Bose Quarks and Non-Leptonic Weak Interactions of Charmed Mesons

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It is shown that recent experiments on charmed meson decays are still consistent with 20-dominance, an extension of SU(3)-8-dominance which follows naturally from assuming Bose statistics for quarks, contrasting with a seemingly general belief against it.

§ 1.

It is well known that the phenomenological rule, SU(3) 8-dominance ([|ΔI| = 1/2 rule], for non-leptonic weak decays of strange particles naturally follows\(^9\) from assuming Bose statistics for quarks appearing in the usual current × current form of weak interaction \(H_w\). Recently it seems to be a general belief that an extension of this, SU(4) 20-dominance (or rather 20-enhancement)\(^{21-23}\) may not be valid for decays of charmed particles,\(^9\) since (A1) a forbidden process by this rule \(D^+\to \overline{K}^0\pi^+\) has been observed\(^9\) as frequently as an allowed process \(D^+\to K^-\pi^+\) and (A2) a large non-leptonic decay rate and a small semi-leptonic branching ratio, which are expected from the 20-enhancement, seems to be rejected by the recent experimental value\(^6\) \(BR(D\to e^+\nu+\cdots) = 9.8 \pm 1.4\%\), which is not small.

The purpose of the present paper is to point out that the 20-dominance (not 20-enhancement) is still consistent with present experiments, contrasting with the general belief. The essential point of our reasoning is as follows: As for (A1), the 20 (SU(4))-dominance (strictly, 6 or 6* (SU(3))-dominance for \(\Delta C = \Delta S = \pm 1\) process\(^7\)) forbids the process \(D^+\to \overline{K}^0\pi^+\) only in the SU(3)-symmetric limit with \(m_\pi = m_\pi\), and there is a good possibility that this process with physical masses \(m_\pi \gg m_\pi\) gets decay strength of nearly allowed order. The situation is quite similar to for the process \(K^0\to 2\pi\), which is forbidden in the symmetric limit but survives with proper strength in the actual case. As for (A2) it may suffice to note that our rule coming from assuming Bose statistics for quarks is not 20-enhancement but 20-dominance, or rather 84-suppression out of usually expected 20- and 84-spurions for the weak interaction.

\(^{*)}\) This work was distributed as the preprint NUP-A-78-11 July 1978.
§ 2.

For completeness first we recapitulate the now-well-known situations concerning relations between quark-statistics and transformation property of the weak Hamiltonian. It is a direct extension to the \( SU(4) \) case of the mechanism\(^*\) first pointed out in the \( SU(3) \) case by us and the other authors. We start from the assumption that the Hamiltonian is given by the usual charged current \( \times \) current form with \( V-A \) type.\(^**\) Then the Hamiltonian has the symmetry property as:

For Bose quark without color we obtain

\[
H_{B_{\infty}}[\bar{q}^a q_a][\bar{q}^b q_b] = \frac{1}{2} \{[\bar{q}^a q_a] [\bar{q}^b q_b] - [\bar{q}^b q_b] [\bar{q}^a q_a]\} \approx T_{[c,d]}^{[a,b]},
\]

where \( \bar{q}^a \) denotes quark with \( a \)-flavor etc., \( [\bar{q} q] = (\bar{q} \Gamma(q) \ q) \), \( [a, b] \) denotes an antisymmetrization over the suffices \( a \) and \( b \), and in deriving (1) the Bose commutation relation for the \( q \)'s and a well-known Fierz-transformation property for Dirac matrices \( \Gamma_i \cdot \Gamma_i \) with \( \Gamma_i = \gamma_\alpha (1 + \gamma_\beta) \) has been used. For Fermi quark without color, similarly we obtain

\[
H_{F_{\infty}}[\bar{q}^a q_a][\bar{q}^b q_b] = \frac{1}{2} \{[\bar{q}^a q_a] [\bar{q}^b q_b] + [\bar{q}^b q_b] [\bar{q}^a q_a]\} \approx T_{[c,d]}^{[a,b]},
\]

where \( \{a, b\} \) denotes a symmetrization. For the usual colored Fermi quark, similarly

\[
H_{col,F_{\infty}}[\bar{q}^a q_a^k][\bar{q}^b q_b^l] = \frac{1}{2} \left[ [\bar{q}^a q_a^k] [\bar{q}^b q_b^l] + [\bar{q}^b q_b^l] [\bar{q}^a q_a^k] \right] \approx T_{[c,d]}^{[a,b]} + T_{[c,d]}^{[a,b]},
\]

where the suffices \( k, l \) denote the color. Thus in the standard colored Fermi quark model, quarks in the Hamiltonian behave, aside from the color freedom, in two ways as "Bosons" and "Fermions".

The tensors in flavor space which appeared in (1), (2) and (3) are decomposed into irreducible parts as

\[
T_{[c,d]}^{[a,b]}: \ 36 = 1 + 15 + 20: \ SU(4),
\]

\[
(9 = 1 + 8: \ SU(3)) \quad (4)
\]

\[
T_{[c,d]}^{[a,b]}: \ 100 = 1 + 15 + 84: \ SU(4).
\]

\[
(36 = 1 + 8 + 27: \ SU(3)) \quad (5)
\]

For the usual charged current in the quartet scheme a \( 15 \)-plet does not exist.\(^21,29\) Thus we see clearly the situation that in the case of Bose (Fermi) quark \( 20 \)-dominance (\( 84 \)-dominance) is valid, while in the usual case of colored Fermi quark the Hamiltonian behaves as a sum of \( 20 \)-plet and \( 84 \)-plet. The \( SU(3) \) content

\* For neutral current the following simple discussion suffers from complex modifications due to the trace part of current.

\** The following transformation properties of \( H \) are valid with \( V+A \) type, too.

\( ^* \)\(^* \)
of the parts with $JC=\Delta S=\pm 1$, which are Cabibbo-enhanced, of $SU(4)\cdot 20$ ($SU(4)\cdot 84$) is an $SU(3)\cdot 6$ or $6^*$ ($SU(3)\cdot 15$ or $15^*$).

§ 3.

Several years ago we proposed a general kinematical framework without the color freedom for unified description of hadrons, the ur-citon scheme, in which Bose-like behavior of quarks is included consistently with the due properties of hadron as a whole, such as Fermi-statistics for baryons and a right anti-particle-conjugation parity for mesons. For these years we have applied it to various problems with successful results. Our main results concerning weak interactions, which are relevant to our present problem, are as follows:

i) In the ur-citon scheme non-leptonic weak interactions among hadrons are unifiedly determined through the usual current-current form of interactions among ur-citons, our Bose quarks. As for decays of hyperons all the $S$-wave and $P$-wave amplitudes are given in terms of only one parameter and was shown to reproduce qualitative experimental behavior except for the $P$-wave amplitude of $\Xi^{-}$.

ii) We proposed a new form of universality of weak interactions, on the basis of the ur-citon scheme, which enables us, by following simple rules, to derive an effective Hamiltonian for arbitrary weak process without any parameter except for a universal weak coupling strength, the Cabibbo angle, and a constant dimensional parameter $\varepsilon$ inherent to our scheme. By its actual application to the decay widths of typical semi-leptonic and non-leptonic processes, it was shown to be valid satisfactorily and the parameter $\varepsilon$ was determined as

$$\varepsilon = 150\sim 300 \text{ MeV}. \quad (6)$$

§ 4.

Now by applying the same framework to our relevant problem we shall examine quantitatively our advocacy for 20-dominance in the case of non-leptonic decays of charmed mesons. Starting from the ur-citon interaction of the form given in (1), effective interactions for the process of a $p\bar{s}$-meson into two $p\bar{s}$-mesons are given unifiedly as

$$H_{\phi\phi}^{[a,b]} = g_{\phi\phi,cd} \sum_{k_{1,2}} H_{1}(k_0, k_1, k_2); \quad (7)$$

$$H_{1} = \frac{1}{\sqrt{2}} \left( \frac{k_1 k_2}{\mu_1 \mu_2} + \frac{k_2 k_3}{\mu_2 \mu_0} \right) \bar{P}_{\phi}^{[e]}(k_1) \bar{P}_{\phi}^{[d]}(k_2), \quad H_{1} = H_{1} (1\leftrightarrow 2),$$

* The relation for $S$-wave decays, $S(\Delta S)$: $S(\Xi^{-})$: $S(\Xi^{-}) = 1: -\sqrt{3}: 2$, which was recently noted to be derivable from 20-dominance, had been derived as one of our various relations. It is notable that this relation comes from merely the symmetry property (1) in $SU(3)$ space and has no such direct physical connection with the $SU(4)$ as usually discussed.
where $P$ and $\bar{P}$ represent initial and final $\rho$-$\pi$-mesons respectively, $k_i, \mu_i$ do a four-momentum and the mass of respective mesons, and $g_{abc,cd}$ is a coupling parameter. It is notable that in the symmetric limit with $\mu_0 = \mu_5 = \mu_6$ all momentum factors in (7) vanish, and we confirm a general theorem which forbids three-$\rho$-meson vertices in the symmetric limit under restriction of CP-conservation. With our relevant process, $H_{[3/2]}^3$ out of (7) is concerned, and $H_1, H_2, H_3$ and $H_4$ ($H_1$ and $H_2$) contribute to the process $D^0 \rightarrow K^- \pi^+$ ($D^+ \rightarrow \bar{K}^0 \pi^+$). From this we get the expression for the decay-width ratio

$$ R = \frac{\Gamma(D^+ \rightarrow \bar{K}^0 \pi^+)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = \frac{(a-b)^2}{(a+c)^2}, \tag{8} $$

where $a$, $b$ and $c$ are momentum factors of $H_1, H_2$ and $H_3$ respectively. It is easy to see that in the SU(3) limit with $m_5 = m_6$, $\Gamma(D^+ \rightarrow \bar{K}^0 \pi^+) = 0$, as $a = b$ in this limit. In Table I we have given numerical values of the ratio $R$ in the three cases; where masses of final mesons are taken to be (a) physical ($m_5 = 140$ MeV, $m_6 = 498$ MeV), (b) SU(3)-symmetric ($m_5 = m_6 = 500$ MeV) and (c) SU(6)-symmetric** ($m_5 = m_6 = 782$ MeV) for calculating $\Gamma(D^0 \rightarrow K^- \pi^+)$; while using always physical masses for $\Gamma(D^0 \rightarrow \bar{K}^0 \pi^+)$. From this it seems (although nothing definite may be said since the value is, to our regret, strongly dependent upon the choices of final $\pi$ and $k$ masses) actually to be a good chance that the forbidden process (in the symmetric limit) $D^+ \rightarrow \bar{K}^0 \pi^+$ gets decay strength of nearly allowed order in the SU(3)-symmetry breaking case, as was conjectured at the beginning.

<table>
<thead>
<tr>
<th>mass of $\pi, K$</th>
<th>physical</th>
<th>SU(3)-symmetric</th>
<th>SU(6)-symmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(D^+ \rightarrow \bar{K}^0 \pi^+)$</td>
<td>0.03</td>
<td>0.30</td>
<td>1.96</td>
</tr>
<tr>
<td>$\Gamma(D^0 \rightarrow K^- \pi^+)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

§ 5.

Here it may be interesting to treat an old typical forbidden process (in the SU(3) symmetric limit) $K^0 \rightarrow 2\pi$ similarly. With the process $K^0(\bar{K}^0) \rightarrow \pi^- \pi^+$, $H_{[3/2]}^3$ + $H_{[1/2]}^3$ out of (7) is concerned, and contributions come from $H_4$ and $H_1$ ($H_1$ and $H_3$).

** Note that the properties of $H_8$ (with Bose quark) under CP are the same as the usual one $H_{col.}$.  
** Note In our previous work our universality was shown to be valid with this SU(6)-symmetric masses.
Following the procedure given in a previous work, we obtain for the decay rate

\[ \Gamma(K_s^0 \to \pi^+\pi^-) = \frac{A^2}{16\pi m_k} \left(1 - \frac{4m_\pi^2}{m_k^2}\right)^{1/2}, \]

where

\[ A = G\varepsilon^{5/2} \cos \theta_e \sin \theta_e m_\pi \sqrt{m_k} \left(2 - \frac{m_k}{m_\pi} \right)^2. \]

Adopting the usual values as \( G = 1.026 \times 10^{-2} \) and \( \cos \theta_e = 0.978 \) in this formula and comparing it with the experimental value \( \Gamma_{\exp}(K_s^0 \to \pi^+\pi^-) = 5.061 \times 10^{-12} \) MeV, we get the value of \( \varepsilon \)

\[ \varepsilon(K_s^0 \to \pi^+\pi^-) = 2.4 \times 10^5 \text{ MeV}, \]

which is in the range of (6) obtained previously from analyses on allowed processes.

§ 6.

By applying our new form of universality for weak interactions also to charmed mesons we can derive effective interactions for any non-leptonic or semi-leptonic decay of charmed mesons. As an example we give the result of calculation of decay for a typical semi-leptonic process \( \Gamma(D^0 \to K^-e^+\nu) \) in unit of \( \Gamma(D^0 \to K^-\pi^-) \) in Table II. In our scheme is valid \( \Gamma(D^- \to \bar{K}^0 e^+\nu) = \Gamma(D^0 \to K^-e^+\nu) \). From these we may say that the semi-leptonic branching ratio is certainly not small in our scheme in conformity with experiments.

<table>
<thead>
<tr>
<th>( \varepsilon ) (MeV)</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma(D^0 \to K^-e^+\nu) )</td>
<td>0.50</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>( \Gamma(D^0 \to K^-\pi^-) )</td>
<td></td>
<td></td>
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</table>

Finally we should like to note that to check the validity of the 20 (SU(4))-dominance for non-leptonic weak interactions or the Bose-like behavior of quarks in the current \( \times \) current form of interactions, is most crucial to examine the part of interactions, \( H \sim \cos \theta \sin \theta \{ [u \bar{d}] [\bar{d} \bar{c}] + [\bar{c} \bar{u}] [\bar{d} \bar{u}] \} \), where \( |D| = 1/2 \) rule (out of \( |D| = 1/2 \) and 3/2 generally expected) is valid in the case of Bose quark. We expect in this case

\[ \frac{\Gamma(D^\pm \to \pi^\pm\pi^0)}{\Gamma(D^0 \to \pi^+\pi^-)} \approx \frac{\Gamma(K_s^0 \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^-)} \approx 1/500. \]

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References

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   T. F. Walsh, invited talk at 1977 Int. Symp. on Lepton and Photon Interactions at high energies (DESY).
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   See also, D. Ito, Soryushiron Kenkyu (Kyoto) 43 (1971), 12.
    See also, T. Hayashi et al., Prog. Theor. Phys. Suppl. Extra Number (1968), 381.

Note added: Recently we received preprints by M. Katuya and Y. Koide, SH-78-05 and -07, where they also insist the validity of 20-dominance from the different viewpoint. It is interesting that their result $\Gamma(D^*\to K^n\pi^+) / \Gamma(D\to K^-\pi^+)$ is similar to ours in the case of physical $\pi$- and $K$-masses in Table I. However, quite recently we were kindly informed by Professor Iizuka that experimentally the ratio is $\Gamma(D^*\to K^n\pi^+) / \Gamma(D\to K^-\pi^+) = 0.70 \pm 0.23$ (H. Schopper, Preprint DESY 77/79), which is close to our value in the case of $SU(3)$-symmetric masses.