becomes appreciable for \( m_q \gtrsim 20 \text{ GeV} \), and for \( m_q = 50 \text{ GeV} \) the weak decay could be the dominant decay mode if the naive estimate of (2.13) and (2.14) is valid for such a heavy object.

This exercise suggests that the heavy quarkonium above the energy regions of PETRA and PEP accelerators may lead to a very interesting interplay of strong, weak and electromagnetic interactions. The chiral structure of the heavy quark will be determined via the weak decays of \( Q \bar{q} \) as well as \( Q \bar{Q} \). Once the chiral structure is fixed, the energy spectrum of \( \mu \) from the quarkonium decay (2.16) may be used for the purpose of the weak "diagnosis" of the heavy quarkonium; the detailed investigation of the Fermi motion and other bound state effects of \( Q \) will provide valuable information on the wave function and also on the potential for the quarkonium system. As an example, the decay \( \mu \) spectrum from a heavy free quark \( t \) at rest is shown in Fig. 3 for \( m_b = 5 \text{ GeV} \) (the \( V - A \) and \( V + A \) cases of quarks are obtained from the \( V + A \) and \( V - A \) of heavy leptons, respectively, as \( m_t > m_b \) for quarks but \( m_L > m_\nu \) for leptons). The Fermi motion and other bound state effects of \( Q \) modify this "bare" spectrum. The weak decay of \( (t \bar{t}) \), if \( m_t \) is really heavy, will also provide a good source of \( (tb) \), which is otherwise not easily produced.

Besides these practical implications, we find it interesting that the very effective OZI rule\(^{10}\) for the strong interaction, if it is operating at higher energies,
can potentially prohibit the “strong” decay of the heavy quarkonium to the level below the weak decay of constituent quarks. This property, i.e., a composite object decays weakly rather than strongly, may be useful when one considers the dynamical origin of flavors.

§ 3. Superheavy Fermions with \( m_F > M_w \)

The massive gauge bosons coupled to non-conserved currents such as in the standard sequential scheme in general lead to a strong interaction for superheavy Fermions with \( m_F > M_w \). This can be illustrated\(^{121}\) by considering the semi-weak decay of the heavy lepton in the doublet (2·1) by assuming \( m_L > M_w \) and \( m_L > m_r \). We plot the decay width

\[
\Gamma (L \rightarrow \nu + W) \approx \left( \frac{g^2}{64\pi} \right) \frac{(m_L^2 - M_w^2)}{m_L^3} \left[ 2 + \left( \frac{m_L}{M_w} \right)^2 \right] \tag{3.1}
\]

in Fig. 4. The decay width of a 1 TeV heavy lepton is about 300 GeV! This strong coupling for the semi-weak process can be easily seen by using the so-called 't Hooft-Feynman gauge for the process (3·1). As is shown in Fig. 5, the unphysical scalar \( S \) (the Nambu-Goldstone boson) has a strong coupling to the superheavy Fermion with a coupling constant \( \sim g (m_F/m_w) \). We note that this property holds independently of the detailed representation contents of the Higgs fields.

When the Higgs system is relatively simple such as in the standard sequential scheme,\(^3\) this strong coupling effect of the unphysical scalar is also reflected in the coupling of physical Higgs scalars via the effective potential. For the purpose of illustration, we recall the one-loop effective potential evaluated by Weinberg\(^{120}\)

\[
V(\varphi) = -\frac{3}{2} M^2 \varphi^2 + f \varphi^4
+ (64\pi^2)^{-1} \text{ Tr } \left( 3 \mu_1^4 \ln \frac{\mu_1}{M_1} + M_4^4 \ln M_4^2 - 4 M_4^4 \ln m_{\varphi}^2 \right), \tag{3.2}
\]

Fig. 4. The semi-weak decay rate of the superheavy Fermion in Eq. (3·1).
\( M_w = 75 \text{ GeV.} \)

Fig. 5. Feynman diagrams for the process in Eq. (3·1) in the 't Hooft-Feynman gauge. The field \( W \) corresponds to the transverse component of \( W. \)
where $\mu$, $M$ and $m$ are, respectively, the zeroth-order vector, scalar and spinor mass matrices for a scalar field vacuum expectation value $\phi$. Although this one-loop effective potential is not quite sufficient to discuss the strong coupling limit we are interested in, it may represent qualitative features correctly. The stability of this effective potential, if literally taken, indicates

$$m_H \gtrsim 2m_F \gg M_W$$

for the standard scheme in the presence of superheavy Fermions; the superheavy Fermions thus provide a driving force to make the Higgs scalar superheavy. As can be easily confirmed by looking at the scalar potential in the standard scheme, the self-interaction of the Higgs scalar becomes superheavy (more generally, it starts to saturate the tree unitarity bound for $m_H \gtrsim 1 \text{ TeV}$). The superheavy Higgs scalar predominantly decays into heavy objects such as weak bosons and superheavy Fermions.

If the heavier Fermions in the standard sequential scheme should continue to multiply, it may be a strong indication of the possible existence of a strongly interacting world of heavy Fermions and heavy Higgs scalars.

Veltman's Constraint

Veltman pointed out some time ago that the low energy weak phenomena place an interesting constraint on the mass spectrum of superheavy Fermions. He observed that the mass terms of superheavy Fermions, if $m_L \gg m_F$, for example, act as a breaking term of the global $SU(2)$ symmetry in the Weinberg-Salam model (i.e., the degeneracy of $W^\pm$ and $W^0$ in the absence of the electromagnetic field). By computing one-loop renormalization effects in the standard scheme, Veltman found the deviation of the parameter $\rho$ from unity

$$\rho = M_W^2/M_Z^2 \cos^2 \theta_W \approx 1 + m_F^2 G_F/8m^2 \sqrt{2}$$

$$\approx 1 + 9.1 \times 10^{-5} m_F^2/m^2$$

(3.4)

with $m_F$ being the proton mass. When $m_F$ is also large, $\rho$ is approximately given by the expression (3.4) with $m_L \rightarrow m_F - m_F$.

On the other hand the recent analysis of weak neutral currents indicates

$$\rho = 0.98 \pm 0.05.$$ (3.5)

Consequently, the superheavy Fermion doublets in the standard scheme, if existed, start to become somewhat degenerate in mass for $m_F \gtrsim 600$ GeV, and they will become more stable than the naive estimate (3.1). If this degeneracy is realized, the unphysical scalar $S$ (the Nambu-Goldstone boson) couples predominantly to the pseudo-scalar current of superheavy weak doublets, and one finds an interesting correspondence with the low energy phenomenology:
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\begin{align}
\begin{pmatrix}
L \\
\nu
\end{pmatrix} & \leftrightarrow \begin{pmatrix}
\nu_l \\
\nu_e
\end{pmatrix}, \\
\tilde{W}, \tilde{Z} & \rightarrow \begin{pmatrix} W, Z, \\ S, \pi, \phi, \sigma \end{pmatrix} \\
\text{Nambu’s superheavy string}^{18} & \text{dual string,}
\end{align}

where \( \tilde{W} \) stands for the transverse component of \( W \) in Fig. 5 and \( \phi \) the physical Higgs scalar. This scaling by a factor of \( \sim 10^3 \) was also noted by Nambu\(^{18} \) in connection with his superheavy 10 TeV string in the Weinberg-Salam model. The possible existence of superheavy Fermions will complete this interesting observation.

Other Implications

In the presence of superheavy Fermions, the GIM-like selection rules are drastically modified. In fact, a simple mixing of Fermions can no more suppress the flavor changing neutral currents, and this effect may also appear among the next generation of heavy Fermions if their mixing with superheavy Fermions is substantial. This is illustrated by considering the model

\begin{equation}
\begin{pmatrix} t \\ b \cos \beta + B \sin \beta \end{pmatrix}_L \begin{pmatrix} T \\ -b \sin \beta + B \cos \beta \end{pmatrix}_L \tag{3.7}
\end{equation}

with \( T \) being a superheavy Fermion. The induced neutral current for

\begin{equation}
(B\bar{b}) \rightarrow (\bar{B}b) \tag{3.8}
\end{equation}

is dominated by an unphysical scalar exchange in Fig. 6. The effective interaction is given by

\begin{equation}
\mathcal{L}_{\text{eff}} \approx \left( \frac{\sin^2 \beta \cos^2 \beta}{\sin^2 \theta_W} \right) \left( \frac{1}{16\pi^2} \right) \left( \frac{M_T}{M_W} \right)^2 \frac{G_F}{2\sqrt{2}} b_T \left( 1 - \gamma_s \right) \bar{b} \bar{T}^a \left( 1 - \gamma_s \right) B. \tag{3.9}
\end{equation}

If the mixing \( \sin \beta \) is not small and \( M_T \) is sufficiently large, (3.9) gives a contribution substantially larger than the “natural” order \( G_F \alpha \). The flavor changing

\begin{equation}
\left( b \cos \beta + B \sin \beta \right)_L \begin{pmatrix} t \\ -b \sin \beta + B \cos \beta \end{pmatrix}_L \begin{pmatrix} \tilde{W} \\ \tilde{Z} \end{pmatrix}_L
\end{equation}

Fig. 6. The lowest order diagram for the flavor changing neutral current in Eq. (3.8).
neutral currents may exhibit rather complicated selection rules for the next genera­
tion of heavy quarks.

Incidentally, the success of the GIM mechanism\textsuperscript{19} for $\Delta S\neq 0$ neutral currents shows that the mixing of the first four light quarks with superheavy quarks, if any, is very small. We also note that (3·9) is obtained by a naive unitary gauge calculation of a box diagram with a cutoff $A$ at $A\approx M_T$ instead of $A=\nu f w$.

§ 4. Discussion

One may feel more or less confident about the general properties of the next generation of heavy Fermions and the pole structure of $W$ and $Z$ bosons discussed in § 2, as the extrapolation from the known physics is relatively small.

On the other hand, the superheavy Fermions and high energy weak interactions in § 3 belong to a completely unknown domain of physics. It should be noted that the weak interaction of superheavy Fermions in some of the gauge models could remain weak instead of becoming strong as in the standard scheme. For example, the vector-like scheme\textsuperscript{20} which is interesting in connection with the dynamical origin of parity violation, could lead to a world of superheavy Fermions with weak interactions of the order of $\alpha$, the fine structure constant. This is because the vector-like scheme in general leads to a left-right symmetric world at high energies. This property when combined with Veltman’s constraint of degenerate Fermions\textsuperscript{10} may imply the nearly conserved weak currents for superheavy Fermions, which do not show any “pathological” behavior of the Higgs mechanism. The price we have to pay for this alternative is that the masses of Fermions may become completely arbitrary parameters independent of the Higgs mechanism (as a result, the effective potential such as (3·3) is not useful for this case). In comparison, the Fermion masses in the Weinberg-Salam model\textsuperscript{11} are, although arbitrary, still generated “dynamically” by the Higgs mechanism.

In conclusion, the next generation of heavier Fermions and their chiral structure will provide crucial information on the possible form of weak interactions in 1 TeV regions.

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Appendix

The decay rate for the process (2·2) can be written as

$$\Gamma = \frac{G^2}{4M} \frac{1}{(2\pi)^3} \int_{m^2}^{M^2} f(u) \frac{u-m^2}{u} \left( \frac{M^2-u}{M^2} \right) du , \quad (A·1)$$
where $M$ and $m$ stand for the masses of $L$ and $\nu$, respectively, and $G$ the Fermi constant. The explicit form of $f(u)$ for $V-A$ and $V+A$ currents will be given later; the total decay rate (A·1) is independent of chiral structure.

The $\mu$ energy spectrum at the rest frame of $L$ is given by

$$\frac{d\Gamma}{dE} = \frac{G^2}{M} \frac{1}{(2\pi)^3} f(u) \left( \frac{u-m^2}{u} \right) E \quad (A·2)$$

for

$$0 \leq E \leq (M^2-m^2)/2M \quad (A·3)$$

and $u=M^2-2ME$.

The $\mu$ energy spectrum from the moving $L$ is given by

$$\frac{d\Gamma}{dE} = \frac{G^2}{M} \frac{1}{(2\pi)^3} \frac{1}{4M^2} \int_{u_{\text{max}}}^{u_{\text{min}}} f(u) \left( \frac{u-m^2}{u} \right) du \quad (A·4)$$

with $\beta$ and $\gamma$ being the Lorentz factors of the moving $L$, and

$$u_{\text{max}} = M^2 - 2ME \gamma \gamma (1-\beta). \quad (A·5)$$

The lower bound $u_{\text{min}}$ is given by

$$u_{\text{min}} = m^2 \quad (A·6)$$

for

$$(M^2-m^2) \gamma (1-\beta)/2M \leq E \leq (M^2-m^2) \gamma (1+\beta)/2M \quad (A·7)$$

and

$$u_{\text{min}} = M^2 - 2ME \gamma (1+\beta) \quad (A·8)$$

for

$$0 \leq E \leq (M^2-m^2) \gamma (1-\beta)/2M. \quad (A·9)$$

The high energy part (A·7) has the scaling property.20

The auxiliary function $f(u)$ is defined by

$$f(u) = 2M_w^4 \left\{ -1 - \frac{2M_w^2-(M^2+m^2-2u)}{(M^2-u)(u-m^2)/u} \ln \left[ 1 - \frac{(M^2-u)(u-m^2)}{uM_w^2} \right] \right.\left. \right\}$$

$$- \frac{(M_w^2+u-m^2)(M_w^2+u-M^2)}{M_w^2[M_w^2-(M^2-u)(u-m^2)/u]} \quad (A·10)$$

for $V-A$ heavy lepton coupling, and

$$f(u) = \frac{2M_w^2(M^2-u)(u-m^2)}{[M_w^2-(M^2-u)(u-m^2)/u]} \quad (A·11)$$

for $V+A$ coupling, respectively, with $M_w$ being the $W$-boson mass.
References

   See also A. D. Linde, JETP Letters 23 (1976), 64.
20) Some of the early attempts toward the direction of vector-like currents are:

Note added in proof: After submitting the present paper, the author learned that C. E. Carlson
Conference] briefly commented on the weak decay of the heavy quarkonium. The GIM mechanism
in the presence of superheavy Fermions was also considered by E. Poggio and H. Schnitzer, Phys.